MULTIPLE SCATTERING OF LIGHT BY A LAYER OF DISCRETE RANDOM MEDIUM. BACKSCATTERING

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Abstract

The problem of multiple scattering of light by a layer of discrete random medium is considered. The medium is supposed to be rarefied with scatterers randomly positioned in the layer. Backscattering of light incident normally to the plane of the layer is considered. The scattering matrix is presented as a sum of three matrices one of them corresponding to incoherent scattering of light and difference of the other two matrices describing coherent scattering. Equations for calculation of these matrices are given.

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Proposed running head: Coherent backscattering by random media

1 Introduction

In scattering of electromagnetic waves by discrete random media a phenomenon of enhancement of backscattering, also referred to as the opposition effect or the weak photon localization, is often observed [1-3]. This phenomenon manifests itself as a nonlinear increase of the brightness of a medium as the scattering angle tends to the exact backscattering direction. When medium is illuminated by non-polarized light such brightness opposition effect can be accompanied by the so-called opposition polarization effect [4,5], which manifests as a specific behavior of linear polarization of scattered light. These two phenomena are characteristic of many atmosphereless cosmic bodies [2,5,6], cometary dust [7], laboratory samples of rough surfaces [8,9], powder-like layers and suspensions of latex particles in water [10-12].

To explain these effects, radiation scattered by a medium is usually presented as a sum of two parts. One of them is incoherent, diffuse radiation corresponding to the first-order-scattering contribution and to the sum of so-called ladder terms of the Bethe-Solpeter equation [13]. The dependence of this part on the characteristic parameters of the medium is described by the vector radiative transfer equation. The second part is coherent one corresponding to the sum of so-called cyclical terms [13]. It is produced by constructive interference of multiply scattered waves traveling in the medium along some trajectories and the reversal to them [1-5]. Accurate computation of the second part should be based on elaborate theoretical techniques. Existing theoretical models of these effects have been derived with significant simplifications [3-5]. The exact theoretical solution of the problem has been obtained only for a semi-infinite medium in the model of point-like scatterers [14]. However, this model does not allow one to describe the characteristics of light scattered by real media, in particular, a bimodal phase angle dependence of the linear polarization observed for the Saturn's rings [15]. In these cases the main physical characteristics of a particulate medium (e.g. particle size, shape, refractive index) should be taken into consideration. In Ref. [16] a rigorous relationship between the contributions of the cyclic and ladder components to the reflection matrix at exactly the backscattering direction has been obtained. However this relationship does not enable one to calculate the angular dependence of the light reflected by a medium.

In this paper we derive equations that describe the incoherent and coherent parts of light reflected by a layer of discrete random medium. The equations have been obtained using the theory of light scattering by systems of spherical particles. However, they are valid for media of chaotically oriented nonspherical particles.

It should be noted that the presentation of radiation reflected by a medium as the above-mentioned parts is possible for rarefied media for which the effects of the near field are small enough. These effects, however, can not be neglected for the systems of densely packed wavelength-size particles [17-19]. A special approach is required for such media, which is not a subject of consideration of this work.

2 Formulation of the problem and basic equations

Let a discrete medium be presented as a homogeneous layer with randomly positioned particles. The incident wave propagates along the z axis of a coordinate system and the its xy plane coincides with the upper boundary of the medium (Fig.1). Consider scattering of light by this medium in the backscattering direction ($z < 0$).

We use the circular polarization (CP) representation which is a convenient base to describe the incident and scattered waves. Transformation to the base of the linear polarization (LP representation) can be found in Refs. [20, 21].

The incident wave can be presented as

$$\mathbf{E}^0 = \mathbf{e}_n(\hat{\mathbf{k}}_0) \exp(i\mathbf{k}_0\mathbf{r}), \tag{1}$$

where $n = \pm 1$, $\mathbf{k}_0$ is the wave vector ($k_0 = 2\pi/\lambda$, λ is the wavelength), $\mathbf{e}_n(\hat{\mathbf{k}}_0)$ is the covariant cyclic basic vector [22] in the above mentioned coordinate system $\hat{\mathbf{k}}_0$. Here and hereafter symbols $\hat{\mathbf{n}}_i$ denote right-handed coordinate systems (x_i, y_i, z_i) with z_i axes along the vectors $\mathbf{n}_i$, and $-\hat{\mathbf{n}}_i$ are the coordinate systems with axes $(x_i, -y_i, -z_i)$.

The field scattered by a medium can be written as

$$\mathbf{E} = \sum_j \mathbf{E}^{(j)}, \tag{2}$$

where $\mathbf{E}^{(j)}$ is the field scattered by particle j. The amplitude scattering matrix t_{pn} for the medium defined as

$$\begin{pmatrix} E_1 \\ E_{-1} \end{pmatrix} = \frac{\exp(ik_0 r)}{-ik_0 r} \begin{pmatrix} t_{11} & t_{1-1} \\ t_{-11} & t_{-1-1} \end{pmatrix} \begin{pmatrix} E_1^0 \\ E_{-1}^0 \end{pmatrix}, \tag{3}$$

where r is the distance between the origin of the coordinate system and the observation point, can be also presented in the form

$$t_{pn} = \sum_j t_{pn}^{(j)}. \tag{4}$$

The scattering matrix $S_{pn\mu\nu}$, which transforms the Stokes parameters of the incident radiation into those of the scattered radiation, is defined by the following expression:

$$S_{pn\mu\nu} = \langle \sum_j t_{pn}^{(j)} t_{\mu\nu}^{*(j)} \rangle + \langle \sum_{j,\sigma \neq j} t_{pn}^{(j)} t_{\mu\nu}^{*(\sigma)} \rangle. \tag{5}$$

Here the angle brackets denote the averaging over the ensemble, the asterisk denotes complex conjugate, and $n, p, \mu, \nu = \pm 1$.

