Light scattering by multi-layered ellipsoidal particles in the quasistatic approximation

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Abstract. The Rayleigh (RA) and Rayleigh-Gans (RGA) approximations are generalized in the form of the quasistatic approximation (QSA). The advantage of the QSA is most conspicuous for non-spherical scatterers. In that case, the RA, RGA, etc. are known to have the applicability range quickly decreasing with a growing particle asphericity, whereas the range of the QSA remains practically independent of the scatterer shape.

In this paper we develop the QSA for multi-layered ellipsoids in the general case of non-confocal layer surfaces. In principle, the approach used should give a solution for ellipsoidal particles with any internal structure of this kind.

Our numerical results for multi-layered ellipsoids show that the QSA is preferable to the RA (and RGA) if the ratio of the maximum to minimum dimensions of the outer (inner) boundary of the layer dominating the scattering process exceeds ∼3, which well coincides with the conclusion drawn earlier for homogeneous spheroids.

We also introduce a special rule of the effective medium theory giving much more accurate results than the known ones for small layered ellipsoids.

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1. Introduction

Studies of small inhomogeneous non-spherical natural particles by optical methods are in growing demand in astrophysics, geophysics, biophysics, etc. Among possible analytical models of these scatterers, that of multi-layered ellipsoidal particles looks to be most universal and reasonable. For instance, ellipsoids well describe the shape of plant and animal cells, bacteria, spores or microalgae, and so on.

A large number of (more or less) exact methods of treating the light scattering by non-spherical inhomogeneous particles have been developed. They give good results in restricted ranges of parameter values and all are very time-consuming computationally. Therefore, a significant role in this area of the light scattering theory is still played by approximate methods, and in particular by approximations developed for particles whose size is small in comparison with the wavelength of incident radiation (see [1, 2, 3] for comprehensive reviews).

The most famous approximation of this kind was introduced by Lord Rayleigh in 1897 for the following limitations:

$$x_{\rm d} \ll 1, \qquad |m| x_{\rm d} \ll 1, \tag{1}$$

where $x_{\rm d} = 2\pi l_{\rm max}/\lambda$, $l_{\rm max}$ is the maximum dimension of a particle, λ the wavelength of incident radiation, m the complex refractive index. In this case an equivalent electrostatic problem is solved. For ellipsoids, one considers the fields parallel to the particle axes and constructs the internal field using the polarizability of the particle [1]. Thus, in the Rayleigh approximation (RA) an anisotropy of the particle shape is taken into account in the same way as a possible anisotropy of the refractive index. For core-mantle ellipsoids, the RA is well known (see, e.g., [1]); for ellipsoids with 3 and more layers, it was recently elaborated by one of the authors [4, 5].

Another useful approach is the Rayleigh-Gans approximation (RGA). It allows a treatment of optically soft particles with small radiation phase variations inside the scatterer

$$|m-1| \ll 1, \qquad |m-1| x_{\rm d} \ll 1. \tag{2}$$

An important assumption here is that each small volume element of the particle is excited only by the incident field [1]. From equations (1)–(2) it is seen that the RGA can be applicable to slightly larger particles than the RA (the constraint on $x_{\rm d}$ is less tight). In the RGA one neglects the polarizability, and the particle shape is totally included in a form-factor. As a result the RGA is not correct in the static limit ($\lambda \to \infty$). Partly to avoid that, Acquista [6] generalized the approximation utilizing the iteration approach of Shifrin [7]. The RGA has been applied to non-spherical layered scatterers, but the range of validity of this method is limited (see [8] and references therein).

It should be noted that the above-mentioned and numerous other approximate methods of the light scattering theory are well developed for homogeneous scatterers, whereas the number of papers considering such methods for inhomogeneous non-spherical particles is rather limited. The only exception is the effective medium theory. This approximate theory substitutes an inhomogeneous particle by a homogeneous one with an average refractive index, but keeps the overall shape of the particle. This strongly simplifies and speeds up calculations, but usually gives results of low or limited accuracy for particles of complex structure, even if they are less than or comparable to the wavelength of incident radiation.

In this paper we develop a quasistatic approximation for the case of multi-layered ellipsoids. General information about the approximation is given in section 2. A description of the techniques used and basic equations are presented in section 3, numerical results are discussed in section 4 and main conclusions are drawn in section 5.

