de Vaucouleurs law

RECHERCHES SUR LES NÉBULEUSES EXTRAGALACTIQUES

I. SUR LA TECHNIQUE DE L'ANALYSE MICROPHOTOMÉTRIQUE DES NÉBULEUSES BRILLANTES

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SOMMAIRE. — *Étude, préliminaire à un programme d'observations étendu, des problèmes techniques de l'analyse microphotométrique des nébuleuses brillantes.*

Dans la première partie sont étudiées les erreurs systématiques et accidentelles provenant de l'atmosphère, de l'instrument, de la plaque photographique et de son traitement, de l'étalonnage photométrique, de l'enregistrement microphotométrique, du dépouillement des enregistrements et de la réduction des mesures ; description des techniques opératoires permettant d'éliminer ou de réduire ces erreurs.

Dans la deuxième partie sont rapportées et discutées les mesures de 3 nébuleuses elliptiques : NGC 3 115 (E7), 3 379 (E0), 4 649 (E2) et d'une spirale : NGC 4 594 (Sa), effectuées sur des clichés obtenus en lumière bleue ($\lambda \sim 0{,}43\mu$) au foyer Cassegrain de 12,25 m du télescope de 0,80 m de l'Observatoire de Haute Provence.

Les profils photométriques, $\log B = f(r)$, *observés suivant les 2 axes principaux de chaque nébuleuse, permettent de calculer la* luminosité intégrée $L_r = \int_0^r B(r)dS$ *et, par une extrapolation généralement inférieure à 20 %, la* luminosité totale $L_T = \int_0^\infty B(r)dS$.

La courbe de luminosité intégrée relative $k(r) = L_r/L_T$ *permet alors de donner une définition, ayant un sens physique précis, des* dimensions des nébuleuses *: j'appelle dimensions* effectives *celles de l'isophote, de demi-axes* a_e *et* b_e, *englobant 50 % de la luminosité totale* ($k = 0{,}50$) *et dimensions* totales *celles* a_T, b_T *correspondant à* $k = 1$, *déterminées par extrapolation.*

Les brillances réduites $\mathcal{B} = B/B_e$ *des nébuleuses elliptiques sont bien représentées en fonction des* dimensions réduites *correspondantes* $\alpha = a/a_e$ *par la formule :*

$$\log \mathcal{B} = -3{,}25\,(\alpha^{1/4} - 1),$$

dont le domaine de validité paraît très étendu.

General form of the de Vaucouleurs law:

$$\lg \frac{I(\alpha)}{I_e} = -\beta(\alpha^{1/4} - 1),$$

where $\alpha = r/r_e$ and β – coefficient ($\beta > 0$).

Let isophotes are homocentric ellipses with ellipticity $\epsilon = 1 - b/a$. Then the total luminosity is

$$L_T = 2\pi I_e r_e^2 (1-\epsilon) \int_0^{+\infty} \exp[-\nu(\alpha^{1/4} - 1)]d\alpha = 8!\pi \frac{e^\nu}{\nu^8}(1-\epsilon) I_e r_e^2,$$

where $\nu = \beta \ln 10$.

Growth curve of the galaxy is

$$k(\alpha) = \frac{L(\leq \alpha)}{L_T} = 1 - \exp(-\nu\alpha^{1/4}) \cdot \Sigma_{n=0}^{n=7} \frac{\nu^n \alpha^{n/4}}{n!}.$$

$r = r_e$ ($\alpha = 1$) $\rightarrow k(1) = 1/2$. Thus, $\nu = 7.66925$ and $\beta = \nu/\ln 10 = 3.33071$.

Therefore, a final form of the de Vaucouleurs law is

$$\lg \frac{I(r)}{I_e} = -3.33071 \left[\left(\frac{r}{r_e} \right)^{1/4} - 1 \right],$$

or, in units of $^m/\Box''$,

$$\mu(r) = \mu_e + 8.32678[(r/r_e)^{1/4} - 1].$$

Total (asymptotic) luminosity:

$$L_T = 7.21457\pi I_e r_e^2 (1-\epsilon) = 22.66523 I_e r_e^2 b/a.$$

Absolute magnitude:

$$M_{Vauc} = \mu_e - 5\lg r_e - 2.5\lg(1-\epsilon) - 39.961,$$

where the effective radius r_e is in kpc.