Let us suppose the scatterers in the medium to be spherical particles with the refractive indexes $\tilde{m_j}$ and radii $\tilde{a_j}$. This supposition is not of fundamental importance. The transformations carried out below are also applicable for systems of randomly oriented nonspherical particles, in particular, for systems of scatterers representing clusters of spherical particles (see Ref. [23]). This supposition allows us to avoid more complicated expressions. Then the field scattered by particle j in the far-field zone can be presented as [17,18,23,24]

$$\mathbf{E}^{(j)} = \frac{\exp(ik_0 r_j)}{-ik_0 r_j} \sum_{LMp} \frac{2L+1}{2} A_{LM}^{(jpn)} D_{Mp}^{*L}(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}) \mathbf{e}_p(\hat{\mathbf{k}}_{sc}). \tag{6}$$

Here r_j is the distance between particle j and the observation point, symbol D denotes the Wigner D function [22]

$$D_{Mp}^L(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}) = \exp(-iM\varphi) d_{Mp}^L(\vartheta) \exp(-ip\psi), \tag{7}$$

φ, ϑ, ψ are the Euler angles determining the rotation from the $\hat{\mathbf{k}}_0$ coordinate system to the $\hat{\mathbf{k}}_{sc}$ one with the z axis parallel to the $\mathbf{k}_{sc}$ vector, $\mathbf{k}_{sc}$ is the wave vector in the scattering direction ($k_{sc} = 2\pi/\lambda$), $\mathbf{e}_p(\hat{\mathbf{k}}_{sc})$ is the covariant cyclic basic vector in the $\hat{\mathbf{k}}_{sc}$ coordinate system. We assume that the scattering angle is the same for all particles.

The coefficients $A_{LM}^{(jpn)}$ are derived from the system of equations [17,18,23,24]

$$A_{LM}^{(jpn)} = a_L^{(jpn)} \exp(i\mathbf{k}_0 \mathbf{R}_j) \delta_{Mn} + \sum_q a_L^{(jpq)} \sum_{s \neq j} \sum_{lm} A_{lm}^{(sqn)} H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}), \tag{8}$$

where $a_L^{(jpn)} = a_L^{(j)} + pnb_L^{(j)}$, $a_L^{(j)}$ and $b_L^{(j)}$ are the Mie coefficients [25], $q = \pm 1$, $\mathbf{R}_j$ is the vector coordinate of particle j (Fig.1), $H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj})$ are coefficients of the addition theorems for the vector Helmholtz harmonics [26,27]. Following Ref. [27], these coefficients can be written as

$$H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}) = \frac{2l+1}{2} (-1)^m \sum_{l_1 m_1} i^{-l_1} h_{l_1}(k_0 r_{js}) D_{m_1 0}^{l_1}(\hat{\mathbf{k}}_0, -\hat{\mathbf{r}}_{sj}) C_{LMl-m}^{l_1 m_1} C_{Lql-q}^{l_1 0}, \tag{9}$$

where $h_l(x)$ is the Hankel spherical function of the first kind, symbols C denote the Clebsh-Gordan coefficients [22], $-\hat{\mathbf{r}}_{sj}$ is the coordinate system with the z axis directed along the vector $-\mathbf{r}_{sj}$ (see Fig.1).

For the case $r_{js} \gg \tilde{a_j} + \tilde{a_s}$ Eq. (9) yields

$$H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}) = \frac{2l+1}{2} \frac{\exp(ik_0 r_{js})}{-ik_0 r_{js}} D_{Mq}^L(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}) D_{mq}^{*l}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}). \tag{10}$$

Expressions (6)-(10) are suitable for complete description of light scattering by systems with a small number of particles. Using these expressions we will derive the equations which describe scattering of light by a medium containing a very large number of particles. To do this, the notion of the effective complex refractive index of the medium should be introduced. Such method is the basis to derive the radiative transfer equation [28].

Let $\varepsilon = Re(\varepsilon) + iIm(\varepsilon)$ be the effective refractive index of the medium. For rarefied media $Re(\varepsilon) \simeq 1$ and the extinction of radiation in media is determined by the value $Im(\varepsilon)$. In the first-order-scattering approximation $2k_0 Im(\varepsilon) = n_0 \langle C_{ext} \rangle$, where n_0 is the mean number of particles in a unit of volume, $\langle C_{ext} \rangle$ is the mean extinction cross section

$$\langle C_{ext} \rangle = \int_0^\infty C_{ext}(\tilde{a}) f(\tilde{a}) d\tilde{a}, \tag{11}$$

where $f(\tilde{a})$ is the size distribution function and $C_{ext}(\tilde{a})$ is the extinction cross section [25] for a particle of radius $\tilde{a}$. A more exact value of the effective refractive index can be calculated from the transport mean free path of photon [3]. The value of ε is supposed to be known. In the medium the exponential multipliers in Eqs. (6), (8), (10) depend on the vector $\mathbf{k} = \varepsilon \mathbf{k}_0$ and

$$\begin{aligned} k_0 r_j &= k_0 r - \mathbf{K}\mathbf{R}_j, \\ \mathbf{K} &= \mathbf{k}_{sc} + \frac{\mathbf{k} - \mathbf{k}_0}{\cos\vartheta}. \end{aligned} \tag{12}$$

In this work we use the representation of matrix (3) analogous to that of this matrix in the theory of light scattering by small particles [25]. In this representation the Stokes parameters of the incident and scattered beams specified with respect to the scattering plane. The basis vectors in this plane [25] are the contravariant cyclic vectors [22] $\mathbf{e}^n(\hat{\mathbf{k}}_0)$ and $\mathbf{e}^p(\hat{\mathbf{k}}_{sc})$. Then we have $\mathbf{e}_n(\hat{\mathbf{k}}_0) = \exp(in\varphi)\mathbf{e}^n(\hat{\mathbf{k}}_0)$ and $\mathbf{e}_p(\hat{\mathbf{k}}_{sc}) = \exp(-ip\psi)\mathbf{e}^p(\hat{\mathbf{k}}_{sc})$. Using these equations we obtain from Eq. (6)

$$t_{pn}^{(j)} = \exp(-i\mathbf{K}\mathbf{R}_j - in\varphi - ip\psi) \sum_{LM} \frac{2L+1}{2} A_{LM}^{(jpn)} D_{Mp}^{*L}(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}). \tag{13}$$