2. Quasistatic approximation

Besides Acquista's approach, another way to generalize the RGA (and RA) was implemented as the quasistatic approximation (QSA). This term is not commonly accepted in the literature and other authors may call it differently. We choose to use this term as it well reflects the nature of the approach. In the QSA the internal field is represented by the incident plane wave (like in the RGA) taking into account the polarizability of the particle (like in the RA).

Apparently, the QSA was first developed for thin cylinders [9] and, independently, for spheroids [10]. Up to now the approximation was utilized rather seldom; there were applications only to single spheroids and their ensembles [11, 12, 13]. The QSA has never been considered for inhomogeneous scatterers.

Obviously the QSA works in particular well for small optically soft particles, in which case it is always preferable to the RGA. Acquista's approach is clearly more complicated than the QSA, and the range of its applicability still needs a special consideration. In contrast, the QSA has been investigated in detail, at least in the case of spheroidal particles [13, 14]. It was demonstrated [13] that the QSA represented the main term of the asymptotic of the scattered field under the conditions: $\frac{b}{a} \ll 1$, $|m|x_{\rm d} = {\rm O}(1)$, where b/a is the ratio of the minor semiaxis to the major one, $x_{\rm d} = 2\pi a/\lambda$ (see also [15]). A consideration of the applicability range of the QSA [14] has shown that it is nearly always larger than that of the RA for $a/b \geq 3$.

Another conclusion from numerical analysis of [14] is that this range expressed in terms of $x_{\rm V} = 2\pi r_{\rm V}/\lambda$ ($r_{\rm V}$ is the radius of a sphere whose volume is equal to that of the spheroid) does not decrease with a/b as it was predicted in [13]. Note that for large a/b the diffraction parameter connected with the linear dimension of a scatterer $x_{\rm d} = 2\pi a/\lambda$ grows as $\sim (a/b)^{2/3}$ for prolate spheroids and as $\sim (a/b)^{1/3}$ for oblate ones. Therefore, when, for example, $x_{\rm V} = 0.01$ and the QSA gives good results, for very large a/b the parameter $x_{\rm d}$ can well exceed 10 and the RA, other similar approximations and even many exact methods will not work at all.

To summarize, it is important to note that the condition of applicability of the quasistatic approximation can be formulated as follows:

$$|m - 1|x_{\rm V} \ll 1. \tag{3}$$

It essentially differs from the conditions of the RA and RGA (see equations (1) and (2), respectively) already in the case of particles with an intermediate asphericity.

3. Description of the approach

We first describe the idea of the approach used to construct the QSA for multi-layered ellipsoids and then give main formulae.

3.1. A brief summary of the procedure

We apply a standard method to obtain a solution to the light scattering problem for scatterers small in comparison with the wavelength of incident radiation (actually satisfying the conditions: $x_{\rm d} \ll 1$, $|m|x_{\rm d} \ll 1$). The method is based on the replacement of such a scatterer by an ideal dipole with the polarizability determined from the electrostatic theory (see [1] for more details). In this theory the incident, internal and scattered fields are connected with corresponding scalar potentials Φ. To find the polarizability, we solve the basic equation of the theory, the Laplace equation, for the potentials along with boundary conditions.

As all layer boundaries are ellipsoidal, the ellipsoidal coordinate systems can be used. Two independent solutions of the Laplace equation $\Phi_{\rm A}, \Phi_{\rm B}$ in such a coordinate system are known in the explicit form. Each of the solutions includes a constant that is determined from boundary conditions.

When the boundaries are confocal, one ellipsoidal system can be introduced for the whole multi-layered ellipsoid. The potentials in the innermost layer (core) and outside the scatterer are similar to those used for homogeneous scatterers. In other layers, superpositions of the potentials $\Phi_{\rm A}$ and $\Phi_{\rm B}$ should be applied.

For non-confocal boundaries, one cannot use the same system even within a layer as its boundaries can be the coordinate surfaces only in different systems. Therefore, each layer is divided into several sublayers with the boundaries stepwisely changing from the inner boundary of the layer to the outer one. An own ellipsoidal system is introduced for each sublayer and a superposition of the potentials $\Phi_{\rm A}$ and $\Phi_{\rm B}$ is used inside it. The coefficients of the superpositions are determined from approximate boundary conditions at the sublayer boundaries.

