The Field of a Vertical Dipole in the Presence of a Perfectly Conducting Body with the Axial Symmetry

Victor G. Farafonov, Andrey A. Loskutov

The authors are with the St. Petersburg University of Aerocosmic Instrumentation, St. Petersburg, Russia. E-mail: vi2087@vi2087.spb.edu

Abstract

A new solution to the problem of diffraction of the field of a dipole located on the axis of symmetry of a perfectly conducting axially symmetric body is presented. It is assumed that the dipole moment is directed along the axis and therefore both incident and scattered fields do not depend on the azimuthal angle. To solve the problem, scalar potentials connected with the azimuthal components of electromagnetic fields are used. Surface integral equations for these potentials are obtained. The potentials are represented by expansions in terms of spherical wave functions. Infinite systems of linear algebraic equations are given for coefficients of these expansions. Under certain conditions on geometry of the problem the systems have unique solutions which can be found by a reduction method. Numerical calculations have demonstrated high efficiency of the approach.

Keywords

Electromagnetic diffraction.

I. Introduction

The theoretical study of diffraction of electromagnetic waves from dipoles by perfectly conducting bodies (such as prolate or oblate spheroids or finite length circular cylinders) has various applications. Among examples are propagation of radio waves along the surface of the Earth, radiation from antennas, etc. For spheres and spheroids, theoretical simulations were mainly based on separation of variables in the spherical or spheroidal coordinate systems with the use of Debye potentials in the first case [1,2] and Abraham potentials which are natural in description of azimuthal components in the second case [3,4]. Another and more general approach has been based on integral equations [5]. Here an important role is played by the T-matrix method [6] where electromagnetic wave fields are represented as expansions with respect to the spherical wave functions and the expansion coefficients are found from infinite systems of linear algebraic equations.

In the present paper we propose a new solution to the task of the axisymmetric diffraction of electromagnetic waves by a perfectly conducting body of revolution. We assume that the wave field is produced by either an electric or magnetic dipole situated on the axis of revolution of the body and the dipole moment is directed along the axis. The solution is based on scalar potentials associated with the azimuthal components of electric and magnetic fields. These scalar potentials are expanded with respect to the spherical wave functions. A direct substitution of these expansions into the boundary condition on the surface of the body can meet substantial difficulties because convergence, known as the Rayleigh hypothesis (that ensures the consistency of the above-mentioned equations up to the surface), occurs only for bodies which do not differ from spheres too much [7]. For example, for both prolate and oblate spheroids the inequality $a/b \le \sqrt{2}$, where a and b are the semiaxes, must be satisfied. The infinite systems of equations arising in the T-matrix method can be solved in the scalar case by reduction for "weakly non-convex" scatterers [8]. This class of scatterers is wider than that for which the Rayleigh hypothesis holds and includes all smooth convex (or, to be precise, analytical) scatterers. For this reason in the present paper the problem is formulated in the form of surface integral equations for potentials. The infinite systems for the coefficients in expansions of the potentials are analogous to those found for the scalar case in [8]. The approach is used to compute the directing diagrams for prolate and oblate spheroids, finite circular cylinders and Chebyshev particles. The computations are tested against the optical theorem and convergence of the method is thoroughly studied numerically. High efficiency of the method is found.

II. Formulation of the problem

Let $\vec{E}^{(j)}$ and $\vec{H}^{(j)}$ be the strengths of the electric and magnetic fields of the incident ($j = 0$) and scattered ($j = 1$) radiation, μ the magnetic permeability and ε the permittivity of the medium outside the perfectly conducting body, $k = \sqrt{\varepsilon\mu}\, k_0$ the wave number in the medium, $k_0 = \omega/c$ the wave number in vacuum, ω the radiation frequency, and c the velocity of light. Further, we assume that $\varepsilon = 1, \mu = 1$, i.e. $k_1 = k_0$.