The solution of the system of equations (8) can be obtained with an iteration procedure [29]. Such presentation of the solution corresponds to the expansion of coefficients $A_{LM}^{(jpn)}$ as a multiple-scattering series

$$\begin{aligned} A_{LM}^{(jpn)}(1) &= a_L^{(jpn)} \exp(i\mathbf{k}\mathbf{R}_j)\delta_{Mn}, \\ A_{LM}^{(jpn)}(2) &= \sum_q a_L^{(jpq)} \sum_{slm} a_l^{(sqn)} H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0, \hat{\mathbf{r}}_{sj}) \exp(i\mathbf{k}\mathbf{R}_s)\delta_{mn}, \\ A_{LM}^{(jpn)}(3) &= \ldots\ldots. \end{aligned} \tag{14}$$

Summing up over all the multiple-scattering w we have $A_{LM}^{(jpn)} = \sum A_{LM}^{(jpn)}(w)$.

3 Scattering matrix for incoherent part

Consider the products

$$\begin{aligned} t_{pn}^{(j)} t_{\mu\nu}^{*(j)} &= \frac{\exp[i\mathbf{R}_j(\mathbf{K}^* - \mathbf{K})]}{4} \exp[i\varphi(\nu - n) + i\psi(\mu - p)] \\ &\times \sum_{LMlm} (2L+1)(2l+1) A_{LM}^{(jpn)} A_{lm}^{*(j\mu\nu)} D_{Mp}^{*L}(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}) D_{m\mu}^{l}(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}). \end{aligned} \tag{15}$$

We take into consideration the following scattering schemes. A wave propagates along some path from a particle σ and then is scattered by a particle j. Another wave propagates from a particle σ_1 along some other path and then is scattered by the particle j. And now we shall calculate products $A_{LM}^{(jpn)}(w) A_{lm}^{*(j\mu\nu)}(w_1)$ (see Eqs. (14)).

Since the particles are randomly positioned significant contribution to these products are made by the scattering schemes for which the phase multipliers depending on the particles coordinates are close to 1. Then we have $\sigma_1 = \sigma$, and the paths of both waves are equal. This case corresponds to incoherent scattering of light. Calculate products $A_{LM}^{(jpn)}(w) A_{lm}^{*(j\mu\nu)}(w)$ for each value w. The series of such products is an iterative solution of the following system of equations:

$$A_{LM}^{(jpn)}A_{L_1M_1}^{*(j\mu\nu)}=a_L^{(jpn)}a_{L_1}^{*(j\mu\nu)}\exp[-2Im(\varepsilon)\mathbf{k}_0\mathbf{R}_j]\delta_{Mn}\delta_{M_1\nu}$$
$$+\sum_{qq_1}a_L^{(jpq)}a_{L_1}^{*(j\mu q_1)}\sum_{s\neq j}\sum_{lml_1m_1}A_{lm}^{(sqn)}A_{l_1m_1}^{*(sq_1\nu)}H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})H_{L_1M_1l_1m_1}^{*(q_1)}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj}). \quad (16)$$

Introduce the following designations

$$\alpha_{L_1M_1}^{(j)(pn)(\mu\nu)}=\frac{1}{4}\sum_{LMlm}(2L+1)(2l+1)A_{LM}^{(jpn)}A_{lm}^{*(j\mu\nu)}(-1)^{M+p}C_{L-Mlm}^{L_1M_1}C_{L-pl\mu}^{L_1N_0}, \quad (17)$$

$$\chi_{L_1}^{(j)(pn)(\mu\nu)}=\frac{1}{4}\sum_{Ll}(2L+1)(2l+1)a_L^{(jpn)}a_l^{*(j\mu\nu)}C_{L-nl\nu}^{L_1M_0}C_{L-pl\mu}^{L_1N_0}, \quad (18)$$

where $M_0=\nu-n, N_0=\mu-p$. Here $\chi_L^{(j)(pn)(\mu\nu)}$ are coefficients of the expansion in series of the Wigner D functions of the scattering matrix components for an isolated spherical particle [20,21]. For both spherical and chaotically oriented nonspherical particles they depend on the angular momentum L_1 and polarization indexes n,p,μ,ν and they have some symmetric properties (see, e.g., Ref. [20]). With Eq. (10) we expand the products of the Wigner D functions in Eqs. (15) and (16) through the Clebsh-Gordan series [22]. Taking Eqs. (17) and (18) into account we get after some algebraic manipulations

$$t_{pn}^{(j)}t_{\mu\nu}^{*(j)}=\exp[i\mathbf{R}_j(\mathbf{K}^*-\mathbf{K})]\exp[i\varphi(\nu-n)+i\psi(\mu-p)]\sum_{LM}\alpha_{LM}^{(j)(pn)(\mu\nu)}D_{MN_0}^{L}(\hat{\mathbf{k}}_0,\hat{\mathbf{k}}_{sc}), \quad (19)$$

where coefficients $\alpha_{LM}^{(j)(pn)(\mu\nu)}$ are the solution of the system of equations

$$\alpha_{LM}^{(j)(pn)(\mu\nu)}=\chi_L^{(j)(pn)(\mu\nu)}\exp[-k_0z_j2Im(\varepsilon)]\delta_{MM_0}+\sum_{qq_1}\chi_L^{(j)(pq)(\mu q_1)}$$
$$\times\sum_{s\neq j}\sum_{L_1M_1}\alpha_{L_1M_1}^{(s)(qn)(q_1\nu)}\frac{\exp[-2Im(\varepsilon)k_0r_{js}]}{(k_0r_{js})^2}D_{MN}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})D_{M_1N}^{L_1}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj}). \quad (20)$$

Here $N=q_1-q$. Note that the coefficients $\alpha_{LM}^{(j)(pn)(\mu\nu)}$ differ from zero only at $M=M_0=\nu-n$.