If the scalar potentials are determined, one can find the polarizability of the particle. In the case of multi-layered ellipsoids it is obtained from a product of several matrices (one matrix for each sublayer). The polarizability tensor allows one to calculate the optical properties of multi-layered scatterers using the standard formulae for homogeneous scatterers.

3.2. Basic equations

In the case of non-confocal layer boundaries of an m-layered particle, an own ellipsoidal coordinate system (ξ, η, ζ) is used for each of them. For instance, the system for the inner boundary of the i-th layer ($i = 2, 3, ..., m$) is connected with the Cartesian coordinate system (x, y, z) as follows:

$$\frac{x^2}{a_{i-1}^2 + \xi_{i-1}} + \frac{y^2}{b_{i-1}^2 + \xi_{i-1}} + \frac{z^2}{c_{i-1}^2 + \xi_{i-1}} = 1, \quad -c_{i-1}^2 < \xi_{i-1} < \infty; \tag{4}$$

$$\frac{x^2}{a_{i-1}^2 + \eta_{i-1}} + \frac{y^2}{b_{i-1}^2 + \eta_{i-1}} + \frac{z^2}{c_{i-1}^2 + \eta_{i-1}} = 1, \quad -b_{i-1}^2 < \eta_{i-1} < -c_{i-1}^2; \tag{5}$$

$$\frac{x^2}{a_{i-1}^2 + \zeta_{i-1}} + \frac{y^2}{b_{i-1}^2 + \zeta_{i-1}} + \frac{z^2}{c_{i-1}^2 + \zeta_{i-1}} = 1, \quad -a_{i-1}^2 < \zeta_{i-1} < -b_{i-1}^2; \tag{6}$$

where a_{i-1}, b_{i-1}, c_{i-1} are the semiaxes of the boundary surface. For the outer boundary of the layer, one should replace $i-1$ with i ($a_{i-1} < a_i$, $b_{i-1} < b_i$, $c_{i-1} < c_i$). The equations of the boundaries look similar: $\xi_{i-1} = 0$ and $\xi_i = 0$, but are given in different coordinate systems because of the non-confocality of the boundary surfaces.

Let us introduce the scalar potentials: Φ_0 for the incident field, Φ_1 for the field in the core, Φ_{m+1} for the scattered field, and $\tilde{\Phi}_i^{(1)}$,$\tilde{\Phi}_i^{(2)}$, two superpositions of the potentials $\Phi_{\rm A}$,$\Phi_{\rm B}$ for the field inside the i-th layer ($i = 2, 3, ..., m$). All the potentials and the superpositions are solutions of the Laplace equation

$$\triangle \Phi = 0. \tag{7}$$

Two superpositions in each layer are used because of the difference of the ellipsoidal coordinate systems connected with the inner and outer boundaries of the layer in the general, non-confocal case.

Each potential includes a constant that can be found from the conditions of continuity of the potentials and their tangential derivatives at the inner and outer boundaries of the layer. The conditions for the i-th layer ($i = 2, 3, ..., m$) are

$$\tilde{\Phi}_{i-1}^{(2)} = \tilde{\Phi}_i^{(1)}, \qquad \varepsilon_{i-1} \frac{\partial \tilde{\Phi}_{i-1}^{(2)}}{\partial \xi_{i-1}} = \varepsilon_i \frac{\partial \tilde{\Phi}_i^{(1)}}{\partial \xi_{i-1}}, \qquad \xi_{i-1} = 0; \tag{8}$$

$$\tilde{\Phi}_i^{(2)} = \tilde{\Phi}_{i+1}^{(1)}, \qquad \varepsilon_i \frac{\partial \tilde{\Phi}_i^{(2)}}{\partial \xi_i} = \varepsilon_{i+1} \frac{\partial \tilde{\Phi}_{i+1}^{(1)}}{\partial \xi_i}, \qquad \xi_i = 0; \tag{9}$$

where ε_i is the permittivity in the layer, and to simplify the formulae, the following notations are used: $\tilde{\Phi}_1^{(2)} = \Phi_1$, $\tilde{\Phi}_{m+1}^{(1)} = \Phi_0 + \Phi_{m+1}$.

