SCATTERING AND POLARIZATION PROPERTIES OF THE NON-SPHERICAL PARTICLES

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ABSTRACT

The albedo and dichroic polarization of spheroidal particles are studied. For the description of the phase function two asymmetry parameters $g_{||}$ and $g_{\perp}$ characterizing the anisotropy in forward/backward and left/right directions are introduced and calculated. The consideration is based on the solution to the light scattering problem by the Separation of Variables Method (Voshchinnikov and Farafonov, 1993).

1. INTRODUCTION

In many scientific and engineering applications prolate and oblate spheroids are appropriate models for real particles. We consider the light scattering by homogeneous spheroids using the Separation of Variables Method (SVM). The optical properties of prolate spheroids of various aspect ratios a/b for several refractive indices m are calculated and the results for the particles of the same volume are compared.

2. GENERAL DEFINITIONS AND METHOD

A spheroid (ellipsoid of revolution) is obtained by the rotation of an ellipse around its major axis (prolate spheroid) or its minor axis (oblate spheroid). The ratio of the major semiaxis a to the minor semiaxis b (i.e. the aspect ratio a/b) characterizes the particle shape which may vary from a nearly spherical one ($a/b \approx 1$) to a needle or a disk ($a/b \gg 1$).

We assume that an incident plane wave has the wavelength λ. Let α denote the angle between the propagation direction and the rotation axis of the spheroid ($0° \leq \alpha \leq 90°$).

For the axial propagation ($\alpha = 0°$), there is no polarization of transmitted radiation due to symmetry. If $\alpha \neq 0°$, two cases of polarization of the incident radiation have to be considered: the electric vector $\vec{E}$ is parallel (TM mode) or perpendicular (TE mode) to the plane defined by the spheroid's rotation axis and the wave propagation vector.

The size parameter is given by

$$x_{\rm V} = \frac{2\pi r_{\rm V}}{\lambda}, \tag{1}$$

where $r_{\rm V}$ is the radius of the sphere whose volume is equal to that of the spheroid. The radius $r_{\rm V}$ for prolate spheroids is defined as

$$r_{\rm V}^3 = ab^2\,. \tag{2}$$

One usually calculates the efficiency factors $Q = C/G$ which are the ratio of the corresponding cross-sections C to the geometrical cross-section G of the prolate spheroid (the area of the particle's shadow)

$$G(\alpha) = \pi b \left(a^2 \sin^2\alpha + b^2 \cos^2\alpha\right)^{1/2}\,. \tag{3}$$

In order to compare the optical properties of the particles of different shapes it is convenient to consider the ratios of the cross-sections for spheroids to the geometrical cross-sections of the equal volume spheres, $C/\pi r_{\rm V}^2$. For a prolate spheroid they can be found as

$$\frac{C}{\pi r_{\rm V}^2} = \frac{[(a/b)^2 \sin^2\alpha + \cos^2\alpha]^{1/2}}{(a/b)^{2/3}} Q\,. \tag{4}$$

The albedo of a particle can be calculated from the extinction and scattering cross-sections

$$\Lambda = \frac{C_{sca}}{C_{ext}}\,. \tag{5}$$

In general, the radiation scattered by aligned spheroidal particles has an azimuthal asymmetry that provokes a non-coincidence of the directions of the radiation pressure force and of the wave-vector of incident radiation (Voshchinnikov, 1990; Il'in and Voshchinnikov, 1998). Another consequence of the azimuthal asymmetry is the anisotropy of the phase function in the left/right direction. The geometry of the phase function in forward/backward and left/right directions may be characterized by two asymmetry parameters $g_{||}$ and $g_{\perp}$, respectively. Expressions for them can be found from the consideration of radiation pressure (Voshchinnikov, 1990; Il'in and Voshchinnikov, 1998)

$$g_{||} = \frac{\mathcal{A}\,(m, x_{\rm V}, a/b, \alpha) \cos\alpha + \mathcal{B}\,(m, x_{\rm V}, a/b, \alpha) \sin\alpha}{Q_{\rm sca}\,(m, x_{\rm V}, a/b, \alpha)}\,, \tag{6}$$