We consider rarefied media in such a way that we can neglect the near-field effects [17-19]. For systems of identical spherical wavelength-size particles these effects are insignificant if the distances between particles are larger than two [19] or four [30] their diameters. For these distances the radial correlation function between particles j and s is [31-33]: $W(\tilde{a}_j,\tilde{a}_s,\rho)=1$ for $\rho\geq k_0(\tilde{a}_j+\tilde{a}_s)$ and $W(\tilde{a}_j,\tilde{a}_s,\rho)=0$ for $\rho<k_0(\tilde{a}_j+\tilde{a}_s)$, where $\rho=k_0r_{js}$. With this taken into account we make the following transformations. We sum up the products $t_{pn}^{(j)}t_{\mu\nu}^{*(j)}$ in Eq. (19) over all j. Following Ref. [31], we average the obtained equation and Eq. (20) over the ensemble including the averaging of coefficients $\alpha_{LM}^{(j)(pn)(\mu\nu)}$ and $\chi^{(j)(pn)(\mu\nu)}$ over sizes of particles. Using designations

$$\alpha_L^{(z)(pn)(\mu\nu)}=\langle\alpha_{LM}^{(j)(pn)(\mu\nu)}\rangle,\quad \chi_L^{(pn)(\mu\nu)}=\langle\chi_L^{(j)(pn)(\mu\nu)}\rangle, \quad (21)$$

where angle brackets denote averaging over particle sizes (see Eq. (11)), after transformations we obtain from Eqs. (19), (20)

$$S_{pn\mu\nu}^{(nc)}=\frac{n_0}{k_0}\sum_L d_{M_0N_0}^L(\vartheta)\int_0^{k_0Z_0}\exp(2z\frac{Im(\varepsilon)}{\cos\vartheta})\alpha_L^{(z)(pn)(\mu\nu)}dz, \quad (22)$$

where $S_{pn\mu\nu}^{(nc)}$ is the scattering matrix for the incoherent part normalized to a unit of the medium boundary, Z_0 is the thickness of the medium layer, n_0 is the mean number of particles in a unit of volume. Coefficients $\alpha_L^{(z)(pn)(\mu\nu)}$ are determined from the equation

$$\alpha_L^{(z)(pn)(\mu\nu)}=\chi_L^{(pn)(\mu\nu)}\exp(-\tau_z)+\frac{2\pi n_0}{k_0^3}\sum_{qq_1}\chi_L^{(pq)(\mu q_1)}$$
$$\times\int\sum_l\alpha_l^{(\rho)(qn)(q_1\nu)}\exp(-\tau_\rho)d_{M_0N}^L(\omega)d_{M_0N}^l(\omega)\sin\omega d\omega d\rho, \quad (23)$$

where ρ, ω are the polar coordinates of the integration point with respect to the point z, $\tau_x = 2Im(\varepsilon)x$. Unlike the scattering angle, the angle ω $(0 \le \omega \le \pi)$ is counted off from the vector $-\mathbf{k}_0$ direction. The upper limit of integration in the integral over ρ is

$$U = \begin{cases} \dfrac{z}{\cos\omega} & \text{for } \omega < \dfrac{\pi}{2} \\ \dfrac{-(Z_0 k_0 - z)}{\cos\omega} & \text{for } \omega > \dfrac{\pi}{2}. \end{cases} \tag{24}$$

Since the solution of Eq. (23) depends slowly on the lower limit of integration we can take this limit equal to zero. As for the coherent part of scattered light, it is determined by an equation with kernel $\sim 1/\rho^2$ (see Eq. (42) in the following section). Therefore we should calculate this limit more exactly. For systems of identical particles the lower limit of integration is equal to the particle diameter. For systems of particles of different sizes this limit depends on indexes L and l. Indeed, according to Ref. [31], we solve system (20) by the method of iterations and then substitute summation over particles with integration over the volume of the medium and average over particle sizes. As a result, we have the following series of transformations which is valid in the case $2k_0 Im(\varepsilon)(\tilde{a}_j + \tilde{a}_s) \ll 1$:

$$\begin{aligned}
& \chi_L^{(j)(pq)(\mu q_1)} \chi_l^{(s)(qn)(q_1\nu)} \int_0^U W(\tilde{a}_j, \tilde{a}_s, \rho) \exp(-\tau_\rho) d\rho \\
= \; & \chi_L^{(j)(pq)(\mu q_1)} \chi_l^{(s)(qn)(q_1\nu)} \int_0^U \exp(-\tau_\rho) d\rho - \chi_L^{(j)(pq)(\mu q_1)} \chi_l^{(s)(qn)(q_1\nu)} \int_0^{k_0(\tilde{a}_j + \tilde{a}_s)} \exp(-\tau_\rho) d\rho \\
\simeq \; & \chi_L^{(j)(pq)(\mu q_1)} \chi_l^{(s)(qn)(q_1\nu)} \int_0^U \exp(-\tau_\rho) d\rho - \chi_L^{(j)(pq)(\mu q_1)} \chi_l^{(s)(qn)(q_1\nu)} k_0(\tilde{a}_j + \tilde{a}_s) \\
\to \; & \chi_L^{(pq)(\mu q_1)} \chi_l^{(qn)(q_1\nu)} \int_0^U \exp(-\tau_\rho) d\rho - \Delta_{Ll} \chi_L^{(pq)(\mu q_1)} \chi_l^{(qn)(q_1\nu)} \\
\simeq \; & \chi_L^{(pq)(\mu q_1)} \chi_l^{(qn)(q_1\nu)} \int_{\Delta_{Ll}}^U \exp(-\tau_\rho) d\rho,
\end{aligned} \tag{25}$$

where the mean correlation distance is

$$\Delta_{Ll} = k_0 \frac{\langle \tilde{a}_j \chi_L^{(j)(pq)(\mu q_1)} \rangle}{\chi_L^{(pq)(\mu q_1)}} + k_0 \frac{\langle \tilde{a}_j \chi_l^{(j)(qn)(q_1\nu)} \rangle}{\chi_l^{(qn)(q_1\nu)}}. \tag{26}$$