The scattering problem can be formulated as follows. The vectors $\vec{E}^{(1)}$ and $\vec{H}^{(1)}$ outside the body satisfy Maxwell's equations

$$\vec{E}^{(1)}(\vec{r}) = -\frac{1}{ik}\vec{\nabla} \times \vec{H}^{(1)}(\vec{r}), \qquad \vec{H}^{(1)}(\vec{r}) = \frac{1}{ik}\vec{\nabla} \times \vec{E}^{(1)}(\vec{r}), \tag{1}$$

the boundary conditions at the surface of the body are

$$\left(\vec{E}^{(0)}(\vec{r}) + \vec{E}^{(1)}(\vec{r})\right) \times \vec{n} = 0, \qquad \vec{r} \in S, \tag{2}$$

and the radiation conditions at infinity

$$\lim_{r\to\infty} r\left(\frac{\partial \vec{E}^{(1)}(\vec{r})}{\partial r} - ik\vec{E}^{(1)}(\vec{r})\right) = 0, \quad \lim_{r\to\infty} r\left(\frac{\partial \vec{H}^{(1)}(\vec{r})}{\partial r} - ik\vec{H}^{(1)}(\vec{r})\right) = 0. \tag{3}$$

Here $\vec{n}$ is an external normal to the surface of the scatterer (S) that occupies the domain $\mathcal{D}$, $\vec{r}$ the radius vector, $r = |\vec{r}|$. The time dependence of the fields is given by the factor $\exp(-i\omega t)$.

We assume also that the scattering body has an axial symmetry and introduce a Cartesian system (x, y, z) with the axis $\vec{z}$ along the symmetry axis of the body. In the spherical polar coordinates (r, θ, φ) the surface S is described by

$$r = r(\theta). \tag{4}$$

We consider the vertical magnetic or electric dipole situated on the rotation axis, i.e.

$$\vec{E}^{(0)}(\vec{r}) = ik\vec{\nabla} \times \left(\vec{i}_z \frac{\exp ik|\vec{r} - \vec{r}_0|}{|\vec{r} - \vec{r}_0|}\right) \tag{5}$$

or

$$\vec{H}^{(0)}(\vec{r}) = ik\vec{\nabla} \times \left(\vec{i}_z \frac{\exp ik|\vec{r} - \vec{r}_0|}{|\vec{r} - \vec{r}_0|}\right), \tag{6}$$

where $(\vec{i}_{\rm x}, \vec{i}_{\rm y}, \vec{i}_{\rm z})$ are the unit vectors of the Cartesian system, $\vec{r_0} = (0, 0, r_0)$ is the dipole position.

The incident electromagnetic radiation does not depend on the azimuthal angle φ (see Eqs. (5)–(6)). The scattered radiation does not depend on the angle φ either because of the commutativity of the operator T corresponding the scattering problem and the operator $L_{\rm z} = \partial/\partial\varphi$. Taking into account the Stratton-Chu formula [5], we represent the operator T in the integral form

$$\begin{aligned}\vec{E}^{(1)}(\vec{r}) = T\vec{E}^{(1)}(\vec{r}) \;&=\; \vec{\nabla} \times \int_S [\vec{n} \times \vec{E}^{(1)}(\vec{r}\,')] G(\vec{r}, \vec{r}\,') dS' - \\ &- \frac{1}{ik}\vec{\nabla} \times \vec{\nabla} \times \int_S [\vec{n} \times \vec{H}^{(1)}(\vec{r}\,')] G(\vec{r}, \vec{r}\,') dS',\end{aligned} \tag{7}$$

where the differential operator $\vec{\nabla}$ acts on the variables $\vec{r}\,'$, and

$$G(\vec{r}, \vec{r}\,') = \frac{\exp ik|\vec{r} - \vec{r}\,'|}{4\pi|\vec{r} - \vec{r}\,'|} \tag{8}$$

is the Green function of the wave equation.

The equality $TL_{\rm z} = L_{\rm z}T$ can be easily proved by integrating the left part by parts over the variable φ' if one keeps in mind the relation (4), no dependence of the metric coefficients of the spherical coordinate system on the azimuthal angle, and the obvious relation $\partial G(\vec{r},\vec{r}')/\partial\varphi' = -\partial G(\vec{r},\vec{r}')/\partial\varphi$. Note that the terms outside the integral which appear upon the integration by parts are equal to zero since all functions in the integral (7) are periodic over φ with the period of 2π.