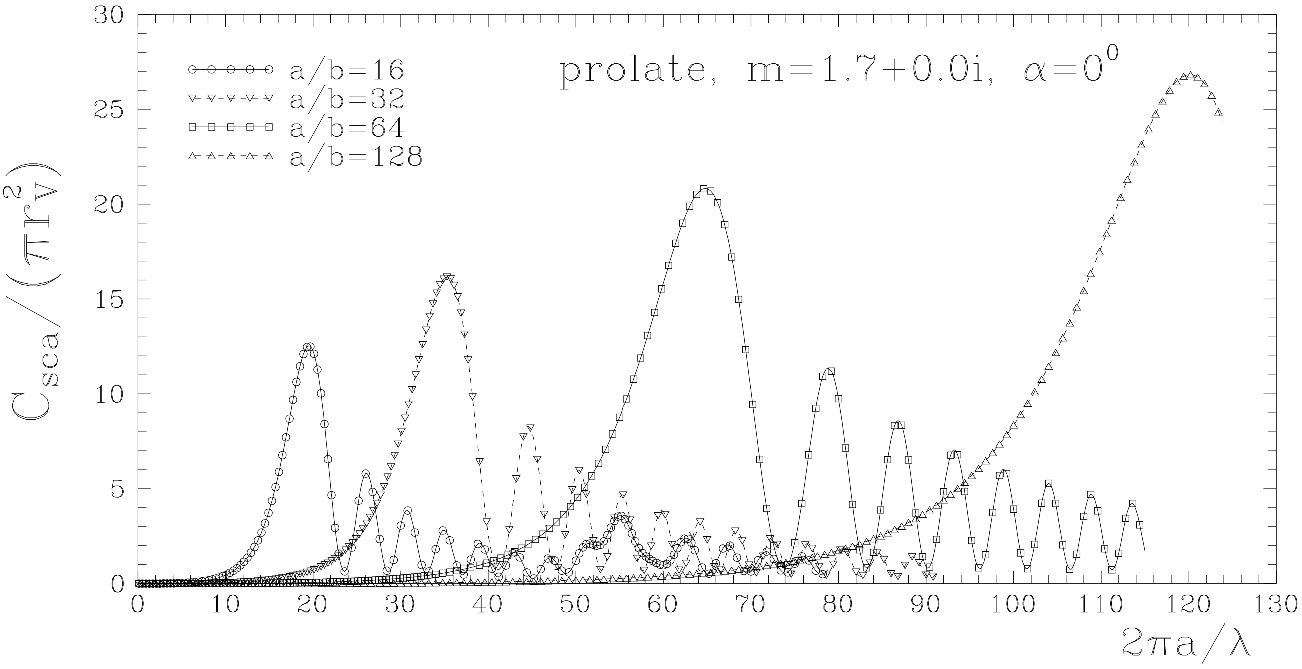

Figure 1: Normalized scattering cross-sections for elongated spheroidal particles in dependence on size parameter $2\pi a/\lambda$. Transformation to the parameter $x_{\rm V}$ is given by the expression $x_{\rm V} = (a/b)^{-2/3}(2\pi a/\lambda)$

$$g_{\perp} = \frac{\mathcal{A}\,(m, x_{\rm V}, a/b, \alpha)\sin\alpha - \mathcal{B}\,(m, x_{\rm V}, a/b, \alpha)\cos\alpha}{Q_{\rm sca}\,(m, x_{\rm V}, a/b, \alpha)}\,, \tag{7}$$

where $Q_{\rm sca}$ is the scattering efficiency factor, and the coefficients $\mathcal{A}$ and $\mathcal{B}$ are

$$\mathcal{A} = \mathcal{K}\,(x_{\rm V}, a/b, \alpha)\int_0^{2\pi}\int_0^{\pi} i(\theta,\varphi)\cos\theta\sin\theta{\rm d}\theta{\rm d}\varphi, \tag{8}$$

$$\mathcal{B} = \mathcal{K}\,(x_{\rm V}, a/b, \alpha)\int_0^{2\pi}\int_0^{\pi} i(\theta,\varphi)\sin^2\theta\cos\varphi{\rm d}\theta{\rm d}\varphi. \tag{9}$$

