COHERENT OPPOSITION EFFECTS FOR SEMI-INFINITE DISCRETE RANDOM MEDIUM IN THE DOUBLE-SCATTERING APPROXIMATION

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Abstract - The rigorous equations of the theory of multiple scattering of light by a layer of disordered medium have been used in the double-scattering approximation for semi-infinite medium to determine the influence of the particle properties on the coherent opposition effects. The effects were found to be strongly dependent on concentration of scatterers in the medium. The polarization opposition effect is more sensitive to the properties of the scatterers than the photometric opposition effect. The interference of waves could result in the negative polarization at the backscattering direction as well as in the positive polarization.

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Proposed running head: Coherent backscattering by random media

1 Introduction

Coherent backscattering of light by disordered medium result in the well-known backscattering enhancement phenomenon which manifests itself as a narrow intensity peak centered at the exact backscattering direction [1,2]. In case of unpolarized incident wave this effect can be accompanied by the polarization opposition effect that may be shown as a narrow peak of negative polarization at a small phase angle [3,4] (the angle between the reflected beam and the backscattering direction). Both of these effects are typical for many atmosphereless cosmic bodies, laboratory samples of powder-like media and rough surfaces [1-6]. Therefore they can be an informative characteristics in various optical applications.

Theoretical description of coherent backscattering of light by disordered medium is too complicated even in the case of the Rayleigh scatterers. The exact vector theory of coherent backscattering for semi-infinite medium composed of non-absorbing point-like scatterers has been obtained only recently [7,8]. Numerical calculations of the polarization opposition effect for such medium are in good agreement with the laboratory data [9]. However the dependence of the opposition effects on the properties of the scatterers (size parameter, real and imaginary parts of refractive index) is studied insufficiently.

The equations for coherent and incoherent (diffuse) components of reflection matrix for a layer of disordered medium composed of chaotically oriented arbitrary scatterers have been obtained recently in Ref.10. These equations are too complicated for numerical calculations. We use these equations in double-scattering approximation to consider the influence of particle properties on the opposition polarization effects for semi-infinite medium.

2 Basic equations

In the case of beam normally incident on a rarefied, homogeneous and isotropic layer of discrete random medium the reflection matrix $S_{pn\mu\nu}$ was presented in Ref.10 as a sum of three matrices $S_{pn\mu\nu} = S^{(nc)}_{pn\mu\nu} + S^{(co)}_{pn\mu\nu} + S^{1}_{pn\mu\nu}$ $(p, n, \mu, \nu = \pm 1)$. Matrix $S^{(nc)}_{pn\mu\nu}$ corresponds to the diffuse part of scattered radiation and can be written as:

$$S^{(nc)}_{pn\mu\nu} = \frac{n_0}{k} \sum_L d^L_{M_0 N_0}(\vartheta) \int_0^{kZ_0} \exp(2z \frac{Im(\varepsilon)}{\cos\vartheta}) \alpha_L^{(z)(pn)(\mu\nu)} dz. \tag{1}$$

Here matrix $S^{(nc)}_{pn\mu\nu}$ is normalized to a unit of surface area, n_0 is the concentration of particles in the medium, $k = 2\pi/\lambda, \lambda$ is the wavelength of the incident radiation, $d^L_{M_0 N_0}(\vartheta)$ is the Wigner function [11], ϑ is the scattering angle (the angle between the incident beam and the scattering direction), Z_0 is the thickness of the medium layer, $M_0 = \nu - n$, $N_0 = \mu - p$, ε is the complex effective refractive index of the medium. Here and hereafter the CP-representation [12,13] (the basis of the circular polarization) is used.

The expansion coefficients $\alpha_L^{(z)(pn)(\mu\nu)}$ are determined from the system of equations [10]:

$$\alpha_L^{(z)(pn)(\mu\nu)} = \chi_L^{(pn)(\mu\nu)} \exp(-\tau_z) + \frac{2\pi n_0}{k^3} \sum_{qq_1} \chi_L^{(pq)(\mu q_1)}$$

$$\times \int \sum_l \alpha_l^{(\rho)(qn)(q_1\nu)} \exp(-\tau_\rho) d^L_{M_0 N}(\omega) d^l_{M_0 N}(\omega) \sin\omega d\omega d\rho, \tag{2}$$