Commutativity of the operators T and L_z for an axially symmetric body means that the diffraction problem can be solved independently for each term of Fourier series representation of the vectors $\vec{E}$ and $\vec{H}$, i.e. in this case the separation with respect to the variable φ occurs.

III. Construction and solution of the integral equations

We introduce the scalar potentials

$$p^{(j)} = E_\varphi^{(j)} \cos\varphi, \qquad q^{(j)} = H_\varphi^{(j)} \cos\varphi, \tag{9}$$

where $E_\varphi^{(j)}$ and $H_\varphi^{(j)}$ are the azimuthal components of $\vec{E}^{(j)}$ and $\vec{H}^{(j)}$ ($j = 0, 1$). Other components of the fields can be expressed via the azimuthal components

$$H_r = \frac{1}{ikr\sin\theta}\frac{\partial(\sin\theta E_\varphi)}{\partial\theta}, \qquad H_\theta = \frac{-1}{ikr}\frac{\partial(rE_\varphi)}{\partial r},$$

$$E_r = \frac{-1}{ikr\sin\theta}\frac{\partial(\sin\theta H_\varphi)}{\partial\theta}, \qquad E_\theta = \frac{1}{ikr}\frac{\partial(rH_\varphi)}{\partial r}. \tag{10}$$

From Maxwell's equations, it follows that the potentials satisfy the scalar Helmholtz equation

$$\Delta p^{(j)} + k^2 p^{(j)} = 0, \qquad \Delta q^{(j)} + k^2 q^{(j)} = 0. \tag{11}$$

The boundary conditions for the scalar potentials are

$$\left.\begin{array}{l} p^{(0)} + p^{(1)} = 0, \\ \frac{\partial(q^{(0)}+q^{(1)})}{\partial n} + \frac{1}{\sqrt{r^2+r_\theta'^2}}\left(1 - \frac{r'_\theta}{r}\cot\theta\right)\left(q^{(0)}+q^{(1)}\right) = 0, \end{array}\right\}_{\vec{r}\in S} \tag{12}$$

where as before $p^{(j)}$ and $q^{(j)}$ for $j = 0$ and 1 correspond to the incident and scattered fields, respectively; r'_θ is the derivative of the function $r(\theta)$. Note that the problem can be solved for the potentials p and q separately.

Thus, the scattering problem is reduced to solution of scalar Eq. (11) with the boundary conditions (12) and the conditions at infinity

$$\lim_{r\to\infty} r\left(\frac{\partial p^{(1)}(\vec{r})}{\partial r} - ikp^{(1)}(\vec{r})\right) = 0, \quad \lim_{r\to\infty} r\left(\frac{\partial q^{(1)}(\vec{r})}{\partial r} - ikq^{(1)}(\vec{r})\right) = 0. \tag{13}$$

Now we reformulate the problem in terms of integral equations. The scalar potential $p^{(1)}$ for the scattered radiation satisfies the Helmholtz equation (11) outside the scatterer (the domain $R^3 \setminus \bar{\mathcal{D}}$) and the condition (13). Therefore, we can write [5]

$$\int_S \left(p^{(1)}(\vec{r}\,') \frac{\partial G(\vec{r},\vec{r}\,')}{\partial n} - \frac{\partial p^{(1)}(\vec{r}\,')}{\partial n} G(\vec{r},\vec{r}\,') \right) dS' = \begin{cases} 0, & \vec{r} \in \mathcal{D}, \\ \\ p^{(1)}(\vec{r}) & \vec{r} \in R^3 \setminus \bar{\mathcal{D}}. \end{cases} \tag{14}$$