In Eqs. (8) – (9), $\mathcal{K}$ is a parameter, $i(\theta,\varphi)$ the dimensionless intensity of scattered radiation (phase function). From a symmetry consideration, it is clear that for spheroids $g_{\perp} = 0$ if $\alpha = 0°$ or $90°$. Note that in all cases the following inequality is valid

$$-1 \leq \sqrt{g_{||}^2 + g_{\perp}^2} \leq 1\,. \tag{10}$$

The dichroic polarization efficiency is defined by the extinction cross-sections for TM and TE modes

$$\frac{P}{\tau} = \frac{C_{ext}^{\rm TM} - C_{ext}^{\rm TE}}{C_{ext}^{\rm TM} + C_{ext}^{\rm TE}}\cdot 100\%\,. \tag{11}$$

This ratio describes the efficiency to polarize light transmitted through an uniform slab consisting of non-rotating particles of the same orientation.

The optical properties of spheroidal particles can be determined by various methods of light scattering theory (see Mishchenko et al., 2000 for a review). We use the SVM's solution developed by Farafonov and numerical code based on it (see Voshchinnikov and Farafonov, 1993 for more details). A comparison of different numerical codes and benchmark results can be found in the paper of Voshchinnikov et al. (2000).

The main problem of the SVM for spheroids is the difficulties with computations of the spheroidal wavefunctions. Especially it is related to very elongated particles with sizes larger than the wavelength because the standard expansions of the prolate spheroidal wavefunctions in series of the Legendre functions do not converge. Farafonov and Voshchinnikov (2000, in preparation) have considered new expansion of the prolate wavefunctions that opens a possibility to calculate the optical properties of particles with $a/b \gg 1$. Firstly, this method was applied to calculations of the radial wavefunctions with the index $m = 1$ that allows us to study the case of axial propagation of radiation. Some results are shown in Fig. 1 for non-absorbing spheroids with the refractive index $m = 1.7 + 0.0i$. As the extinction and scattering cross-sections were the same (with 6 and more digits), the values presented are expected to be correct.

3. NUMERICAL RESULTS

We present some results illustrating the behaviour of the optical properties of prolate spheroidal particles in a fixed orientation.

3.1 Albedo

The integral scattering properties of particles are characterized by their albedo. This quantity depends on the particle size and, in general, on the particle shape. Figure 2 shows the size dependence of the albedo for spheroids with $m = 1.3 + 0.05i$ and $m = 1.7 + 0.7i$ and the aspect ratios $a/b = 2$ and 10. The calculations were made for prolate particles and $\alpha = 0°$ (we adopt that the incident radiation is non-polarized). In this case in comparison with others (oblate spheroids, oblique incidence of radiation) the largest deviations of the ratio $\Lambda(\mathrm{spheroid})/\Lambda(\mathrm{sphere})$ from unity occur (see Voshchinnikov et al., 2000 for discussion).