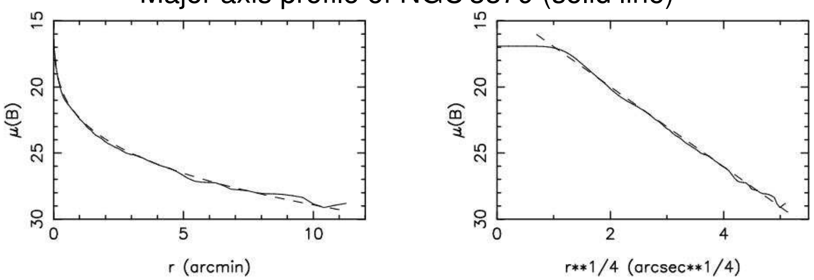
Mean surface brightness within r_e is

$$\langle I \rangle_e = 3.61 I_e \quad \text{or} \quad \langle \mu \rangle_e = \mu_e - 1.39.$$

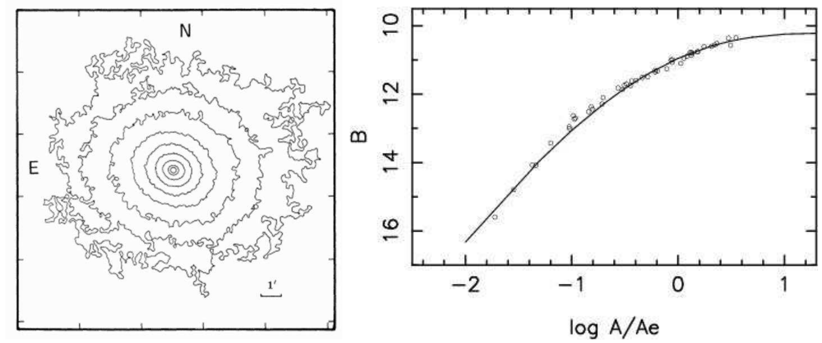
Total luminosity, expressed through $\langle I \rangle_e$, is $L_T = 2\pi \langle I \rangle_e r_e^2\, b/a$.
Central surface brightness of the de Vaucouleurs model is

$$I_0^b = 10^{3.33} I_e = 2140 I_e.$$

Major axis profile of NGC 3379 (solid line)



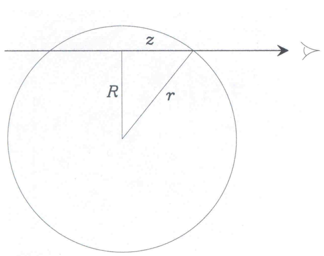
Dashed line – approximation with $\mu_e(B) = 22.24$ and $r_e = 56.''8$ (2.7 kpc).
De Vaucouleurs law fits the s.b. profile within $\Delta\mu \sim 10^m$ with error $\pm 0.^m08$.



Growth curve for NGC 3379. Open circles – aperture measurements, solid line – approximation by standard curve $k(\alpha)$ for the de Vaucouleurs law with $B_T = 10.20$.

Deprojection of the de Vaucouleurs law

So far we have discussed observed surface brightness profile $I(\mathbf{R})$, that is 3D distribution of light (stars) projected onto the plane of the sky. The question is whether we can, from this measured quantity, infer the real 3D distribution of light, $j(\mathbf{r})$ in a galaxy. If $I(\mathbf{R})$ is circularly symmetric, we can assume that $j(\mathbf{r})$ will be spherically symmetric, and from the following figure it is apparent that:



$$I(R) = \int_{-\infty}^{\infty} dz\, j(r) = 2\int_R^{\infty} \frac{j(r)rdr}{\sqrt{r^2 - R^2}}.$$

This is an Abel integral equation for j as a function of I, and its solution is:

$$j(r) = -\frac{1}{\pi}\int_r^{\infty} \frac{dI}{dR}\frac{dR}{\sqrt{R^2 - r^2}}.$$

3D density distribution: assuming $\mathrm{M}/L = const \rightarrow \rho(r)$.

Example of analytical approximation (Mellier & Mathez 1987):

$$\rho(r) = \rho_0\, r^{-0.855}\exp(-r^{1/4}).$$

Therefore,

$$\mathrm{M}(\leq r) = \mathrm{M}_0\, \gamma(8.58, r^{1/4}),$$

where $\mathrm{M}_0 = 16\pi\rho_0(r_e/\nu^4)^3$ and $\mathrm{M}_{\mathrm{tot}} = 1.65 \cdot 10^4\, \mathrm{M}_0$.