where ρ, ω are the polar coordinates of integration point with respect to the point z, the angle ω $(0 \leq \omega \leq \pi)$ is counted off from the backscattering direction, $\tau_x = 2Im(\varepsilon)x$, $N = q_1 - q$ $(q, q_1 = \pm 1)$. Coefficients $\chi_L^{(pn)(\mu\nu)}$ are the expansion coefficients of the scattering matrix elements for isolated particle in series of the Wigner D function [12]. For spherical particles they can be written as:

$$\chi_{L_1}^{(pn)(\mu\nu)} = \frac{1}{4} \sum_{Ll} (2L+1)(2l+1) \langle a_L^{(pn)} a_l^{*(\mu\nu)} \rangle C_{L-nl\nu}^{L_1 M_0} C_{L-pl\mu}^{L_1 N_0}, \tag{3}$$

where $a_L^{(pn)} = a_L + pnb_L$, a_L, b_L are the Mie coefficients [14], angle brackets denote averaging over particle properties, symbols C denote the Clebsh-Gordan coefficients [11], the asterisk denotes complex conjugation. The Eqs.(1), (2) are one of forms of the well-known vector radiative transfer equation.

Expression for describing of coherent part of scattering radiation has the form

$$S_{pn\nu\mu}^{(co)} + S_{pn\nu\mu}^{1} = \frac{n_0}{k} \int_0^{kZ_0} G^{(z)(pn)(\mu\nu)} \exp[-2zIm(\varepsilon)(1 - \frac{1}{\cos\vartheta})] dz, \tag{4}$$

where $S_{pn\nu\mu}^{(co)}$ corresponds to the coherent part of scattering radiation, matrix $S_{pn\nu\mu}^{1}$ corresponds to some part of the incoherent radiation. Matrices $S_{pn\nu\mu}^{(co)}$ and $S_{pn\nu\mu}^{1}$ are normalized similarly to the matrix $S_{pn\mu\nu}^{(nc)}$. Matrix $G^{(z)(pn)(\mu\nu)}$ is defined by the equation [10]:

$$\begin{aligned} G^{(z)(pn)(\mu\nu)} &= \langle t_{pn} t_{\nu\mu}^{*} \rangle + \frac{2\pi n_0}{k^3} \sum_{qq_1 LM} \zeta_{LM}^{(pq)(\mu q_1)} \int \sum_{lm} \beta_{lm}^{(\rho)(qn)(q_1\nu)} \\ &\times \; d_{MN}^{L}(\omega) d_{mN}^{l}(\omega) \exp(-\tilde{\tau}_\rho) F_{m-M}(\rho, \vartheta, \omega) d\rho \sin\omega d\omega. \end{aligned} \tag{5}$$

Here t_{pn} is the amplitude scattering matrix for isolated particle,

$$\begin{aligned} \zeta_{L_1 M_1}^{(pq)(\mu q_1)} &= \sum_{Ll} \frac{(2L+1)(2l+1)}{4} \langle a_L^{(pq)} a_l^{*(\mu q_1)} \rangle C_{Lm_1 l-\mu}^{L_1 M_1} C_{L-ql q_1}^{L_1 N} (-1)^l d_{m_1-p}^{L}(\vartheta), \\ F_m(\rho, \vartheta, \omega) &= \sum_l i^{-l} (2l+1) j_l(2\rho\cos\frac{\vartheta}{2}) d_{m0}^{l}(\frac{\vartheta}{2}) d_{m0}^{l}(\omega), \\ \tilde{\tau}_\rho &= \rho Im(\varepsilon)\Big(2 - \cos\omega(1 - \frac{1}{\cos\vartheta})\Big) + i\rho\cos\omega[Re(\varepsilon) - 1](1 + \frac{1}{\cos\vartheta}), \end{aligned} \tag{6}$$

where $m_1 = M_1 + \mu$, $j_l(x)$ is the spherical Bessel function. Coefficients $\beta_{LM}^{(z)(pn)(\mu\nu)}$ are determined from the systems of equations [10]:

$$\begin{aligned} \beta_{LM}^{(z)(pn)(\mu\nu)} &= (-1)^L \zeta_{LM}^{*(\nu\mu)(np)} + \frac{2\pi n_0}{k^3} \sum_{qq_1} \chi_L^{(pq)(\mu q_1)} \int \sum_{lm} \beta_{lm}^{(\rho)(qn)(q_1\nu)} \\ &\times \; d_{MN}^{L}(\omega) d_{mN}^{l}(\omega) \exp(-\tilde{\tau}_\rho) F_{m-M}(\rho, \vartheta, \omega) d\rho \sin\omega d\omega. \end{aligned} \tag{7}$$