When using this correlation distance, Eq. (23) becomes complicated. For rarefied media such complication seems unjustified. We can use an approximation in which the correlation function is the same for particles of different diameters [33], namely, $W(\tilde{a}_j, \tilde{a}_s, \rho) = 1$ for $\rho \ge 2k_0 \langle \tilde{a} \rangle$ and $W(\tilde{a}_j, \tilde{a}_s, \rho) = 0$ for $\rho < 2k_0 \langle \tilde{a} \rangle$. Therefore, the lower limit of the integral over ρ is $2k_0 \langle \tilde{a} \rangle$.

Note that coefficients $\chi_L^{(pq)(\mu\nu)}$ and $\alpha_L^{(z)(pn)(\mu\nu)}$ depend on differences of the indexes n, p, ν, μ (see Egs. (17), (18)). Thus we can rewrite Egs. (22), (23) in the following form:

$$S_{pn\mu\nu}^{(nc)} = \frac{k_0^2}{2\pi} \xi_0 \sum_L d_{M_0 N_0}^L(\vartheta) \int_0^{\tau_0} \exp(\frac{\tau}{\cos\vartheta}) \alpha_L^{(\tau)(M_0 N_0)} d\tau, \tag{27}$$

where $\xi_0 = \pi n_0 / k_0^3 Im(\varepsilon)$, $\tau_0 = 2k_0 Z_0 Im(\varepsilon)$, $M_0 = \nu - n$, $N_0 = \mu - p$,

$$\begin{aligned}
\alpha_L^{(\tau)(Mm)} &= \chi_L^{(Mm)} \exp(-\tau) + \xi_0 \sum_{N_1} \chi_L^{(N_1 m)} \\
&\times \int \sum_l \alpha_l^{(x)(MN_1)} \exp(-\frac{x}{\cos\omega}) d_{MN_1}^L(\omega) d_{MN_1}^l(\omega) \tan\omega \, d\omega dx.
\end{aligned} \tag{28}$$

Here $M, m, N_1 = 2, 0, -0, -2$, $U = \tau$ for $\omega < \pi/2$ and $U = \tau - \tau_0$ for $\omega > \pi/2$, the lower limit of the integral over x is $4k_0 Im(\varepsilon) \langle \tilde{a} \rangle \cos\omega$.

As an example, for the diffuse reflection coefficient $I^{(nc)}$ we have from Eqs. (3), (5)

$$I^{(nc)} = \frac{-1}{2k_0^2 \cos\vartheta} \sum_{pn} S_{pnpn}^{(nc)}. \tag{29}$$

The transformations carried out above are also valid for chaotically oriented nonspherical particles. Eqs. (22), (23), (27), (28) are also suitable in this case if the coefficients $\chi_L^{(pn)(\mu\nu)}$ are replaced with those corresponding to nonspherical particles. Note that Eqs. (23), (28) do not depend on scattering angle.

4 Scattering matrix for coherent part

The coherent part of the light scattered by a medium is known to be produced by constructive interference of conjugate pairs of waves propagating along the following paths [1-5]. For the wave (1): source of light $\rightarrow$ particle $s_1 \rightarrow$ particle $s_2 \rightarrow \ldots \rightarrow$ particle $s_{w-1} \rightarrow$ particle $j \rightarrow$ detector. For the wave (2): source of light $\rightarrow$ particle $j \rightarrow$ particle $s_{w-1} \rightarrow \ldots \rightarrow$ particle $s_2 \rightarrow$ particle $s_1 \rightarrow$ detector. The interference of such pairs of waves results in the opposition enhancement of backscattering [1-3]. This coherent part of scattered light is contained in the second term of Eq. (5). The first term of this equation also contains a coherent part. This part corresponds to propagation of waves along the following loop-like trajectories. For the wave (1): source of light $\rightarrow$ particle $j \rightarrow$ particle $s_1 \rightarrow \ldots \rightarrow$ particle $s_{w-1} \rightarrow$ particle $j \rightarrow$ detector. For the wave (2): source of light $\rightarrow$ particle $j \rightarrow$ particle $s_{w-1} \rightarrow \ldots \rightarrow$ particle $s_1 \rightarrow$ particle $j \rightarrow$ detector. The interference of such waves is constructive for all scattering angles. A particular case of such paths is the case when loop-like paths degenerate to polylines, i.e. when both waves propagate along the following path: source of light $\rightarrow$ particle $j \rightarrow$ particle $s_1 \rightarrow \ldots \rightarrow$ particle $s_{v-1} \rightarrow$ particle $s_v \rightarrow$ particle $s_{v-1} \rightarrow \ldots \rightarrow$ particle $s_1 \rightarrow$ particle $j \rightarrow$ detector. This case corresponds to a part of incoherent scattered light which is described by Eqs. (22), (23). Note that this part corresponds to uneven numbers of scattering starting from the third time.

With this remarks, we derive an equation for the sum $\sum_\sigma t_{pn}^{(j)} t_{\mu\nu}^{*(\sigma)}$, supposing that σ can take the value j. This equation also describes the above-mentioned part of the incoherent scattered radiation. To describe this part of radiation we derive another equation.