Sersic law

Sersic profile is a generalization of the de Vaucouleurs profile:

$$I(r) = I_0\, e^{-\nu_n \alpha^{1/n}},$$

where I_0 – central surface brightness, $\alpha = r/r_e$, $n > 0$ and a constant ν_n is chosen so that half the total luminosity predicted by the law comes from $r \leq r_e$.
Also, this profile can be written as

$$\frac{I(r)}{I_e} = \exp\left[-\nu_n \left(\left[\frac{r}{r_e}\right]^{1/n} - 1\right)\right],$$

where $I_e = I_0\, e^{-\nu_n}$.
When $n = 4$ ν_4=7.66925 the Sersic law transforms to the de Vaucouleurs law.

In units of $^m/\square''$:

$$\mu(r) = \mu_0 + \frac{2.5\nu_n}{\ln 10}\left(\frac{r}{r_e}\right)^{1/n} \qquad (*)$$

If $n = 4$ (*) $\rightarrow \mu(r) = \mu_e + 8.32678[(r/r_e)^{1/4} - 1]$.

Effective surface brightness for the Sersic law ($\mu_e = \mu(r_e)$) is $\mu_e = \mu_0 + 2.5\nu_n/\ln 10$.

Luminosity within r:

$$L(\leq r) = \frac{2\pi n}{\nu_n^{2n}}\, \gamma(2n, \nu_n \alpha^{1/n})\, I_0 r_e^2,$$

where $\gamma(\eta, x) = \int_0^x e^{-t} t^{\eta-1} \mathrm{d}t$ – incomplete gamma function.
Total (asymptotic) luminosity:

$$L_T = \frac{2\pi n}{\nu_n^{2n}} \Gamma(2n)\, I_0 r_e^2,$$

where $\Gamma(\eta) = \gamma(\eta, \infty)$ – gamma function.

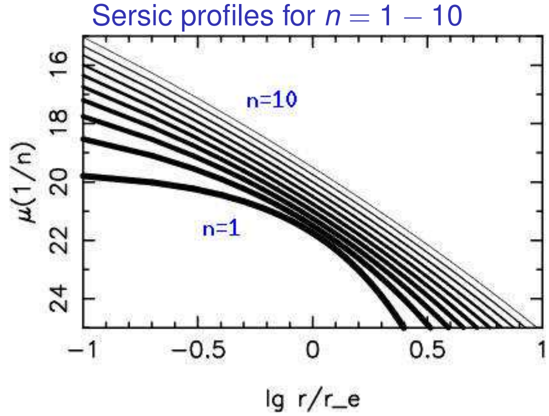
Growth curve: $k(\alpha) = \frac{L(\leq\alpha)}{L_T} = \frac{\gamma(2n, \nu_n \alpha^{1/n})}{\Gamma(2n)}$.

Table: The values of ν_n (Ciotti & Bertin 1999)

n	ν_n	n	ν_n
1	1.67834699	6	11.6683632
2	3.67206075	7	13.6681146
3	5.67016119	8	15.6679295
4	7.66924944	9	17.6677864
5	9.66871461	10	19.6676724

Analytical approximation (Ciotti & Bertin 1999):
$\nu_n = 2n - \frac{1}{3} + \frac{4}{405n} + \frac{46}{25515n^2} + O(n^{-3})$.

Sersic profiles for $n = 1 - 10$



Luminous ellipticals, cD galaxies – $n \sim 4$ or even ≥ 4, dwarf E – $n \sim 1$.

Hubble-Reynolds formula

The first model used to describe the surface brightness profiles of elliptical galaxies (Reynolds 1913):

$$I(r) = \frac{4I(r_0)}{(1+r/r_0)^2}, \qquad (I(r) \propto r^{-2} \;\; at \;\; r >> r_0)$$

where r_0 – characteristic radius of the distribution, $I(r_0)$ – surface brightness at r_0 from the nucleus.
Total luminosity of circular galaxy within $\leq r$ is

$$L(\leq r) = 8\pi I(r_0) r_0^2 \int_0^{\alpha} \frac{x dx}{(1+x)^2} = 8\pi I(r_0) r_0^2 \left[\ln(1+\alpha) - \frac{\alpha}{1+\alpha}\right],$$

where $\alpha = r/r_0$. As one can see, $r \to \infty \quad L(\leq r) \to \infty$.