Matrix $S_{pn\mu\nu}^{(co)}$ describes the interference of conjugate pairs of waves propagating along the same trajectories in a medium but in reversal directions [1-7] including loop-like trajectories [10]. The case when these loop-like trajectories degenerate to polylines was considered in Ref.10 as a part of incoherent radiation which was described by matrix $S_{pn\mu\nu}^{1}$. But more detailed consideration shows that this part of scattered radiation is coherent part. Therefore $S_{pn\mu\nu}^{1}$ should correspond to single scattering (see the first term in the right side of the Eq.(5)).

Matrices (1) and (4) are written in the CP-representation. The transformation from CP to LP-representation (the basis of linear polarization) can be found in Refs.12,13. For example, using determination of the Stokes parameters in Ref.12, the components of the diffuse reflection matrix $R^{(nc)}_{ij}$ can be written as:

$$\begin{aligned}
R^{(nc)}_{11} &= U\sum_{pn} S^{(nc)}_{pnpn}, \qquad R^{(nc)}_{21} = -U\sum_{pn} S^{(nc)}_{pn-pn}, \qquad R^{(nc)}_{22} = U\sum_{pn} S^{(nc)}_{pn-p-n}, \\
R^{(nc)}_{33} &= U\sum_{pn} S^{(nc)}_{pn-p-n} i^{p-n}, R^{(nc)}_{43} = U\sum_{pn} S^{(nc)}_{pnp-n} i^{p-n+1}, R^{(nc)}_{44} = U\sum_{pn} S^{(nc)}_{pnpn} i^{p-n}. \qquad (8)
\end{aligned}$$

Here $U = -1/2k^2\cos\vartheta$. Similar relations can be written for the components of the coherent reflection matrix $R^{(co)}_{ij}$.

In special case of the exact backscattering ($\vartheta = \pi$) the Eqs.(6) yields [10]:

$$F_m(\rho,\pi,\omega) = \delta_{m0}, \qquad \zeta^{(pn)(\mu\nu)}_{LM} = -(-1)^L \chi^{(pn)(\mu\nu)}_L \delta_{MN_0}. \qquad (9)$$

In this case, as follows from the Eqs.(2),(4) and (5), $S^{(nc)}_{pn\mu\nu} = S^{(co)}_{pn\nu\mu} + S^1_{pn\nu\mu}$. Thus from the Eqs.(1),(4) and (8) we can obtain Mishchenko's relations [15]:

$$\begin{aligned}
2R^{(co)}_{11} &= R^{(nc)}_{11} + R^{(nc)}_{22} - R^{(nc)}_{33} + R^{(nc)}_{44} - 2R^1_{11}, \\
2R^{(co)}_{22} &= R^{(nc)}_{11} + R^{(nc)}_{22} + R^{(nc)}_{33} - R^{(nc)}_{44} - 2R^1_{22}, \\
2R^{(co)}_{33} &= -R^{(nc)}_{11} + R^{(nc)}_{22} + R^{(nc)}_{33} + R^{(nc)}_{44} - 2R^1_{33}, \\
2R^{(co)}_{44} &= R^{(nc)}_{11} - R^{(nc)}_{22} + R^{(nc)}_{33} + R^{(nc)}_{44} - 2R^1_{44}, \qquad (10)
\end{aligned}$$

where matrix R^1_{ij} corresponds to single scattering.

If the size parameter of scatterers isn't too large, numerical solution of system (2) doesn't have peculiarity. The numerical tests of the diffuse matrix (1) are in good agreement with those from the vector radiative transfer equation. Unlike the system (2), the system (7) depends on scattering angle ϑ. At scattering angle $\vartheta \neq \pi$ numerical solution of this system is too complicated because of frequent oscillation of the function $F_m(\rho,\vartheta,\omega)$. In this work we use the double-scattering approximation for semi-infinite medium. In this approach the expressions for scattering matrices are simplified considerably. For example Eqs. (1),(2) are reduced to:

$$S^{(nc)}_{pn\mu\nu} = -\frac{n_0\cos\vartheta}{2kIm(\varepsilon)(1-\cos\vartheta)} \sum_L d^L_{M_0N_0}(\vartheta) A^{(pn)(\mu\nu)}_L, \qquad (11)$$

where

$$\begin{aligned}
A^{(pn)(\mu\nu)}_L &= \chi^{(pn)(\mu\nu)}_L + \frac{\pi n_0}{k^3 Im(\varepsilon)} \sum_{qq_1} \chi^{(pq)(\mu q_1)}_L \sum_l \chi^{(qn)(q_1\nu)}_l \Big(\int_0^{\pi/2} d^L_{M_0N}(\omega) d^l_{M_0N}(\omega) \frac{\cos\vartheta \sin\omega d\omega}{\cos\vartheta - \cos\omega} \\
&+ \int_0^{\pi/2} d^L_{M_0N}(\pi-\omega) d^l_{M_0N}(\pi-\omega) \frac{\sin\omega d\omega}{1+\cos\omega} \Big). \qquad (12)
\end{aligned}$$

The first term in the last equation corresponds to the single scattering, the second one is related to the second-order scattering.

To determine the coherent matrix components in this approach, we use the expressions:

$$\begin{aligned}
F_m(\rho,\vartheta,\omega) &= \sum_l i^{-l}(2l+1) j_l(2\rho\cos\frac{\vartheta}{2}) d^l_{m0}(\frac{\vartheta}{2}) d^l_{m0}(\omega) \\
&= i^{-m} J_m(\rho\sin\vartheta\sin\omega)\exp[-i\rho\cos\omega(1+\cos\vartheta)], \\
\int_0^\infty J_m(bx)\exp(-ax)dx &= \frac{b^m}{\sqrt{a^2+b^2}(a+\sqrt{a^2+b^2})^m}, \quad (m > -1, Re(a) > Im(b)). \qquad (13)
\end{aligned}$$

Here $J_m(x)$ is the Bessel function. Thus from Eqs.(4) and (5) one can obtain:

$$S^{(co)}_{pn\nu\mu} = -\frac{\pi n_0^2 \cos\vartheta}{k^4 Im(\varepsilon)(1-\cos\vartheta)} \sum_{LMlmqq_1} \zeta^{(pq)(\mu q_1)}_{LM} (-1)^l \zeta^{*(\nu q_1)(nq)}_{lm} i^{-|m-M|} B^{(pn))(\mu\nu)}_{LMlm}, \tag{14}$$

where

$$\begin{aligned}
B^{(pn)(\mu\nu)}_{LMlm} &= \int_0^{\pi/2} d^L_{MN}(\omega) d^l_{mN}(\omega) \frac{c^{|m-M|}\sin\omega d\omega}{\sqrt{c^2+f^2}(f+\sqrt{c^2+f^2})^{|m-M|}} \\
&+ \int_0^{\pi/2} d^L_{MN}(\pi-\omega) d^l_{mN}(\pi-\omega) \frac{c^{|m-M|}\sin\omega d\omega}{\sqrt{c^2+f^{*2}}(f^*+\sqrt{c^2+f^{*2}})^{|m-M|}}, \\
c &= \sin\vartheta \sin\omega, \\
f &= 2Im(\varepsilon) + \cos\omega \Big(Im(\varepsilon)(1-\frac{1}{\cos\vartheta}) \\
&+ i[Re(\varepsilon)-1](1+\frac{1}{\cos\vartheta}) + i(1+\cos\vartheta) \Big).
\end{aligned} \tag{15}$$

As follows from the above presented formulae, at the exact backscattering direction ($\vartheta = \pi$) the intensity and the interference term of the second-order scattered waves are numerically equal to each other. At the other scattering angles ($\vartheta \neq \pi$) the matrix (14) depends strongly on the properties of the medium.

3 Calculations and discussion

The coherent backscattering mechanism for the effect of the negative polarization was considered in details in Refs.3, 4. It was demonstrated that in case of the positive polarization for the isolated scatterers, the interference of second-order scattering waves results in the negative polarization peak in the backscattering direction [3,4]. However positive polarization is a peculiar property of the Rayleigh scatterers. The angular dependence of polarization for the particles whose diameter is of the order of the wavelength becomes too complicated. Therefore, the interference of second-order scattering waves can result in positive polarization as well as in negative one.

