SIMPLE RELATIONSHIPS BETWEEN RADIATIVE AND MICROPHYSICAL CHARACTERISTICS OF CLOUDY MEDIA

A. A. Kokhanovsky
Institute of Physics
70 Skarina Avenue, Minsk 220072, Belarus

ABSTRACT

This paper is devoted to the derivation of simple relationships between reflection and transmission matrices of cloudy media and their microphysical characteristics (average size of droplets, their concentration and refractive index). Such relations are of importance for the parametrization of radiative and polarization characteristics of clouds. They can be used not only for the solution of direct problems but for the development of semi-analytical inversion algorithms as well.

1. INTRODUCTION

The influence of clouds on radiative fluxes in the earth atmosphere has been studied by many authors both experimentally and theoretically. The knowledge obtained has been applied to the solution of inversion problem, namely for the determination microstructure, thickness and liquid water path of cloudy media.

However, cloudy media not only absorb and scatter radiation. They also can be considered as major sources (along with aerosols and gases) of polarized light in the terrestrial atmosphere. They introduce a new property in coming solar light, namely the preferential direction of oscillation of an electic vector in a light beam.

The polatization arises mostly due to a single scattering of light beam by a droplet. Multiple scattering of the beam will lead to the randomisation both photons` directions and polatization states. This will cause the decrease of light polarization due to increase in the optical thickness of a cloud. Another effect of multiple scattering is the introduction of ellipticity in scattered light beams. It should be pointed out that the solar light scattered by a spherical droplet becomes partially linearly polarized. The ellipticity of a singly scattered light is equal to zero.

Both polarizarion and radiative characteristics of cloudy media can be studied with modern radiative transfer codes. However, it is of importance to have approximate formulae for optical chracteristics of cloudy media. This is due to the fact that that simple solutions allow to make rapid estimations of results expected. They also of importance for the development of semi-analytical inversion schemes.

2. TRANSMISSION AND REFLECTION MATRICES

Asymptotic theory for thick layers can be used both for studies of polarization signatures of light fields both inside of clouds and on their boundaries. We are concern only with the polarization characteristics of reflected light, which can be used for a global coverage from instruments on board of different satellites. Thus, we will limit ourselves by studies of the reflection matrix $\hat{R}(\mu,\mu_0,\psi)$ of a cloudy medium. This matrix depends on the optical thickness of a cloud τ, microstructure of a cloud, cosine μ_0 of the solar angle ϑ_0, cosine μ of the viewing angle ϑ and the relative azimuth $\psi=\varphi-\varphi_0$, where φ_0 and φ are azimuths of the Sun and observer respectively. The reflection matrix $\hat{R}(\mu,\mu_0,\psi)$ can be used to find the Stokes vector $\vec{S}_r(I,Q,U,V)$ of the reflected light for arbitrary incident light beams. Namely, it follows:

$$\vec{S}_r(\mu,\varphi)=\frac{1}{\pi}\int_0^1 d\mu\ \mu\int_0^{2\pi} d\varphi \hat{R}(\mu,\mu,\varphi-\varphi)\vec{S}_0(\mu,\varphi) \qquad (1)$$

where $\vec{S}_0(\mu,\varphi)$ represents the stokes vector of the incident light beam. A similar expression could be written for the transmission matrix.

One can consider the Sun a source of light incident from only one direction μ_0,φ_0. Then it follows:

$$\vec{S}_0(\mu,\varphi)=\frac{1}{\mu_0}\delta(\mu-\mu_0)+\delta(\varphi-\varphi_0)\pi\vec{F}, \qquad (3)$$

where a vector $\vec{F}$ is normalized so that the first element of $\pi\vec{F}$ is the net flux of solar beam per unit area of a cloud layer. Eq. (1) becomes in this case:

$$\vec{S}_r(\mu,\varphi)=\hat{R}(\mu,\mu_0,\varphi-\varphi_0)\vec{F}, \qquad (4)$$

where only first element of the column vector $\vec{F} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$ differs from zero.