To calculate the matrix for coherent part of scattered light, we use the reciprocity principle (see, e.g., Ref. [16]). Let the incident wave be scattered first by particle σ, propagate along some path in the medium to particle j and then be scattered by particle j in the direction of vector $\mathbf{k}_{sc}$. Insert the term of series (14) corresponding to this case into Eg. (13) and denote obtained expression as $t_{pn}^{(j)}(w, \hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}, -\mathbf{K}, \mathbf{k})$. Derive the expression for the case when the incident wave propagates along vector $-\mathbf{k}_{sc}$, is scattered first by particle j, travels along the same path but in the reversal direction and then is scattered by particle σ in the direction of vector $-\mathbf{k}_0$. As an example for $w = 2$ we get

$$\begin{aligned}
& t_{pn}^{(j)}(2, \hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}, -\mathbf{K}, \mathbf{k}) = \exp(-i\mathbf{K}\mathbf{R}_j - in\varphi - ip\psi) \\
\times & \sum_{LMqlm} \frac{2L+1}{2} a_L^{(jpq)} a_l^{(\sigma qn)} H_{LMlm}^{(q)}(\mathbf{k}_0, \mathbf{r}_{\sigma j}) \exp(i\mathbf{k}\mathbf{R}_\sigma) D_{Mp}^{*L}(\hat{\mathbf{k}}_0, \hat{\mathbf{k}}_{sc}) \delta_{mn} \\
= & \exp[i\mathbf{k}\mathbf{R}_\sigma + in(\pi - \varphi) + ip(\pi - \psi)] \sum_{lm} \frac{2l+1}{2} D_{mn}^{*l}(-\hat{\mathbf{k}}_{sc}, -\hat{\mathbf{k}}_0) \\
\times & \sum_q a_l^{(\sigma qn)} \sum_{LM} a_L^{(jpq)} H_{lmLM}^{(q)}(-\mathbf{k}_{sc}, \mathbf{r}_{j\sigma}) \exp(-i\mathbf{K}\mathbf{R}_j) \delta_{Mp} \\
= & t_{np}^{(\sigma)}(2, -\hat{\mathbf{k}}_{sc}, -\hat{\mathbf{k}}_0, \mathbf{k}, -\mathbf{K}).
\end{aligned} \tag{30}$$

In derivation of this equality the following properties of the Wigner D function have been used [22,28]:

$$\begin{aligned}
\delta_{Mm} &= \sum_{m_1} D_{Mm_1}^L(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) D_{mm_1}^{*L}(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2), \\
D_{Mm}^L(\hat{\mathbf{n}}_2, \hat{\mathbf{n}}_1) &= \sum_{m_1} D_{m_1 M}^{*L}(\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_2) D_{m_1 m}^L(\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_1), \\
D_{Mm}^L(\hat{\mathbf{n}}_2, \hat{\mathbf{n}}_1) &= (-1)^{M-m} D_{Mm}^{*L}(-\hat{\mathbf{n}}_2, -\hat{\mathbf{n}}_1), \\
D_{Mm}^L(\hat{\mathbf{n}}_2, \hat{\mathbf{n}}_1) &= D_{mM}^{*L}(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2).
\end{aligned} \tag{31}$$

Equality (30) is valid for any w. Taking these equalities into account, one can see that the derivation of system of equations for the sum $\sum_\sigma t_{pn}^{(j)} t_{\mu\nu}^{*(\sigma)}$ is similar to those of the system (16). We can write

$$\sum_\sigma t_{pn}^{(j)} t_{\nu\mu}^{*(\sigma)} = \frac{1}{4}\exp[i\varphi(\mu-n)+i\psi(\nu-p)]\exp[-2\mathbf{R}_j Im(\mathbf{K}^*+\mathbf{k})]$$
$$\times \sum_{LMlm}(2L+1)(2l+1)A_{LM}^{(jpn)}A_{lm}^{*(j\mu\nu)}D_{Mp}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{k}}_{sc})D_{m\mu}^{l}(-\hat{\mathbf{k}}_{sc},-\hat{\mathbf{k}}_0), \qquad (32)$$

where products $A_{LM}^{(jpn)}A_{lm}^{*(j\mu\nu)}$ are determined from the system of equations

$$A_{LM}^{(jpn)}A_{L_1M_1}^{*(j\mu\nu)} = a_L^{(jpn)}a_{L_1}^{*(j\mu\nu)}\delta_{Mn}\delta_{M_1\nu} + \sum_{qq_1} a_L^{(jpq)}a_{L_1}^{*(j\mu q_1)}$$
$$\times \sum_{s\neq j}\sum_{lml_1m_1} A_{lm}^{(sqn)}A_{l_1m_1}^{*(sq_1\nu)}H_{LMlm}^{(q)}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})H_{L_1M_1l_1m_1}^{*(q_1)}(-\hat{\mathbf{k}}_{sc},\hat{\mathbf{r}}_{sj})\exp(i\mathbf{r}_{js}(\mathbf{K}^*+\mathbf{k})). \qquad (33)$$

Introducing following designations

$$G^{(j)(pn)(\mu\nu)} = \exp[i(\mu-n)\varphi+i(\nu-p)\psi]\sum_{LMlm}\frac{(2L+1)(2l+1)}{4}$$
$$\times A_{LM}^{(jpn)}A_{lm}^{*(j\mu\nu)}D_{Mp}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{k}}_{sc})D_{m\mu}^{l}(-\hat{\mathbf{k}}_{sc},-\hat{\mathbf{k}}_0),$$
$$B_{LM}^{(sqn)}B_{lm}^{*(sq_1\nu)} = \exp(i\nu\psi-in\varphi)\sum_{M_1}A_{LM}^{(sqn)}A_{lM_1}^{*(sq_1\nu)}D_{mM_1}^{*l}(\hat{\mathbf{k}}_0,-\hat{\mathbf{k}}_{sc}),$$
$$\zeta_{L_1M_1}^{(j)(pq)(\mu q_1)} = \exp(i\mu\varphi-ip\psi)\sum_{LMl}\frac{(2L+1)(2l+1)}{4}$$
$$\times a_L^{(jpq)}a_l^{*(j\mu q_1)}C_{L-Ml-\mu}^{L_1M_1}C_{L-qlq_1}^{L_1N}(-1)^{l+M+q}D_{Mp}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{k}}_{sc}), \qquad (34)$$