Modified Hubble law

(or modified Hubble-Reinolds law)

$$I(r) = \frac{I_0}{1+(r/r_0)^2}, \qquad (I(r) \propto r^{-2} \;\; at \;\; r >> r_0)$$

and

$$L(\leq r) = \pi r_0^2 \ln[1 + (r/r_0)^2].$$

Again, $r \to \infty \quad L(\leq r) \to \infty$.
Modified Hubble law corresponds to a simple analytical form for 3D distribution:

$$j(r) = \frac{j_0}{[1+(r/r_0)^2]^{3/2}},$$

where $j_0 = I_0/2r_0$.

Hubble-Oemler law

$$I(r) = \frac{I_0}{(1+r/r_0)^2} e^{-r^2/r_t^2}$$

For $r < r_t$ the surface brightness changes as $I(r) \propto r^{-2}$.

For $r > r_t$ the surface brightness profile decays very quickly and predicts a finite total luminosity.

In the limit $r_t \to \infty$ this one reduces to the Hubble-Reynolds law.

King formula

$$I(r) = K[(1+[r/r_c]^2)^{-1/2} - (1+[r_t/r]^2)^{-1/2}]^2,$$

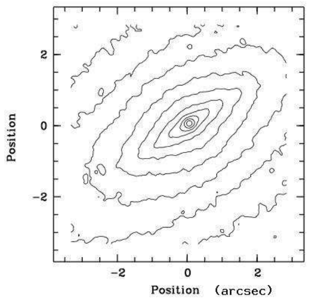
where r_c is the "core" radius ($\frac{I(r=0)}{I(r=r_c)} = 2$), r_t is the "tidal" radius and K – the scale factor.

This formula gives a very good representation of star counts in tidally-limited globular clusters and low-density spheroidal galaxies.

Jaffe law, Hernquist law etc.

Centers of early-type galaxies

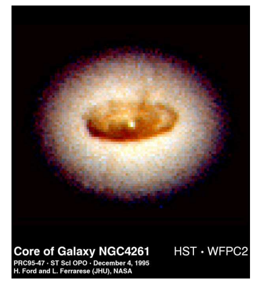
The HST observations of early-type galaxies reveal that the central parts have surface brightness distibutions that are different from the extrapolation of traditional fitting formulae derived from ground-based observations.



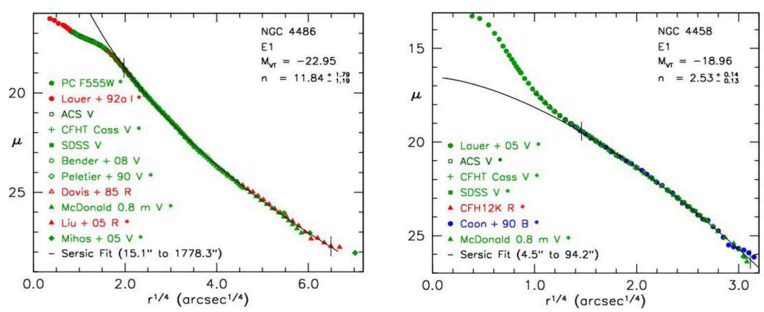
NGC 3115 (HST, F555W filter)
$\Delta\mu = 0.^m44/\square''$

The surface brightness profiles generally consist of two distinct regions:
a steep power-law regime – $I(r) \propto r^{-\beta}$ – at large radius, and
a shallower power law – $I(r) \propto r^{-\gamma}$ – at small radius.

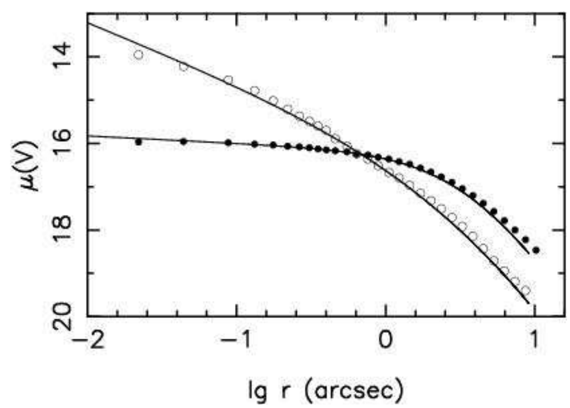
Classification:
$\gamma < 0.3$ – “core” galaxies (shallow inner slope),
$\gamma > 0.5$ – power-law galaxies.