For non-confocal ellipsoids, we split each layer into $2n$ sublayers (numbered with the index j) with n sublayers located between the inner boundary and the middle of the layer, and n sublayers between the middle and the outer boundary. The surfaces of the sublayers have the semiaxes

$$a_{i,j}^2 = a_i^2 + \frac{j-1}{2n} \triangle a_i^2, \qquad a_{i+1,j}^2 = a_{i+1}^2 - \frac{j-1}{2n} \triangle a_i^2, \qquad \triangle a_i^2 = a_{i+1}^2 - a_i^2, \tag{10}$$

and in the same way one can get other semiaxes $b_{i,j}$, $b_{i+1,j}$, $c_{i,j}$, $c_{i+1,j}$, where $i = 1, 2, ..., m-1$; $j = 1, 2, ..., n+1$.

Let us consider, for example, the external electric field parallel to the z-axis. The conditions of continuity at the sublayer boundaries are:
a) between the bottom and the middle of the layer (at $\xi_{i-1,j+1} = 0$ and $\xi_{i-1,j} = \frac{1}{2n}\triangle c_i^2, j = 1, 2, ..., n-1$)

$$\tilde{\Phi}^{(1)}_{i,j+1} = \tilde{\Phi}^{(1)}_{i,j}, \tag{11}$$

$$\frac{\partial\tilde{\Phi}^{(1)}_{i,j+1}}{\partial\xi_{i-1,j+1}} = \frac{\partial\tilde{\Phi}^{(1)}_{i,j}}{\partial\xi_{i-1,j}}, \tag{12}$$

b) in the middle of the layer (at $\xi_{i-1,n} = \frac{1}{2n}\triangle c_i^2$ and $\xi_{i,n} = -\frac{1}{2n}\triangle c_i^2$)

$$\tilde{\Phi}^{(2)}_{i,n} = \tilde{\Phi}^{(1)}_{i,n}, \tag{13}$$

$$\frac{\partial\tilde{\Phi}^{(2)}_{i,n}}{\partial\xi_{i,n}} = \frac{\partial\tilde{\Phi}^{(1)}_{i,n}}{\partial\xi_{i-1,n}}, \tag{14}$$

c) between the middle and the top of the layer (at $\xi_{i,j+1} = 0$ and $\xi_{i,j} = -\frac{1}{2n}\triangle c_i^2, j = 1, 2, ..., n-1$)

$$\tilde{\Phi}^{(2)}_{i,j+1} = \tilde{\Phi}^{(2)}_{i,j}, \tag{15}$$

$$\frac{\partial\tilde{\Phi}^{(2)}_{i,j+1}}{\partial\xi_{i,j+1}} = \frac{\partial\tilde{\Phi}^{(2)}_{i,j}}{\partial\xi_{i,j}}, \tag{16}$$

where $i = 1, 2, ..., m+1$. These conditions are approximate as the potential superpositions on the left and right sides of the equations are considered at surfaces that are close but do not coincide because of the difference of the ellipsoidal coordinate systems with the indices j and $j \pm 1$.

Both independent solutions of the Laplace equation in the ellipsoidal coordinate system have a well-known analytical form (see, e.g., [1]). Their substitution into the superpositions and then into the boundary conditions (11)–(16) leads to a system of algebraic equations for constants involved in the potentials. The solution of the system for the constant in the potential of the scattered field can be expressed using two quantities A_1 and A_2 which are as follows:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 1 & L^z_m \\ \epsilon_m & \epsilon_m(L^z_m - 1) \end{pmatrix} \cdot \Psi \cdot \begin{pmatrix} (\epsilon_1 - 1)L^z_1 + 1 \\ -(\epsilon_1 - 1)\delta_1 \end{pmatrix}, \tag{17}$$

where $\epsilon_i = \varepsilon_i/\varepsilon_{i+1}$ is the relative permittivity, L^z_i and δ_i are the geometrical factor and the fraction of volume inside the outer boundary of the i-th layer, respectively. The matrix Ψ is a product of 2×2 matrices Ω_i and Γ_i for each layer ($i = 2, 3, ..., m-1$)

$$\Psi = \prod_{i=2}^{m-1}(\Omega_i\Gamma_i)\Omega_1, \tag{18}$$

and Ω_i is a product of rather simple 2×2 matrices Λ_{ij} each depending only on the parameters of j-th sublayer of the i-th layer (see [5] for more details).