One can see that the knowledge of matrices $\hat{R}(\mu, \mu_0, \varphi - \varphi_0)$ and $\hat{T}(\mu, \mu_0, \varphi - \varphi_0)$ is of primary importance for understanding radiative and polarization characteristics of light fluxes, which emerge from a cloud body.

3. ASYMPTOTIC THEORY

The optical thickness of most of water clouds is much larger then 1. Some mixed and crystalline clouds could be also extended in the vertical direction for many hundred meters. Thus, the asymptotic vector theory is quite applicable for cloudy media. The reflection matrix $\hat{R}(\mu, \mu_0, \varphi)$ and transmission matrix $\hat{T}(\mu, \mu_0)$ of a homogeneous plane-parallel turbid layer of the large optical thickness τ can be written in the following form:

$$\hat{R}(\mu, \mu_0, \varphi) = \hat{R}_\infty(\mu, \mu_0, \varphi) - f\hat{T}(\mu, \mu_0), \tag{1}$$

$$\hat{T}(\mu, \mu_0) = t\vec{K}(\mu)\vec{K}^T(\mu_0). \tag{2}$$

Here $\mu = \cos\vartheta, \mu_0 = \cos\vartheta_0, \vartheta_0$ and ϑ are incidence and observation angles respectively, φ is the relative azimuth of incident and reflected or transmitted light beams and

$$t = \frac{m\exp(-k\tau)}{1 - f^2}. \tag{3}$$

Matrices $\hat{R}(\mu, \mu_0, \varphi)$ and $\hat{T}(\mu, \mu_0)$ can be used to calculate the Stokes vectors of reflected and transmitted light beams under arbitrary illumination of a scattering layer.

Note, that the vector $\vec{K}^T(\mu_0)$ is transpose to the vector $\vec{K}(\mu_0)$. Constants *f, m* and the vector $\vec{K}(\mu)$ can be found from following relations:

$$f = s\exp(-k\tau) \tag{4}$$

$$m = 2\int_0^1 d\mu\mu\left[\vec{P}^T(\mu)\vec{P}(\mu) - \vec{P}^T(-\mu)\vec{P}(-\mu)\right],$$

$$\vec{K}(\mu) = m^{-1}\left[\vec{P}(\mu) - 2\int_0^1 d\xi\xi\hat{R}_\infty(\mu,\xi)\vec{P}(-\xi)\right],$$

where

$$s = 2\int_0^1 d\mu\mu\vec{K}^T(\mu)\vec{P}(-\mu),$$

and

$$\hat{R}_\infty(\mu,\xi) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\hat{R}_\infty(\mu,\xi,\varphi)$$

is the azimuthally averaged reflection matrix of the semi-infinite medium with the same local optical characteristics as those for a finite layer under study. Note, that the vector $\vec{K}(\mu)$ also occurs in the so-called Milne problem, i. e. the problem of finding the angular dependence and the polarization characteristics of light emerging from a semi-infinite turbid layers with sources of radiation located deep inside the medium. The constant *k* is the smallest discrete eigenvalue, which is real and nondegenerate, obeying to the following equation:

$$(1 - ku)\vec{P}(u) = \frac{\omega_0}{2}\int_{-1}^1 d\xi\hat{Z}(u,\xi)\vec{P}(\xi),$$

which is called the vector characteristic equation of the radiative transfer theory. Here *u* is the cosine of the observation angle, ω_0 is the single scattering albedo, which is equal to the ratio of scattering and extinction coefficients, $\hat{Z}(u,\xi)$ is the azimuth-averaged phase matrix and $\vec{P}(\xi)$ is correspondent eigenvector.

Eqs. (1), (2) are very general and can be applied to optically thick disperse media with arbitrary local optical characteristics. However, their application to the solution of practical problems is difficult due to the necessity to solve integral equation (9) and perform integrations (5) - (8). The reflection matrix $\hat{R}_\infty(\mu,\xi,\varphi)$ obeys to the integral equation as well.