we can rewrite Eq. (32) and Eq. (33) as

$$\sum_\sigma t_{pn}^{(j)}t_{\nu\mu}^{*(\sigma)} = \exp[-2\mathbf{R}_j Im(\mathbf{K}^*+\mathbf{k})]G^{(j)(pn)(\mu\nu)},$$
$$G^{(j)(pn)(\mu\nu)} = t_{pn}^{(j)}t_{\nu\mu}^{*(j)} + \sum_{qq_1LM}\zeta_{LM}^{(j)(pq)(\mu q_1)}$$
$$\times \sum_{slm}\beta_{lm}^{(s)(qn)(q_1\nu)}D_{MN}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})D_{mN}^{l}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})\frac{\exp(\tau_{js})}{(k_0r_{js})^2}, \qquad (35)$$

where $N=q_1-q$, $\tau_{js}=-2Im(\varepsilon)k_0r_{js}+i\mathbf{r}_{js}(\mathbf{K}^*+\mathbf{k})$, and coefficients $\beta_{lm}^{(s)(qn)(q_1\nu)}$ are determined from the system of equations

$$\beta_{LM}^{(j)(pn)(\mu\nu)} = (-1)^L\zeta_{LM}^{*(j)(\nu\mu)(np)} + \sum_{qq_1}\chi_L^{(j)(pq)(\mu q_1)}$$
$$\times\sum_s\sum_{lm}\beta_{lm}^{(s)(qn)(q_1\nu)}D_{MN}^{*L}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})D_{mN}^{l}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}_{sj})\frac{\exp(\tau_{js})}{(k_0r_{js})^2}. \qquad (36)$$

Coefficients $\beta_{LM}^{(j)(pn)(\mu\nu)}$ have the form of coefficients $\alpha_{LM}^{(j)(pn)(\mu\nu)}$ if we replace in Eq. (17) $A_{LM}^{(jpn)}A_{L_1M_1}^{*(j\mu\nu)}$ by $B_{LM}^{(jpn)}B_{L_1M_1}^{*(j\mu\nu)}$.

Using formula [17]

$$\exp[i\mathbf{r}(\mathbf{k}_{sc}+\mathbf{k}_0)] = \sum_{lm} i^l(2l+1)j_l(2rk_0\cos\frac{\vartheta}{2})D_{m0}^{l}(\varphi,\frac{\vartheta}{2},0)D_{m0}^{*l}(\hat{\mathbf{k}}_0,\hat{\mathbf{r}}), \qquad (37)$$

averaging Eqs. (35), (36) over ensemble, using designations similar to Eq. (21) we obtain

$$S_{pn\nu\mu}^{(co)} + S_{pn\nu\mu}^{1} = \frac{n_0}{k_0}\int_0^{k_0Z_0} G^{(z)(pn)(\mu\nu)}\exp[-2zIm(\varepsilon)(1-\frac{1}{\cos\vartheta})]dz, \qquad (38)$$

where matrix $S^{(co)}_{pn\nu\mu}$ corresponds to the coherent part of scattered light, and matrix $S^{1}_{pn\nu\mu}$ corresponds to the incoherent part of light as mentioned in the beginning of this section,

$$\begin{aligned}
G^{(z)(pn)(\mu\nu)} &= \langle t_{pn}t^{*}_{\nu\mu}\rangle + \frac{2\pi n_0}{k_0^3}\sum_{qq_1LM}\zeta^{(pq)(\mu q_1)}_{LM}\int\sum_{lm}\beta^{(\rho)(qn)(q_1\nu)}_{lm} \\
&\times\; d^{L}_{MN}(\omega)d^{l}_{mN}(\omega)\exp(-\tau_\rho)F_{m-M}(\rho,\vartheta,\omega)d\rho\sin\omega d\omega, \\
\zeta^{(pq)(\mu q_1)}_{L_1M_1} &= \sum_{Ll}\frac{(2L+1)(2l+1)}{4}\langle a^{(pq)}_{L}a^{*(\mu q_1)}_{l}\rangle C^{L_1M_1}_{Lm_1l-\mu}C^{L_1N}_{L-qlq_1}(-1)^l d^{L}_{m_1-p}(\vartheta), \\
F_m(\rho,\vartheta,\omega) &= \sum_{l} i^{-l}(2l+1)j_l(2\rho\cos\frac{\vartheta}{2})d^{l}_{m0}(\frac{\vartheta}{2})d^{l}_{m0}(\omega), \\
\tau_\rho &= \rho Im(\varepsilon)[2-\cos\omega(1-\frac{1}{\cos\vartheta})] + i\rho\cos\omega[Re(\varepsilon)-1](1+\frac{1}{\cos\vartheta}) \\
&\simeq \rho Im(\varepsilon)[2-\cos\omega(1-\frac{1}{\cos\vartheta})].
\end{aligned} \tag{39}$$

Here ρ,ω are the polar coordinates of the integration point with respect to the point z, $m_1 = M_1+\mu$, angle brackets denote averaging over particle sizes, and coefficients $\beta^{(z)(pn)(\mu\nu)}_{LM}$ are determined from the system of equations

$$\begin{aligned}
\beta^{(z)(pn)(\mu\nu)}_{LM} &= (-1)^L\zeta^{*(\nu\mu)(np)}_{LM} + \frac{2\pi n_0}{k_0^3}\sum_{qq_1}\chi^{(pq)(\mu q_1)}_{L}\int\sum_{lm}\beta^{(\rho)(qn)(q_1\nu)}_{lm} \\
&\times\; d^{L}_{MN}(\omega)d^{l}_{mN}(\omega)\exp(-\tau_\rho)F_{m-M}(\rho,\vartheta,\omega)d\rho\sin\omega d\omega.
\end{aligned} \tag{40}$$