$\gamma \approx 0$ – core



Major-axis brightness profiles of Virgo ellipticals (V passband) – Kormendy (2009).



Surface brightness profiles for NGC 596 (open circles) – power-law nucleus, and NGC 1399 (solid circles) – galaxy with a core. Solid lines represent Nuker law fits (see further).

Nuker law

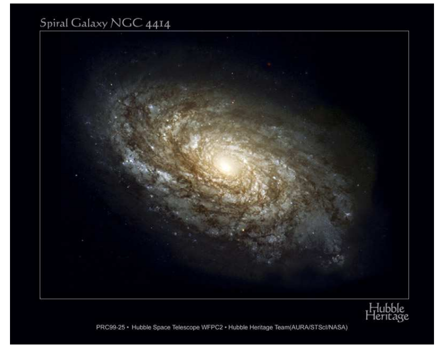
To parametrize the HST brightness profiles, Lauer et al. (1995) introduced general empirical double power law (the "Nuker" law):

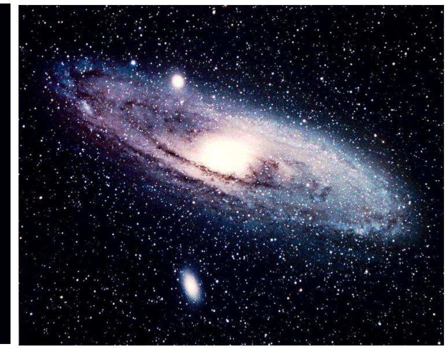
$$I(r) = 2^{\frac{\beta-\gamma}{\alpha}} I_b \left(\frac{r_b}{r}\right)^{\gamma} \left[1 + \left(\frac{r}{r_b}\right)^{\alpha}\right]^{\frac{\gamma-\beta}{\alpha}},$$

where α, β, γ, I_b, r_b – parameters.
The break radius, r_b, is the radius at which the steep outer profile, $I(r) \propto r^{-\beta}$, "breaks" to become the inner shallow profile, $I(r) \propto r^{-\gamma}$, and $I_b = I(r_b)$.

The Nuker law contains many simpler fitting formulae as special cases:
The Hubble-Reynolds law corresponds to $\alpha = 1$, $\beta = 2$, $\gamma = 0$;
The modified Hubble law – $\alpha = 2$, $\beta = 2$, $\gamma = 0$.

Disk galaxies





Radial distribution

Disks of spiral galaxies are known to show profiles described well by the "exponential law" (Patterson 1940, de Vaucouleurs 1959, Freeman 1970):

$$I(r) = I_0 \, e^{-r/h}$$

or

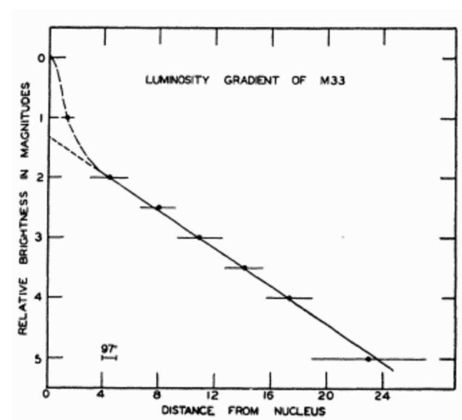
$$\mu(r) = \mu_0 + 1.0857 \, r/h,$$

where h – exponential scale length, I_0 or μ_0 – central surface brightness of the disk.

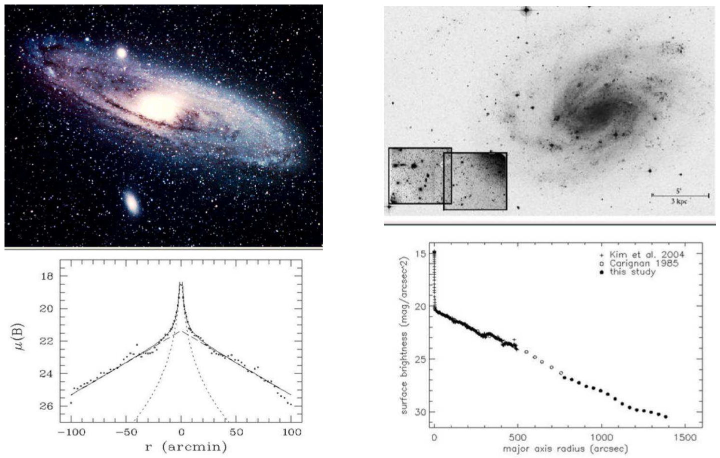
$\mu - r$ plane: exponential disk looks like straight line.