The scalar potential $p^{(0)}$ for the incident radiation satisfies the Helmholtz equation (11) inside the scatterer (the domain $\mathcal{D}$), and one has [5]

$$\int_S \left(p^{(0)}(\vec{r}\,') \frac{\partial G(\vec{r},\vec{r}\,')}{\partial n} - \frac{\partial p^{(0)}(\vec{r}\,')}{\partial n} G(\vec{r},\vec{r}\,') \right) dS' = \begin{cases} -p^{(0)}(\vec{r}), & \vec{r} \in \mathcal{D}, \\ \\ 0, & \vec{r} \in R^3 \setminus \bar{\mathcal{D}}. \end{cases} \tag{15}$$

From Eqs. (14)–(15) keeping in mind the boundary conditions (12) we get the integral equation for the scalar potential of the scattered radiation

$$p^{(1)}(\vec{r}) = -\int_S \frac{\partial \left(p^{(0)}(\vec{r}\,') + p^{(1)}(\vec{r}\,') \right)}{\partial n} G(\vec{r},\vec{r}\,')\, dS', \qquad \vec{r} \in R^3 \setminus \bar{\mathcal{D}}. \tag{16}$$

The equation for the potential $q^{(1)}$ is obtained in a same way as Eq. (16)

$$\begin{aligned} q^{(1)}(\vec{r}) \;=\; & \int_S \Bigg[\left(q^{(0)}(\vec{r}\,') + q^{(1)}(\vec{r}\,') \right) \frac{\partial G(\vec{r},\vec{r}\,')}{\partial n} + \frac{1}{\sqrt{(r')^2 + [(r')'_{\theta'}]^2}} \cdot \\ & \cdot (1 - \frac{(r')'_{\theta'}}{r'} \mathrm{ctg}\theta') \left(q^{(0)}(\vec{r}\,') + q^{(1)}(\vec{r}\,') \right) G(\vec{r},\vec{r}\,') \Bigg] dS', \qquad \vec{r} \in R^3 \setminus \bar{\mathcal{D}}, \end{aligned} \tag{17}$$

We expand both scalar potentials in terms of the spherical wave functions with the index $m = 1$ since their dependence on the azimuthal angle φ is given by the factor $\cos\varphi$ (see Eq. (9)). The potentials $p^{(0)}$ and $q^{(0)}$ for the magnetic dipole (5) are represented as follows:

$$\begin{aligned} p^{(0)} = \frac{1}{2\pi} \left[\int_0^{2\pi} \left(\vec{E}^{(0)}(\vec{r}), \vec{i}_\varphi \right) \mathrm{d}\varphi \right] \cos\varphi = \\ -\frac{k^2}{r_0} \sum_{l=1}^{\infty} (2l+1) j_l(kr_<)\, h_l^{(1)}(kr_>)\; P_l^1(\cos\theta)\; \cos\varphi, \\ q^{(0)} = 0, \end{aligned} \tag{18}$$

where $r_< = \min\{r, r_0\}$, $r_> = \max\{r, r_0\}$, $j_l(kr)$ and $h_l(kr)$ are the spherical Bessel and Hankel function of the first kind, respectively; $P_l^m(\cos\theta)$ is the associated Legendre function. In the case of the electric dipole (6), the equations for the potentials are similar to Eqs. (18) after the replacement of $p^{(0)}$ by $q^{(0)}$ and $q^{(0)}$ by $p^{(0)}$.

The potentials for the scattered radiation are described by the expansions

$$\begin{matrix} p^{(1)} \\ q^{(1)} \end{matrix} = \sum_{l=1}^{\infty} \begin{matrix} a_l^{(1)} \\ b_l^{(1)} \end{matrix} \; h_l^{(1)}(k_1 r) \; P_l^1(\cos\theta) \; \cos\varphi. \tag{19}$$

The Green function is also expanded in terms of the spherical wave functions [9]

$$\begin{aligned} G(\vec{r}, \vec{r}') &= \frac{ik_1}{4\pi} \sum_{m=0}^{\infty} \sum_{l=m}^{\infty} (2 - \delta_{0m})\,(2l+1)\,\frac{(l-m)!}{(l+m)!}\, j_l(k_1 r_<)\, h_l^{(1)}(k_1 r_>)\, P_l^m(\cos\theta) \\ &\quad \times P_l^m(\cos\theta')\; \cos m(\varphi - \varphi'), \end{aligned} \tag{20}$$

where

$$\delta_{0m} = \begin{cases} 1, & m = 0, \\ 0, & m \neq 0, \end{cases} \tag{21}$$

and (r', θ', φ') are the coordinates of the point on the scatterer surface defined by the radius vector $\vec{r}'$.