For confocal surfaces [4], $\Lambda_{ij} = 1$ and

$$\Psi = \prod_{i=2}^{m-1} \begin{pmatrix} (\epsilon_i - 1)L_i^z + 1 & \frac{(\epsilon_i-1)}{\epsilon_i} L_i^z (L_i^z - 1) \\ -(\epsilon_i - 1)\delta_i & -(\epsilon_i - 1)(L_i^z - 1) + 1 \end{pmatrix}. \tag{19}$$

Two notes should be made here. First, the equations (17)–(18) can be rewritten in a recursive form. For example, for confocal case instead of (18) one gets

$$\Psi^{(N+1)} = (\Omega_N \Gamma_N) \cdot \Psi^{(N)}, \tag{20}$$

and instead of (17)

$$\begin{pmatrix} A_{1,j}^{N+1} \\ A_{2,j}^{N+1} \end{pmatrix} = \begin{pmatrix} 1 & L_j^{(N+1)} \\ \tilde{\epsilon}_{N+1} & \tilde{\epsilon}_{N+1}(L_j^{(N+1)} - 1) \end{pmatrix} \cdot (\Omega_N \Gamma_N) \cdot \begin{pmatrix} 1 - L_j^{(N)} & \frac{L_j^{(N)}}{\tilde{\epsilon}_N} \\ 1 & \frac{1}{\tilde{\epsilon}_N} \end{pmatrix} \cdot \begin{pmatrix} A_{1,j}^{N} \\ A_{2,j}^{N} \end{pmatrix}, \tag{21}$$

where $\tilde{\epsilon}_{N+1} = \varepsilon_{N+1}$, $\tilde{\epsilon}_N = \varepsilon_N$ (the refractive index outside the particle is assumed to be 1). However, our experience shows that the equations in the explicit form are more convenient for use than those in the recursive one.

Second, the equations (10)–(21) can easily be generalized for an arbitrary internal structure of multi-layered ellipsoidal particles by inclusion of rotation and a shift of the coordinate systems in the sublayers.

Now, for the potential of the scattered radiation defined via A_1, A_2, the polarizability is [4]

$$\alpha_z = \frac{4\pi a_m b_m c_m}{3} \frac{A_2 - A_1}{(A_2 - A_1)L_m^z + A_1}. \tag{22}$$

Other components of the polarizability tensor in the Cartesian system connected with the scatterer can be obtained in a similar way. The determined polarizability allows all optical characteristics of scatterers to be calculated [1, 16].

In this paper we consider cross-sections. It is well known that the absorption cross-section for the QSA is the same as for the RA; but the scattering cross-section differs by the form-factor $G(u)$ [16]. For x- and y-polarized incident plane waves, one gets the following formulae:

$$\begin{aligned} C_{\mathrm{sca,x}} = \frac{k_0^4}{16\pi^2} \int_0^{2\pi} \int_0^{\pi} & \left[|\tilde{\alpha}_{11} \sin\phi - \tilde{\alpha}_{12} \cos\phi|^2 \right. \\ & \left. + |(\tilde{\alpha}_{11} \cos\phi - \tilde{\alpha}_{12} \sin\phi) \cos\theta - \tilde{\alpha}_{13} \sin\theta|^2 \right] G^2(u) \sin\theta d\theta d\phi, \end{aligned} \tag{23}$$

$$\begin{aligned} C_{\mathrm{sca,y}} = \frac{k_0^4}{16\pi^2} \int_0^{2\pi} \int_0^{\pi} & \left[|\tilde{\alpha}_{22} \cos\phi - \tilde{\alpha}_{12} \sin\phi|^2 \right. \\ & \left. + |(\tilde{\alpha}_{22} \sin\phi - \tilde{\alpha}_{12} \cos\phi) \cos\theta - \tilde{\alpha}_{23} \sin\theta|^2 \right] G^2(u) \sin\theta d\theta d\phi, \end{aligned} \tag{24}$$

where ϕ, θ are the spherical angles in the laboratory system, $\tilde{\alpha}$ is the polarizability in this system (its connection with the polarizability α in the particle system is described, e.g., in [16]), $k_0 = 2\pi/\lambda_0$, λ_0 is the wavelength of incident radiation in vacuum, the form-factor $G(u)$ can be expressed as a function of ϕ, θ, k_0 and the semiaxes of the ellipsoid [16].