Examples

Shirley Patterson, Harvard College Observatory Bulletin No. 914, pp.9-10, 1940

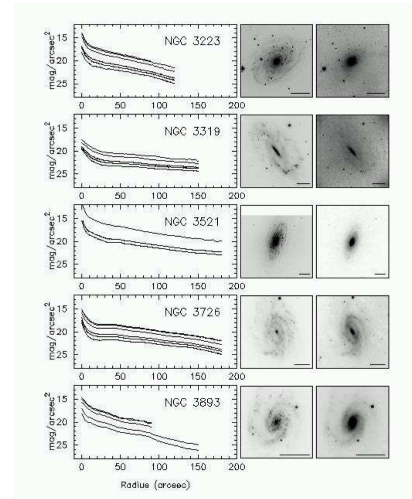


Examples



NGC 300: exponential disk is traced up to $10h$!

Examples



Kassin (2006)

Luminosity within r from the center

$$L(\leq r) = 2\pi I_0 h^2 [1 - (1 + r/h) e^{-r/h}],$$

total luminosity

$$L_T = 2\pi I_0 h^2.$$

Absolute luminosity of exponential disk

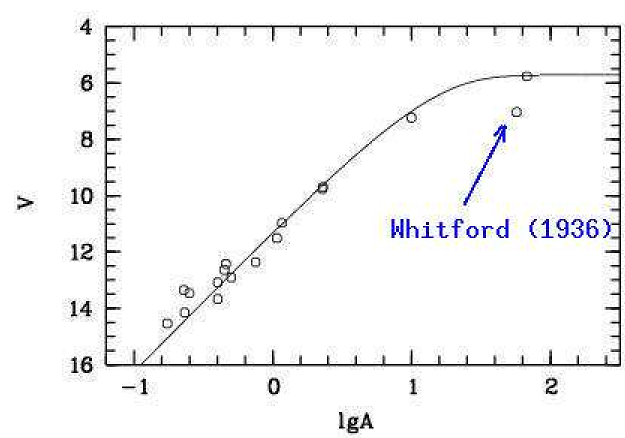
$$M_{exp} = \mu_0 - 5\lg h - 38.57,$$

where exponential scale length is in kpc.

Growth curve

$$k(\alpha) = \frac{L(\leq \alpha)}{L_T} = 1 - (1 + \alpha) e^{-\alpha},$$

$\alpha = r/h$.



Aperture photometry of M 33 (circles). Solid line is the growth curve for exponential disk with $h = 9'$ and $V_T = 5.72$.

Effective radius of exponential disk: $r_e = 1.67835\, h$,
effective surface brightness: $I_e = I_0 e^{-1.678} = 0.187\, I_0$ or

$$\mu_e = \mu_0 + 1.822.$$

In terms of effective parameters we can write total luminosity as $L_T = 3.80332\pi I_e r_e^2$.

Mean surface brightness within effective radius is
$\langle I \rangle_e = 0.355\, I_0$ or
$\langle \mu \rangle_e = \mu_0 + 1.124$.

Edge-on ($i = 90^{\mathrm{o}}$) transparent disk:

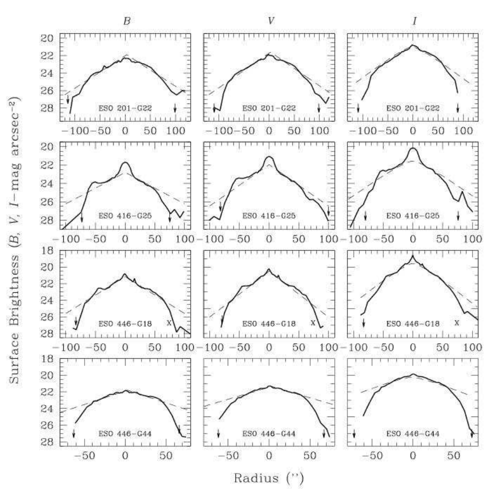
$$I(r) = I_0\, \frac{r}{h}\, \mathrm{K}_1\left(\frac{r}{h}\right),$$

where K_1 is the modified Bessel function.