We demonstrate this by calculations of the angular dependence of linear polarization degree P ($P = -R_{21}(\vartheta)/R_{11}(\vartheta)$) and normalized intensity I ($I = R_{11}(\vartheta)/R_{11}(\pi)$) for semi-infinite medium composed of identical spherical particles under different filling factors ξ. Filling factor is related to concentration of scatterers n_0 in the medium by the expression:

$$\xi = \frac{4}{3}\pi \tilde{a}^3 n_0, \tag{16}$$

where $\tilde{a}$ is the radius of the scatterer. The following value of the effective refractive index of the medium is used $\varepsilon = 1 + in_0 C_{ext}/2k$, where C_{ext} is the optical extinction cross section. For comparison we presented also the angle dependence of the linear polarization $P_0 = -s_{21}/s_{11}$ for isolate particles , where s_{ij} are the Mie scattering matrix elements.

Fig.1 corresponds to the medium composed of spherical particles with $\tilde{x} = k\tilde{a} = 3$ and the complex refractive index $\tilde{m} = 1.35 + 0i$. Scattering matrix element s_{11} for such particles is very asymmetric ($s_{11}(\vartheta = 0^0)/s_{11}(\vartheta = 180^0) \simeq 194$) and the most of radiation is scattered in direction of angles $\vartheta < 60^0$. The polarization of the interference part of scattered radiation is determined by behavior of P_0 in ranges of angles $\vartheta < 60^0$ and $\vartheta > 120^0$. Average value of P_0 in these scattering angle ranges is positive (see Fig.1). Therefore the interference of second-order scattered waves results in the negative polarization near the opposition direction. Note, that our data are in a qualitative accordance with the data for the semi-infinite medium composed of non-absorbing Rayleigh scatterers [9].

Fig.2 corresponds to scattering particles with $\tilde{x} = 4.5$ and the refractive index $\tilde{m} = 1.33+0i$. For such particles $s_{11}(\vartheta = 0^0)/s_{11}(\vartheta = 180^0) \simeq 162$. Average value of P_0 in range of $\vartheta < 60^0$ and in range of $\vartheta > 120^0$ is negative. In this case the interference of the second-order scattered waves results in a positive peak of polarization in the backscattering direction.

Fig. 3 shows another polarization behavior of the reflected radiation. It corresponds to scattering particles with $\tilde{x} = 3$ and the refractive index $\tilde{m} = 1.5 + 0.5i$. In this case $s_{11}(\vartheta = 0^0)/s_{11}(\vartheta = 180^0) \simeq 350$. Average polarization P_0 in range of angles $\vartheta < 60^0$ is positive, and in range of $\vartheta > 120^0$ is negative. The interference of waves for semi-infinite medium results in more complicated dependence of the polarization.

The photometric opposition effect always manifests itself as a typical sharp peak of intensity centered in the exact backscattering direction. Width of the peak depends significantly on the filling factor (Figs.1-3).

Note that presented data are obtained in the assumption that the orders of scattering higher than the second one are polarized weakly. However in some case (for example, when $Im(\tilde{m}) \simeq 0$) multiple scattering intensity can be considerable and influence essentially on the polarization pattern.

It is necessary to note also the polarization behavior for rarefied media. As shown in Fig.1 the peak of negative polarization is asymmetric. Such polarization behavior differs from the symmetric peak of negative polarization observed for many atmosphereless celestial bodies [6]. This difference is caused by near-field effect [16]. The role of this effect increases with an increase of the phase angle [17]. It leads to symmetric polarization curve in backscattering direction for closely packed media.

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Figure captions

Fig. 1. Coherent (1, 2) and diffuse (3) components of scattering radiation for medium with $\tilde{x} = 3$ and $\tilde{m} = 1.35 + 0i$. Curve 1 corresponds to $\xi = 0.001$, curve 2 corresponds to $\xi = 0.01$. s_{21} is the Mie matrix element.

Fig.2. The same as in Fig.1, but for $\tilde{x} = 4.5$ and $\tilde{m} = 1.33 + 0i$.

Fig.3. The same as in Fig.1, but for $\tilde{x} = 3$ and $\tilde{m} = 1.5 + 0.5i$. Curve 1 corresponds to $\xi = 0.002$, curve 2 corresponds to $\xi = 0.01$.