We substitute the expansions (18)–(20) into Eq. (16). From orthogonality of the wave functions over some spherical surface that is totally outside the scatterer, one can derive the following system of linear algebraic equations:

$$a_l^{(1)} = \sum_{n=1}^{\infty} (\alpha_{ln}^{(1)}\; a_n^{(1)} + \alpha_{ln}^{(0)}\; a_n^{(0)}), \quad l = 1, 2, \dots \;, \tag{22}$$

where

$$\begin{aligned} \alpha_{ln}^{(1)} &= -\frac{i(2l+1)}{2l(l+1)} \int_0^{\pi} \Big(k^2 r^2 h_n^{(1)'}\,(kr)\, P_n^1\,(\cos\theta) + \\ &\quad + k r'_\theta \sin\theta P_n^{1'}\,(\cos\theta)\, h_n^{(1)}\,(kr) \Big)\, j_l\,(kr)\, P_l^1\,(\cos\theta) \sin\theta d\theta. \end{aligned} \tag{23}$$

The expressions for the matrix elements $\alpha_{ln}^{(0)}$ coincide with those for $\alpha_{ln}^{(1)}$ after the replacement of $h_n(kr)$ by $j_n^{(1)}(kr)$. The coefficients of the expansions for the incident radiation are (see Eq. (18))

$$a_l^{(0)} = -\frac{k^2}{r_0}(2l+1) h_l^{(1)}(kr_0). \tag{24}$$

For the electric dipole we get the systems

$$b_l^{(1)} = \sum_{n=1}^{\infty} \left(\beta_{ln}^{(1)}\, b_n^{(1)} + \beta_{ln}^{(0)}\, b_n^{(0)} \right), \quad l = 1, 2, ..., \tag{25}$$

where

$$\begin{aligned} \beta_{ln}^{(1)} \;=\; & \frac{i(2l+1)}{2l(l+1)} \int_0^{\pi} \Big\{ k^2 r^2 h_l^{(1)}(kr)\; j_n'(kr)\, P_l^1(\cos\theta)\, P_n^1(\cos\theta)\; \sin\theta\, + \\ & + k r_\theta' \sin^2\theta P_l^1(\cos\theta)\, P_n^{1'}(\cos\theta)\; h_l^{(1)}(kr)\; j_n(kr) + (kr\sin\theta - \\ & - k r_\theta' \cos\theta)\, h_l^{(1)}(kr)\, j_n(kr)\, P_l^1(\cos\theta)\, P_n^1(\cos\theta) \Big\}\, d\theta, \end{aligned} \tag{26}$$

and

$$b_l^{(0)} = -\frac{k^2}{r_0}(2l+1) h_l^{(1)}(kr_0). \tag{27}$$

Note that the prime means the differentiation with respect to the argument.

The infinite systems can be solved explicitly in the case of a spherical body

$$a_l^{(1)} = -\frac{j_l(ka)}{h_l^{(1)}(ka)} a_l^{(0)}, \quad l = 1, 2, ... \tag{28}$$

for the magnetic dipole and

$$b_l^{(1)} = -\frac{[ka j_l(ka)]'}{\left[ka h_l^{(1)}(ka)\right]'} a_l^{(0)}, \quad l = 1, 2, ... \tag{29}$$

for the electric dipole.

The infinite system (22) coincides with that given in [8] for the scalar case. Thus, the system can be solved by a reduction method under certain conditions (see the next section for more details).