$r/h << 1$: $\quad I(r) \approx I_0[1 + (r^2/2h^2)\ln(r/2h)]$
$r/h >> 1$: $\quad I(r) \approx I_0\sqrt{\pi r/2h}\, e^{-r/h}\left[1 + \frac{3}{8r/h}\right]$

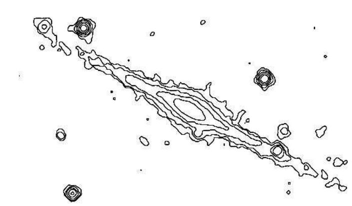
Real stellar disks are not infinite. Exponential distribution typically extends out to about 5 radial scale lengths, beyond which disks are often truncated.

Examples of truncated disks



de Grijs et al. (2001)

Vertical structure

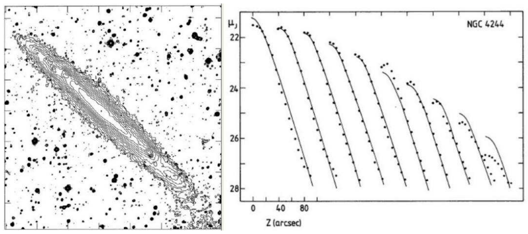


UGC 11859 (B-band)

Standard model to describe vertical surface brightness distribution in edge-on galaxies is isothermal self-gravitating sheet (e.g. van der Kruit & Searle 1981):

$$I(z) \;=\; I_0\ \mathrm{sech}^2(z/z_0),$$

where z_0 – vertical scale (scale height).



van der Kruit & Searle (1981)

In the framework of the model, vertical scale z_0 is connected with σ_z (the dispersion in the velocities in the z-direction) and with $\rho(r,z)$ (the space density of stars), $\Sigma(r)$ (the projected density of stars):

$$\sigma_z^2(r) = 2\pi G\rho(r,0)z_0^2 = \pi G\Sigma(r)z_0.$$

Some galaxies demonstrate vertical density profiles more sharply peaked near $z = 0$ than the $\mathrm{sech}^2(z/z_0)$ model. Such data can be modelled better by exponential law:

$$I(z) \;=\; I_0\, e^{-|z|/h_z},$$

where h_z – exponential scale height.

At $z/z_0 << 1$ $\quad \mathrm{sech}^2(z/z_0) = \exp(-z^2/z_0^2)$,
at $z/z_0 >> 1$ $\quad \mathrm{sech}^2(z/z_0) = 4\exp(-2\,z/z_0)$
and, therefore, $\mathrm{sech}^2(z/z_0)$ and exponential model give approximately the same distribution with $z_0 \;=\; 2\,h_z$.

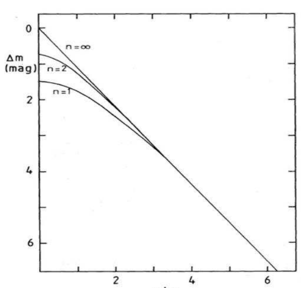
Vertical velocity dispersion of an exponential disk is
$\sigma_z^2(r) = 4\pi G h_z \Sigma(r)\left(1 - \frac{1}{2}e^{-|z|/h_z}\right)$.

van der Kruit (1988) proposed more general law

$$\rho(z) = 2^{-2/n}\,\rho_0\,\mathrm{sech}^{2/n}(nz/2z_0) \quad (n > 0).$$

The case $n = 1$ corresponds to the isothermal distribution
$\rho(z) = (\rho_0/4)\,\mathrm{sech}^2(z/z_0)$,

while the limiting case of $n = \infty$ is the exponential
$\rho(z) = \rho_0\, e^{-z/z_0}$.



The density distributions as a function of distance z from the symmetry plane for the three models. The parameters have been chosen to give the same density at large z. The vertical scale is in magnitudes

3D disks

3D structure of disks:

$$I(r,z) = I(0,0)\,e^{-r/h}\,\mathrm{sech}^2(z/z_0) \qquad (r \le r_{max})$$
$$I(r,z) \;=\; 0 \qquad (r > r_{max})$$

If $i = 0^{\mathrm{o}}$ (face-on disk)

$$I_0^{face-on} \;=\; I(0,0)\int_{-\infty}^{+\infty}\mathrm{sech}^2(z/z_0)dz \;=\; 2z_0 I(0,0).$$

For edge-on disk ($i = 90^{\mathrm{o}}$)

$$I_0^{edge-on} \;=\; 2hI(0,0).$$

Therefore,
$I_0^{edge-on} \;=\; I_0^{face-on}\,h/z_0$ or $\mu_0^{edge-on} = \mu_0^{face-on} - 2.5\,\mathrm{lg}\,\frac{h}{z_0}$.