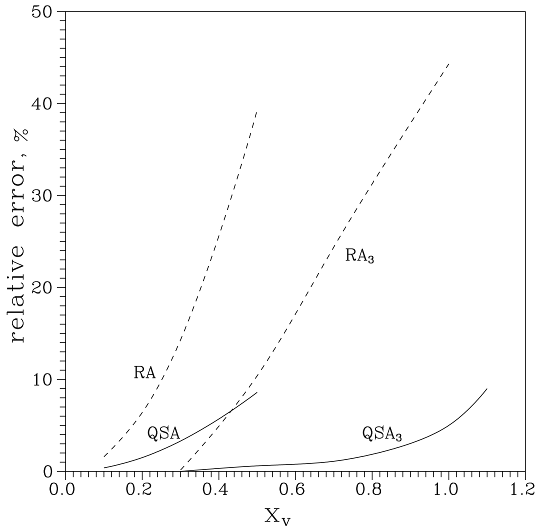

Figure 1. Relative errors of the cross-section $C_{\rm sca}$ calculated with the RA and QSA for a homogeneous ellipsoid with the semiaxis ratio a:b:c=6:2:1 and $m = 1.5$ and for a three-layer ellipsoid (curves denoted by RA_3 and QSA_3) with $(a : b : c, m) = (4{:}2{:}1, 1.33)$ for the mantle, $(6.5{:}3{:}1, 1.7 + 0.3i)$ for the layer, and $(15.8{:}7{:}1, 1)$ for the core; the volume fractions are: $V_{\rm c}/V_{\rm tot}$ =0.22 (core) and $V_{\rm l}/V_{\rm tot}$ =0.34 (layer).

4. Numerical results

We have created a code that realizes the approach and performed calculations of the optical properties of ellipsoidal particles of different shapes and internal structures. The results were compared with those obtained with the code based on the discrete dipole approximation [17] and specially modified by us.

It was found that to treat non-confocal particles with an accuracy of δ one usually needed essentially less than $1/\delta$ sublayers. Therefore the approach is not time-consuming. Another test – application to homogeneous ellipsoids – showed that accuracy of the QSA was higher and the region of its applicability was wider than those of the RA provided any aspect ratio of the particles was larger than about 3 (see figure 1 for an illustration).

For layered particle, the situation is more complicated. Not all layers give comparable contributions to the process of scattering. Nearly always there is a main layer. Our experience indicates that, to a first approximation, it can be identified as a layer with the maximum product $|m-1|\,V$, where m and V are the refractive index and the volume of the layer. It is the shape of the outer boundary of the main layer which is important for the QSA efficiency. If the asphericity of this boundary is intermediate or large (the ratio of the maximum to minimum dimensions exceeds 2–3), the QSA is always preferable to the RA This conclusion is illustrated by figure 2 where results obtained for different core-mantle ellipsoids are presented.

The inner boundary of the main layer also plays a role. It is less important simply because typically its surface area is essentially smaller than that of the outer boundary.

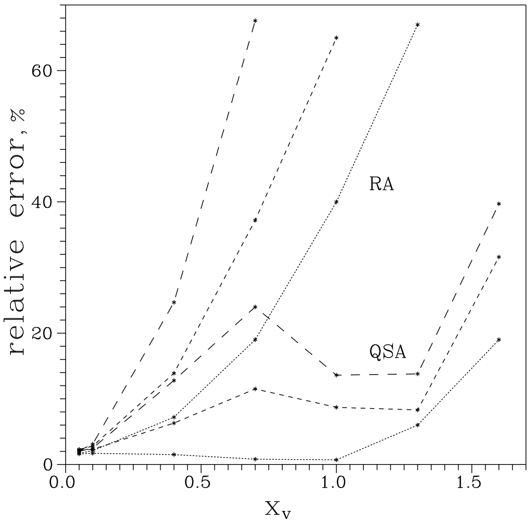

Figure 2. Relative errors of the cross-section $C_{\rm sca}$ calculated with the RA and QSA for core-mantle ellipsoids with different semiaxis ratio a:b:c=4:4:1 (dotted lines), 4:2:1 (short-dashed lines), and 4:1:1 (long-dashed lines). The aspect ratios for the mantle and the core are the same. The refractive indices are: $m_{\rm m}$ =1.3 (mantle) and $m_{\rm c}$ =1.7 (core). The core volume fraction is $V_{\rm c}/V_{\rm tot} = 0.125$.