Taking into account Eqs. (9)–(10) and (19), we obtain the expressions for the directional diagram of the scattered radiation ($r \to \infty$)

$$\vec{E^{(1)}} = \frac{\exp ikr}{-ikr} \vec{F(\theta)}, \tag{30}$$

where

$$|\vec{F(\theta)}|^2 = |\sum_{l=1}^{\infty} i^{-l} a_l^{(1)} P_l^1(\cos\theta)\,|^2 \tag{31}$$

for the magnetic dipole and

$$|\vec{F(\theta)}|^2 = |\sum_{l=1}^{\infty} i^{-l} b_l^{(1)} P_l^1(\cos\theta)\,|^2 \tag{32}$$

for the electric dipole.

IV. Numerical results and discussion

We have developed a computer code that calculates the directing diagrams of scattered radiation for axisymmetric perfectly conducting bodies. The code was carefully tested in several ways.

First, in the case of the plane wave incidence on a perfectly conducting scatterer the so-called optical theorem should hold, i.e. the extinction cross-section should be equal to the scattering one [10]. When the scatterer is axially symmetric, the above-mentioned identity should be satisfied for each term of the Fourier expansion with respect to the azimuthal angle φ. In particular it is true for the zero order terms $C_{\text{ext},0}$ and $C_{\text{sca},0}$ which are independent of φ. Then the coefficients in the expansions of the potentials $p^{(0)}, q^{(0)}$ for TE and TM modes are [11]

$$a_l^{(0)} = b_l^{(0)} = -i^l \, \frac{2l+1}{l(l+1)} \, P_l^1(\cos\alpha), \tag{33}$$

where α is the angle between the direction of propagation of the incident wave and the axis of symmetry. The corresponding cross-sections are [10]

$$C_{\text{ext},0} = -\frac{4\pi}{k_1^2} \, \text{Re} \, \left[\sum_{l=1}^{\infty} i^{-l} b_l^{(1)} P_l^1 \left(\cos\alpha\right) \right], \tag{34}$$

$$C_{\text{sca},0} = \frac{4\pi}{k_1^2} \, \sum_{l=1}^{\infty} \frac{l(l+1)}{2l+1} \, |b_l^{(1)}|^2. \tag{35}$$

where b_l are found from Eqs. (25) for TM mode. We tested whether the equation $C_{\text{ext},0} = C_{\text{sca},0}$ was satisfied by the computed wave fields.

Second, convergence of the results of computations should occur with an increase of the number of used terms N (i.e. the infinity limit in the sums over l in (34)–(35) is replaced by N).

We computed $C_{\text{ext},0}$ and $C_{\text{sca},0}$ for spheroids with the semiaxes a and b ($a > b$), circular cylinders with the length D and the radius R and for Chebyshev particles (see Tables 1 and 2). The latters are considered as perturbed spheres whose surfaces are described by

$$r(\theta) = R \left[1 + \epsilon \cos(m\theta)\right], \tag{36}$$

where R is the radius of a unperturbed sphere, ϵ stands for the strength of perturbation and m is the number of maxima.

In the case when a plane wave of TM mode is incident at the angle $\alpha = 90^o$ to the axis of symmetry, we found that the efficiency of computations of the extinction and scattering cross-sections for oblate spheroid with $a/b = 5$ is approximately similar to that for prolate one with the same value of a/b. For $a/b = 2$ the situation is much alike.

It follows from [8,12] that for an analytic scatterer (i.e. when the function $r(\theta)$ in (4) is analytic) singularities of the analytic continuation of the scattered wave field inside the scatterer are described by the equation

$$r(\theta) + isr'(\theta) = 0, \tag{37}$$

where $s = 1$ or -1, θ is considered as a complex variable. Let us denote by σ_1 the distance from the coordinate system origin (that is assumed to be inside the scatterer) to the most distant singularity of the scattered field. If one considers an inner problem for the same analytic scatterer, singularities of the analytic continuation of the wave field outside the scatterer satisfy according to [8] the same equation (37). We denote by σ_2 the distance between the origin and that singularity of the wave field which is less distant from it. If the condition

$$\sigma_2 > \sigma_1 \tag{38}$$

holds (which defines the so-called "weakly non-convex" scatterers), the system of equations (25) can be solved by a reduction method.