For double exponential disk we have

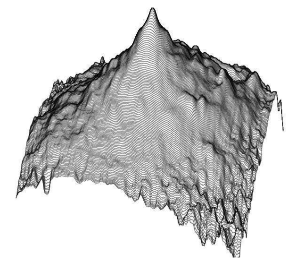
$$I(r,z) = I(0,0)\, e^{-r/h - |z|/h_z}$$

and $$\mu_0^{edge-on} = \mu_0^{face-on} - 2.5 \lg \frac{h}{h_z}.$$

Therefore, for **transparent** disks the observed values of $\mu_0^{edge-on}$ must be brighter than $\mu_0^{face-on}$.
$h/z_0 \approx 5$ for real galaxies $\rightarrow \Delta\mu \approx 1.^m5 - 2^m$.

Diameters of **transparent** edge-on disks must be larger than for face-on disks (measured within the same isophote). For instance, for typical disk with $\mu_0^{face-on} = 21.7$ (B filter) and $h/z_0 = 5$

$$\frac{D_{26}(i = 90^{\circ})}{D_{26}(i = 0^{\circ})} \approx 1.7.$$



NGC 5055 (g-band, mag.)

Real galaxies are multi-component systems:
– bulge (de Vaucouleurs or Sersic law),
– disk (exponential disk),
– bar (e.g., Freeman's bar:
$I_{bar}(x,y) = I_{0,bar}\sqrt{1 - (x/a_{bar})^2 - (y/b_{bar})^2}$ – flattened elliptical disk)

Also:
– lenses,
– inner and outer rings,
– spiral arms etc.

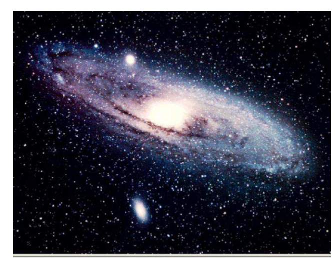
Simplest case: two-component galaxy, consisting of de Vaucouleurs bulge and exponential disk.
Total luminosity: $L_T = L_{bul} + L_{disk} = 2\pi(3.6073 I_e r_e^2 + I_0 h^2)$.
Bulge-to-disk ratio:

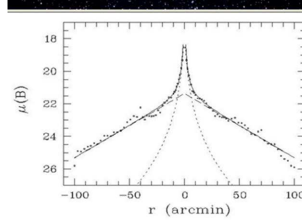
$$B/D = 3.6073\, \frac{I_e}{I_0} \left(\frac{r_e}{h}\right)^2.$$

Growth curve:

$$k(r) = \frac{B/D}{1 + B/D} k_{bul}(r) + \frac{1}{1 + B/D} k_{disk}(r).$$

Photometric decomposition of galaxies





Direct methods: analysis of 1D cuts or 2D images

Software:
GIM2D (Simard 1998),
GALFIT (Peng et al. 2002),
BUDDA (de Souza et al. 2004).

Other methods:
– Iterative decomposition method proposed by Kormendy (1977), in which one solves for the disk parameters in a region where disk light dominates, and likewise for the bulge parameters. At each iteration, the light from the component being kept fixed is subtracted from the total surface brightness profile before the other component is solved for. The process is iterated until convergence is achived.

– Kent (1986) presented a completely different approach: he made no assumption on the fitting laws for either component. He assumed that each one is characterized by elliptical isophotes of constant, and essentially different, flattenings. Then, an iterative process calculated the bulge and disk profiles. (Does not work for face-on galaxies – bulge and disk have roughly the same flattening.)

– Colorimetric decomposition (statistical). Let the color index of the disk is K_D, of the bulge – K_B, and of the whole galaxy is K_T. Then,

$$B/D = -\frac{1 - 10^{0.4(K_D - K_T)}}{1 - 10^{0.4(K_B - K_T)}}$$

Example: normal Sa spiral galaxy with
$B - V = K_T = +0.74$,
$B - V = K_D = +0.5$ (disk),
$B - V = K_B = +1.0$ (bulge).

Therefore, $B/D = 0.73$ (standard value for Sa galaxies is 0.68).