The problem of finding matrices $\hat{R}(\mu, \mu_0, \varphi)$ and $\hat{T}(\mu, \mu_0)$ in Eqs. (1), (2) can be simplified for a broad class of so-called weakly absorbing media, which are characterized by a small value of the probability of photon absorption $\beta = 1 - \omega_0$. It was obtained as $\beta \to 0$ that

$$k = \sqrt{3(1-g)(1-\omega_0)}, \tag{10}$$

$$m = \frac{8k}{3(1-g)}, \tag{11}$$

$$s = 1 - \frac{4k\alpha}{3(1-g)}, \tag{12}$$

$$\hat{R}_\infty(\mu,\mu_0,\varphi) = \hat{R}_\infty^0(\mu,\mu_0,\varphi) - \frac{4k}{3(1-g)}\vec{K}_0(\mu)\vec{K}_0^T(\mu_0),$$

where $\vec{K}_0(\mu)$ and $\hat{R}_\infty(\mu,\mu_0,\varphi)$ are the escape vector and the reflection matrix respectively of the nonabsorbing medium with the same phase matrix as a finite layer under study,

$$g = \frac{1}{2}\int_0^\pi p(\theta)\sin\theta\cos\theta d\theta \tag{14}$$

is the asymmetry parameter of the phase function $p(\theta)$ and

$$\alpha = 3\int_0^1 d\mu\mu^2 S(\mu), \tag{15}$$

where $S(\mu) = \vec{K}_0^T(\mu)\vec{e}_1$, $\vec{e}_1$ is the unit column vector. It should be pointed out, that integral (15) is close to one for all types of phase matrices[1–3]. Thus, will neglect small differences $\Delta = 1-\alpha$ in the discussion, which follows. Note that the first moment of the function $S(\mu)$ does not depend on the phase matrix at all. This is due to the normalization condition[2] :

$$2\int_0^1 d\mu\mu\ S(\mu) = 1 \tag{16}$$

Thus, Eqs. (10) - (15) allow to reduce the complexity of Eqs. (1), (2). Only functions $\vec{K}_0(\mu)$ and $\hat{R}_\infty^0(\mu,\mu_0,\varphi)$ for a nonabsorbing medium should be found numerically in this more simple case. They depend on the phase matrix of a random medim in question. However, they do not depend on the single scattering albedo ω_0 and the optical thickness τ. Another interesting feature of these functions is their weak dependence on the microstructure parameters (size, shape, and chemical composition of particles) of a medium under study in the broad range of angular parameters μ, μ_0, φ. This is due to the randomization both polarization states and propagation directions of photons in highly scattering semi-infinite layers irrespectively to the local optical properties of a medium, where they propagate.

4. APPROXIMATE EQUATIONS FOR WEAKLY ABSORBING MEDIA

Numerical calculations show that expansions (11) - (15) can be applied only for very small values of $\beta = 1-\omega_0\ (\beta \sim 10^{-4})$. This limits the power of the asymptotic theory for a real life problems in a great extent. Different methods to overcome this problem have been used. The most straight forward approach calls for the derivation of higher order terms in expansions (10)-(13). However, we will use here another method, which has initially been applied in the scalar radiative transfer theory. According to this alternative approach expansions (11), (12) are substituted by exponential functions:

$$m = 1 - e^{-2y}, \tag{17}$$

$$s = e^{-y}, \tag{18}$$

where

$$y = \frac{4k}{3(1-g)} \tag{19}$$

and we assumed that $\alpha = 1$.
Expansions (17), (18) transform to exact asymptotic results (11), (12) as $y \to 0$, providing that $\alpha - 1$. However, they do allow to calculate radiative and polarization characteristics for more wide class of absorbing and light-scattering media (up to $\beta \approx 0.05 \div 0.1$, depending on the characteristic in question).
Thus, Eqs. (1), (2) can be transformed to the following forms:

$$\hat{R}(\mu,\mu_0,\varphi) = \hat{R}_\infty(\mu,\mu_0,\varphi) - \hat{T}(\mu,\mu_0)\exp(-y-k\tau), \tag{20}$$

$$\hat{T}(\mu,\mu_0) = t\vec{K}_0(\mu)\vec{K}_0^T(\mu_0), \tag{21}$$

where we accounted for Eqs. (17), (18) and the approximate equality

$$m\vec{K}(\mu)\vec{K}^T(\mu_0) = (1-e^{-2y})\vec{K}_0(\mu)\vec{K}^T(\mu_0). \tag{22}$$

The value of

$$t = \frac{1-e^{-2y}}{e^{k\tau} - e^{-2y-k\tau}} \tag{23}$$

is the global transmittance, defined as[1]

$$t = 4\int_0^1 \mu d\mu \int_0^1 \mu_0 d\mu_0 T_{11}(\mu,\mu_0).$$

(24)

Eq. (24) follows from Eq. (21) accounting for the integral relation [1]

$$2\int_0^1 \mu d\mu K_{01}(\mu)d\mu = 1\,, \tag{25}$$

where $K_{01}(\mu)$ is the first component of the escape vector $\vec{K}_0(\mu)$.

Note, that the global transmittance (24) could be transformed to the following simple form [6]:

$$t = \frac{shy}{sh(x+y)}\,, \tag{26}$$

where

$$x = k\tau\,. \tag{27}$$

Eqs. (20), (21) are much simpler than initial asymptotic formulae (1), (2). They allow to calculate the reflection and transmission matrices of thick weakly absorbing disperse media by simple means , if the solution of the problem for a semi-infinite medium with the same phase matrix as for a finite layer under consideration is available. This reduction of a general problem to the case of a semi-infinite medium is of importance for the radiative transfer theory.

It follows from results presented that the reflection and transmission matrices of cloudy media depend mostly on the asymmetry parameter, extinction and absorption coefficients of a cloudy media. The phase function is of a secondary importance for most observation geometries. These local optical characteristics can be found with following equations, which are valid up to 2.2 micrometers:

$$\sigma_{ext} = \frac{3c}{2\rho a_{ef}}\left(1 + \frac{1.1}{(k_2 a_{ef})^{2/3}}\right),$$

$$g = 0.88 + \frac{1}{2(ka_{ef})^{2/3}} - 0.15\alpha a_{ef}\,,$$

$$\sigma_{abs} = 1.25\alpha\left(1 - \alpha a_{ef}\right)\left(1 + 0.34(1 - \exp(-8\lambda / a_{ef}))\right),$$

where the effective radius is equal to the ratio of third and second moments of the particle size distribution, $k = 2\pi/\lambda, \alpha = 4\pi\chi/\lambda$, c is the liquid water content, ρ is the density of water, χ is the imaginary part of the refractive index of particles and λ is the wavelength. Our calculations show that the accuracy of Eq. (A2.1) is better than 5% for water droplets with the effective radius in the range 4-16 micrometers at λ < 2.2 μm. It is up to 10% in the same spectral range for values of the absorption coefficient and co-asymmetry parameter *1-g*. It is smaller than 5% for both *1-g* and σ_{abs} at λ < 1.2 μm. The dependence of $y(\lambda)$, obtained with Eqs. (A2.1)-(A2.3), , and Mie calculations for spherical polydispersions of water droplets with the gamma particle size distribution is presented in Fig. A2.1. One can see that the accuracy is high. The error is smaller than 7% up to $\lambda = 1.8\mu m$. The error of geometrical optics equation increases for clouds for small $\left(a_{ef} = 4\mu m\right)$ and large $\left(a_{ef} = 16\mu m\right)$ droplets. However, even in this case it is rather high(see Fig. A2.1).

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