An empirical factor characterizing the relative importance of the QSA over the RA could look as follows: $l^{\rm o}_{\rm max}/l^{\rm o}_{\rm min} + V^{\rm i}/V^{\rm o} \cdot l^{\rm i}_{\rm max}/l^{\rm i}_{\rm min}$, where $l_{\rm max}$ and $l_{\rm min}$ are the maximum and minimum dimensions of a surface, the superscripts o and i are related to the outer and inner boundaries of a layer, respectively.

An example of importance of the inner boundary is probably provided by the three-layer ellipsoid examined in figure 1. Due to the largest contribution to the volume-averaged refractive index and the phase shift of the central ray (see also [18]) the main layer of this particle is the middle one. Its outer boundary has the semiaxis ratio that is very close to that of the homogeneous ellipsoid also presented in the figure, but the difference between the RA and QSA for the ellipsoids is very large. We believe that it is the effect of significant asphericity (and a relatively large ratio $V^{\rm i}/V^{\rm o}$) of the inner boundary of the main layer of the thee-layer ellipsoid.

At last, it should be emphasized that in equation (22) we can introduce

$$\varepsilon_{\rm eff} = A_2/A_1 \qquad (25)$$

and consider this expression as a special rule of the effective medium theory (EMT) for multi-layered ellipsoidal particles. The EMT is used to replace long (or even impossible) calculations of the optical properties of inhomogeneous particles by calculations for homogeneous ones with an averaged refractive index (see, e.g., [1]). We compared the rule (25) with the standard ones for confocal core-mantle spheroids (for multi-layered spheres it was done in [19]). In this case, we had a possibility to compute the light scattering with high accuracy utilizing our code based on the separation of variables method [20].

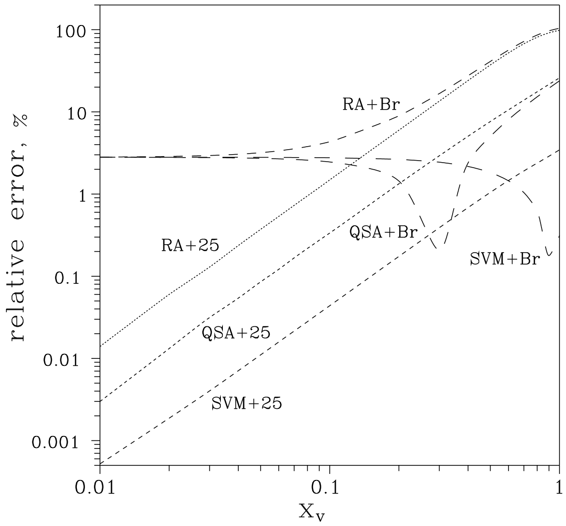

Figure 3. The same as in figure 2, but for core-mantle spheroids with $(a : b, m)$ = (4:1, 1.3 + 0.01i) for the mantle and (6.7:1, 1.7 + 0.03i) for the core. The results were calculated with the code [21] for homogeneous spheroids (SVM) and the approximations (RA and QSA) involving the Bruggeman rule (Br) and the rule (25) and checked using the core-mantle code [20].

It is found that in the region of small diffraction parameter values (x_V <0.1), equation (25) provides a very good approximation, in contrast to other rules of the EMT (cf. curves marked by SVM+25 and SVM+Br in Fig.2). Note that the standard rules of the EMT, like the Bruggeman one used in Fig.2, give a wrong limit at $x_V \to 0$, since, unlike the rule (25), they do not involve information on the internal structure of the scatterer.

5. Conclusions

The quasistatic approximation is extended to the case of multi-layered ellipsoids with practically arbitrary internal structure of this kind. The solution can be formulated in a recursive way.

Numerical results obtained show that the approximation is preferable to the Rayleigh approximation (and the Rayleigh-Gans one) when the boundaries of the layer mostly contributing to the scattering process have intermediate or large asphericity.

A special rule of the effective medium theory that gives much more accurate results than existing rules is introduced for layered ellipsoids.

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