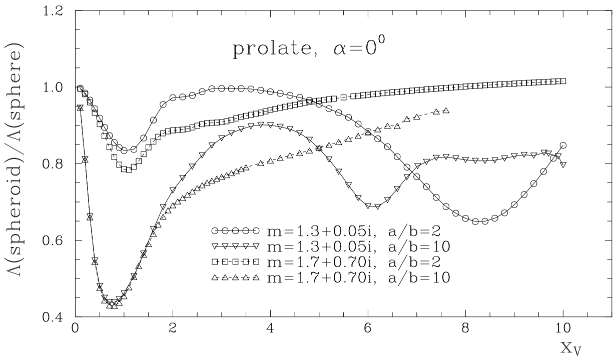

Figure 2: Albedo of spheroidal particles normalized relative to albedo of spherical particles with the same refractive index

It is seen that *the albedo for large non-spherical particles becomes close to that of spheres.* Our calculations made for particles with different absorption show that the distinction of the albedo for spheres and spheroidal particles remains rather small (within $\sim 20\,\%$) if the ratio of the imaginary part of the refractive index to its real part $k/n \gtrsim 0.2 - 0.3$.

3.2 Asymmetry parameter

Another characteristic of scattered radiation is the asymmetry parameter describing the spatial distribution of scattered radiation around a particle. Usually the anisotropy in forward/backward direction is only considered. However, the radiation scattered by any aligned non-spherical particle possesses also an anisotropy in the right/left direction in the case of oblique incidence of radiation (see Fig. 3).

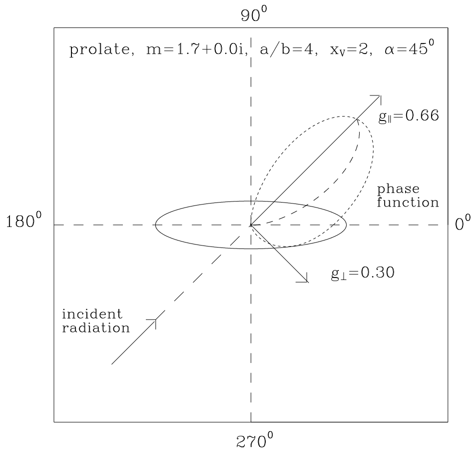

Figure 3: Geometry of light scattering by a prolate spheroid. The wave-vector of incident non-polarized radiation forms the angle $\alpha = 45°$ with the rotation axis of the spheroid. Short-dashed curve shows the angular distribution of the scattered radiation (phase function). The part of scattered radiation which is symmetric relative to the direction of incident radiation is plotted by long-dashed curve. The radial and transversal asymmetry factors and their values are indicated

As it is seen from Fig. 4 both asymmetry factors $g_{||}$ and $g_{\perp}$ change with a/b and α. The values of radial asymmetry factor $g_{||}$ decrease with a growth of α when the path of radiation reduces from $2a$ ($\alpha = 0°$) to $2b$ ($\alpha = 90°$). The transversal asymmetry factor $g_{\perp}$ can be rather large and even exceeds the radial one. Because the geometry of light scattering by very elongated spheroids approaches that of infinite cylinders,[1] *such particles scatter more radiation "to the side" than in forward direction.*

The size dependence of asymmetry factors is plotted in Fig. 5. It shows that the variations of $g_{||}(x_{\rm V})$ for spheroids with $a/b = 2$ are rather similar to those of spheres. However, the radiation scattered by spheroids has a noticeable azimuthal dependence which is absent for spherical particles. If $x_{\rm V} \gtrsim 4$ the azimuthal anisotropy of scattered radiation reduces and $g_{\perp}$ drops.

3.3 Polarization

If a volume contains aligned non-spherical particles, the initially non-polarized incident radiation will be partially polarized after having passed the volume. The simplest and at the same time extreme case of particles' alignment is the perfect alignment of non-rotating particles (picked fence orientation). The maximum polarization usually occurs when the major axes of the particles are perpendicular to the direction of the incident radiation ($\alpha = 90°$).

The behaviour of the polarization efficiency P/τ for non-absorbing and absorbing spheroids is shown in Fig. 6. It is clearly seen that a relatively large particles produce *no polarization independent of their shape*. For absorbing particles, it occurs at smaller $x_{\rm V}$ values than for non-absorbing particles. This effect should depends on the

[1] In this case the scattered radiation forms the conical surface with the opening angle 2α.