To calculate $S^{1}_{pn\mu\nu}$ matrix we use the system of equations (20). Taking the remarks made in this section into account, after manipulations similar to those of the previous section, we obtain

$$S^{1}_{pn\mu\nu} = \frac{n_0}{k_0}\sum_{L} d^{L}_{M_0N_0}(\vartheta)\int_0^{k_0Z_0}\exp[-2zIm(\varepsilon)(1-\frac{1}{\cos\vartheta})]\gamma^{(z)(pn)(\mu\nu)}_{L}dz, \tag{41}$$

where $\gamma^{(z)(pn)(\mu\nu)}_{L}$ is the solution of equation

$$\begin{aligned}
&\gamma^{(z)(pn)(\mu\nu)}_{L} = \chi^{(pn)(\mu\nu)}_{L} + \frac{2\pi n_0}{k_0^3}\sum_{qq_1l}\chi^{(pq)(\mu q_1)}_{L}\chi^{(-qn)(-q_1\nu)}_{l}(-1)^l \\
&\times\int\sum_{L_1}(-1)^{L_1}\gamma^{(\rho)(q-q)(q_1-q_1)}_{L_1}\frac{\exp[-4\rho Im(\varepsilon)]}{\rho^2}d^{L}_{M_0N}(\omega)d^{l}_{M_0N}(\omega)\sin\omega d\omega d\rho.
\end{aligned} \tag{42}$$

Here $M_0 = \nu - n, N_0 = \mu - p, N = q_1 - q$.

5 Discussion

First consider the case of the exact backscattering direction ($\vartheta = \pi$). As can be seen from Eqs. (18), (39) in this case $F_m(\rho,\pi,\omega) = \delta_{m0}$, $\zeta^{(pn)(\mu\nu)}_{LM} = -(-1)^L\chi^{(pn)(\mu\nu)}_{L}\delta_{MN_0}$ and we have $S^{(nc)}_{pn\mu\nu} = S^{(co)}_{pn\nu\mu} + S^{1}_{pn\nu\mu}$. Matrix $S^{1}_{pn\nu\mu}$ is defined only for uneven number of scattering. Thus we can expect that for strongly absorbing media $S^{(nc)}_{pn\mu\nu} \simeq S^{(co)}_{pn\nu\mu}$ (see Ref. [34]).

The case $\vartheta \neq \pi$ we consider in the double-scattering approximation. In this approximation we have

$$\begin{aligned}
G^{(z)(pn)(\mu\nu)} &= \langle t_{pn}t^{*}_{\nu\mu}\rangle + \frac{2\pi n_0}{k_0^3}\sum_{qq_1LM}\zeta^{(pq)(\mu q_1)}_{LM}\sum_{lm}\zeta^{*(\nu q_1)(nq)}_{lm}(-1)^l \\
&\times\int d^{L}_{MN}(\omega)d^{l}_{mN}(\omega)\exp(-\tau_\rho)F_{m-M}(\rho,\vartheta,\omega)d\rho\sin\omega d\omega, \\
\gamma^{(z)(pn)(\mu\nu)}_{L} &= \chi^{(pn)(\mu\nu)}_{L}.
\end{aligned} \tag{43}$$

Some results of numerical calculations in this case are given in Refs. [23],[30]. The integral

$$I = \int_{2k_0\langle\ \tilde{a}\ \rangle}^{U} \exp(-\tau_\rho) F_m(\rho, \vartheta, \omega) d\rho \tag{44}$$

determines the dependence of coefficients $G^{(z)(pn)(\mu\nu)}$ on the scattering angle. The first term of function $F_m(\rho, \vartheta, \omega)$ contributes essentially to this integral. Then

$$I \simeq \int_{2k_0\langle\ \tilde{a}\ \rangle}^{U} \exp(-\tau_\rho) \frac{\sin\xi}{\xi} d\rho, \tag{45}$$

where $\xi = 2\rho\cos(\vartheta/2)$. The integral (45) is a function which tends to 0 rapidly as ϑ decreases from π. This function describes a narrow interference peak centered exactly at the backscattering direction $\vartheta = \pi$. In the intensity of scattered light this peak corresponds to the opposition effect. It can be shown [23,30] that the width of interference peak decreases rapidly with the decrease of $Im(\varepsilon)$ (i.e. with the decrease of n_0) and it decreases considerably less with the increase of $2k_0\langle\ \tilde{a}\ \rangle$. Such dependence of the interference peak on the lower limit of the integral over ρ shows that substitution of the lower limit (26) by $2k_0\langle\ \tilde{a}\ \rangle$ is acceptable. The dependence of this peak on the properties of particles is determined by coefficients $\chi_L^{(pn)(\mu\nu)}$ and $\zeta_{LM}^{(pn)(\mu\nu)}$. More detailed conclusions about dependence of this peak on the characteristic parameters of the medium can be obtained from numerical solution of the equations.

6 Conclusion

The equations given in the previous sections describe both the incoherent and the coherent parts of light scattered by a layer of discrete random medium. These equations have been derived for the case of spherical particles. However, the transformations made for derivation of the equations are similar to the case of randomly oriented nonspherical particles, in particular, for systems of randomly oriented clusters of spherical particles [23,30]. Thus these equations are valid for nonspherical particles if the coefficients $\chi_L^{(pn)(\mu\nu)}$ and $\zeta_{LM}^{(pn)(\mu\nu)}$ are replaced by the corresponding coefficients for nonspherical particles. To derive these equations the mean distance between particles was supposed to be much larger than the characteristic sizes of particles. This enabled us to neglect the near field effects. The numerical calculations for randomly oriented bispheres with wavelength-sized components has shown that it is possible to neglect these effects if the mean distance between particles is larger then their double diameters [19], whereas analysis of the near field properties for such particles shows that this distance should be larger than four diameters of particles [30].

The equations obtained in this work describe backscattering of light by rarefied media. A complete description of light scattering by such medium required that the corresponding equations for the forward-scattering direction should be obtained. We plan to do it in future work.

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Figure captions

Fig. 1. The coordinate system used.