For the outer scattering problem, the Rayleigh hypothesis holds under the condition [7]

$$\sigma_1 < \min\{r(\theta)\}, \tag{39}$$

in which case expansion of the scattered wave field with respect to the spherical wave functions converges outside the surface of the scatterer.

For prolate and oblate spheroids, the location of singularities of the scattered wave fields is the segment between the focuses and the focal circle, respectively. From Table 1 it is seen that for the spheroids which are sufficiently aspherical ($a/b = 5$), the sums for the cross-sections (34) and (35) diverge. At the same time the results for $N = 14$ or $N = 16$ look rather reliable. For $a/b = 2$, convergence is observed for sufficiently large values of N.

For finite length cylinders, the agreement of the results of computations with the optical theorem is worse. This may be an effect of non-analyticity of the scatterers (the presence of edges).

Chebyshev particles are non-convex for $\epsilon > 1/(m^2+1)$. We examined the non-rayleigh and weakly non-convex scatterers (see Table 1) as well as the rayleigh and nonconvex scatterers (see Table 2). Note that in the last case ($m = 10$) the condition (39) is not satisfied for $\epsilon > 0.0663$.

For a point source $C_{\mathrm{ext},0} \neq C_{\mathrm{sca},0}$. Here the speed of convergence of $C_{\mathrm{sca},0}$ with N, being a bit slower than for a plane wave, is qualitatively the same. It depends on the distance between the source and the scatterer, and the larger it is, the better the convergence is. Note that the source must be situated outside the smallest sphere centered at the origin and inside which the scatterer lies.

Figs. 1–3 present the directing diagrams of the scattered wave fields for axisymmetric perfectly conducting bodies of different shapes. In all the cases the main maximum is in the direction $\theta \simeq 140$–160°. The diagrams for spheroids and finite circular cylinders with the same length-to-width ratio and the same length multiplied by the wave number are similar though the main maxima for cylinders are higher and a secondary maximum can appear.

Fig. 1 The directing diagrams of the scattered wave fields for prolate spheroids (solid line) and finite circular cylinders (dashed line): a) $ka = 5, a/b = 2, kr_0 = 6$ and $kL/2 = 5, L/D = 2, kr_0 = 6$; b) $ka = 5, a/b = 5, kr_0 = 6$ and $kL/2 = 5, L/D = 5, kr_0 = 6$.

Fig. 2 Same as in Fig. 1 but for oblate spheroids (solid line) and finite circular cylinders (dashed line): a) $ka = 5, a/b = 2, kr_0 = 6$ and $kD/2 = 5, D/L = 2, kr_0 = 6$; b) $ka = 5, a/b = 5, kr_0 = 6$ and $kD/2 = 5, D/L = 5, kr_0 = 6$.

Fig. 3 Same as in Fig. 1 but for spheres and Chebyshev particles: a) $kR = 5, kr_0 = 6$ (solid line), $kR = 5, \epsilon = 0.01, m = 10, kr_0 = 6$ (short-dashed line), $kR = 5, \epsilon = -0.01, m = 10, kr_0 = 6$ (long-dashed line); b) $kR = 5, kr_0 = 6$ (solid line), $kR = 5, \epsilon = 0.05, m = 10, kr_0 = 6$ (short-dashed line), $kR = 5, \epsilon = -0.05, m = 10, kr_0 = 6$ (long-dashed line).

A comparison of the results for spheres and Chebyshev particles shows that for relatively small perturbation of the sphere in the direction of the source ($\epsilon = 0.05, m = 10$) the diagrams suffer a dramatic change. A similar perturbation in the orthogonal direction ($\epsilon = -0.05, m = 10$) results only in arising of a small secondary maximum. At the same time a weaker perturbation ($|\epsilon| = 0.01, m = 10$) provides rather small effect in the diagrams.

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Victor Farafonov was born in Kirov, Russia, on October 11, 1953. He received the M.S. and Ph.D. degrees in mathematical physics from the St.Petersburg State University, Russia in 1976 and 1981, respectively. From 1981 he has been employed as an Assistant Lecturer and a Professor with the St.Petersburg State University of Aerocosmic Instrumentation, Russia, where he is presently the Chairman of Applied Mathematics. His research interests include the scattering and diffraction of electromagnetic waves and the theory of special functions.