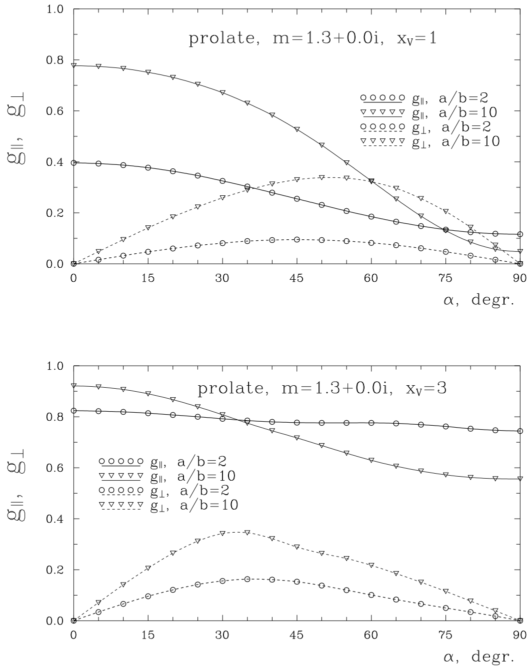

Figure 4: Angular dependence of the asymmetry factors of scattered radiation

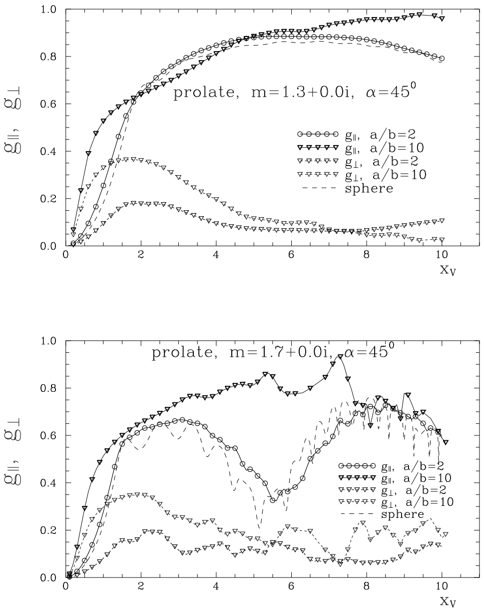

Figure 5: Size dependence of the asymmetry factors of scattered radiation

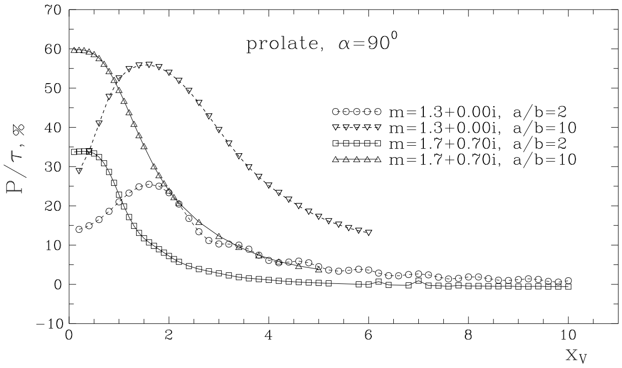

Figure 6: Size dependence of the polarization efficiency

imaginary part of the refractive index as well.

4. CONCLUSIONS

The albedo of large non-spherical particles exhibits only a weak dependence on the particle shape if the ratio of the imaginary part of the refractive index to its real part $k/n \gtrsim 0.2 - 0.3$.

The radiation scattered by aligned spheroidal particles has an azimuthal asymmetry and its geometry may be described by two asymmetry parameters $g_{||}$ and $g_{\perp}$ showing the deviations from the symmetric scattering in forward/backward and left/right directions, respectively. The transversal asymmetry factor $g_{\perp}$ can be rather large and even exceeds the radial one, therefore, very elongated spheroids scatter more radiation "to the side" than in forward direction.

Particles larger than a certain minimum size do not polarize the transmitted radiation independent of their shape.

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