TABLE I

THE EXTINCTION CROSS-SECTION $C_{\mathrm{ext},0}$ AND THE SCATTERING CROSS-SECTION $C_{\mathrm{sca},0}$ IN DEPENDENCE ON THE NUMBER OF TERMS USED IN CALCULATION N (PROLATE SPHEROIDS $2\pi a/\lambda = 5, a/b = 5$; FINITE CIRCULAR CYLINDERS $2\pi L/\lambda = 5, L/D = 5$; CHEBYSHEV PARTICLES $2\pi R/\lambda = 5, m = 10, \epsilon = 0.05$).

	prolate spheroids		finite cylinders		Chebyshev particles	
N	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$
6	43.43	43.47	45.6	46.3	20.7	20.5
8	44.803	44.805	44.4	44.6	24.4	24.7
10	44.90626	44.90630	44.20	44.24	24.16	24.02
12	44.911224	44.911225	44.12	44.16	24.96	25.13
14	44.911395	44.911395	44.097	44.110	25.22	25.49
16	44.911400	44.911400	44.099	44.087	25.30	25.35
18	44.91138	44.91140	44.12	44.09	24.94	24.84
20	44.9117	44.9115	44.14	44.11	25.00	25.02
22	44.909	44.911	44.15	44.13	24.85	24.81
24	44.91	44.90	44.01	44.13	24.80	24.74
26	46.3	65.5	44.4	94.1	24.77	24.76
28	-	-	-	-	24.85	24.88
30	-	-	-	-	24.836	24.831
32	-	-	-	-	24.877	24.889
34	-	-	-	-	24.891	24.906
36	-	-	-	-	24.898	24.901
38	-	-	-	-	24.875	24.866
40	-	-	-	-	24.880	24.882

TABLE II

THE EXTINCTION CROSS-SECTION $C_{\mathrm{ext},0}$ AND THE SCATTERING CROSS-SECTION $C_{\mathrm{sca},0}$ IN DEPENDENCE ON THE NUMBER OF TERMS USED IN CALCULATION N (OBLATE SPHEROIDS $2\pi a/\lambda = 5, a/b = 2$; FINITE CIRCULAR CYLINDERS $2\pi D/\lambda = 5, D/L = 2$; CHEBYSHEV PARTICLES $2\pi R/\lambda = 5, m = 10, \epsilon = 0.01$).

	oblate spheroids		finite cylinders		Chebyshev particles	
N	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$	$C_{\mathrm{ext},0}$	$C_{\mathrm{sca},0}$
6	6.353	6.354	5.18	5.63	19.43	19.42
8	6.302783	6.302780	4.94	4.75	19.69	19.70
10	6.3055345	6.3055346	5.20	5.23	19.67	19.66
12	6.3054241	6.3054241	5.224	5.222	19.688	19.689
14	6.3054273	6.3054272	5.233	5.229	19.691	19.693
16	6.3054272	6.3054272	5.245	5.252	19.6971	19.6974
18	6.3054272	6.3054272	5.258	5.261	19.6965	19.6963
20	6.3054272	6.3054272	5.262	5.259	19.69667	19.69673
22	6.3054272	6.3054272	5.265	5.266	19.69654	19.69653
24	6.3054272	6.3054272	5.271	5.273	19.69652	19.69651
26	6.3054272	6.3054272	5.2742	5.2735	19.696462	19.696461
28	6.3054272	6.3054272	5.2747	5.2746	19.696467	19.696471
30	6.3054272	6.3054272	5.2778	5.2790	19.696466	19.696467
32	6.3054272	6.3054272	5.2806	5.2807	19.696467	19.696469
34	6.3054272	6.3054272	5.2808	5.2804	19.696467	19.696469
36	6.3054272	6.3054272	5.2820	5.2826	19.696468	19.696469
38	6.3054272	6.3054272	5.2843	5.2846	19.696468	19.696469
40	6.3054272	6.3054272	5.2848	5.2845	19.696468	19.696469

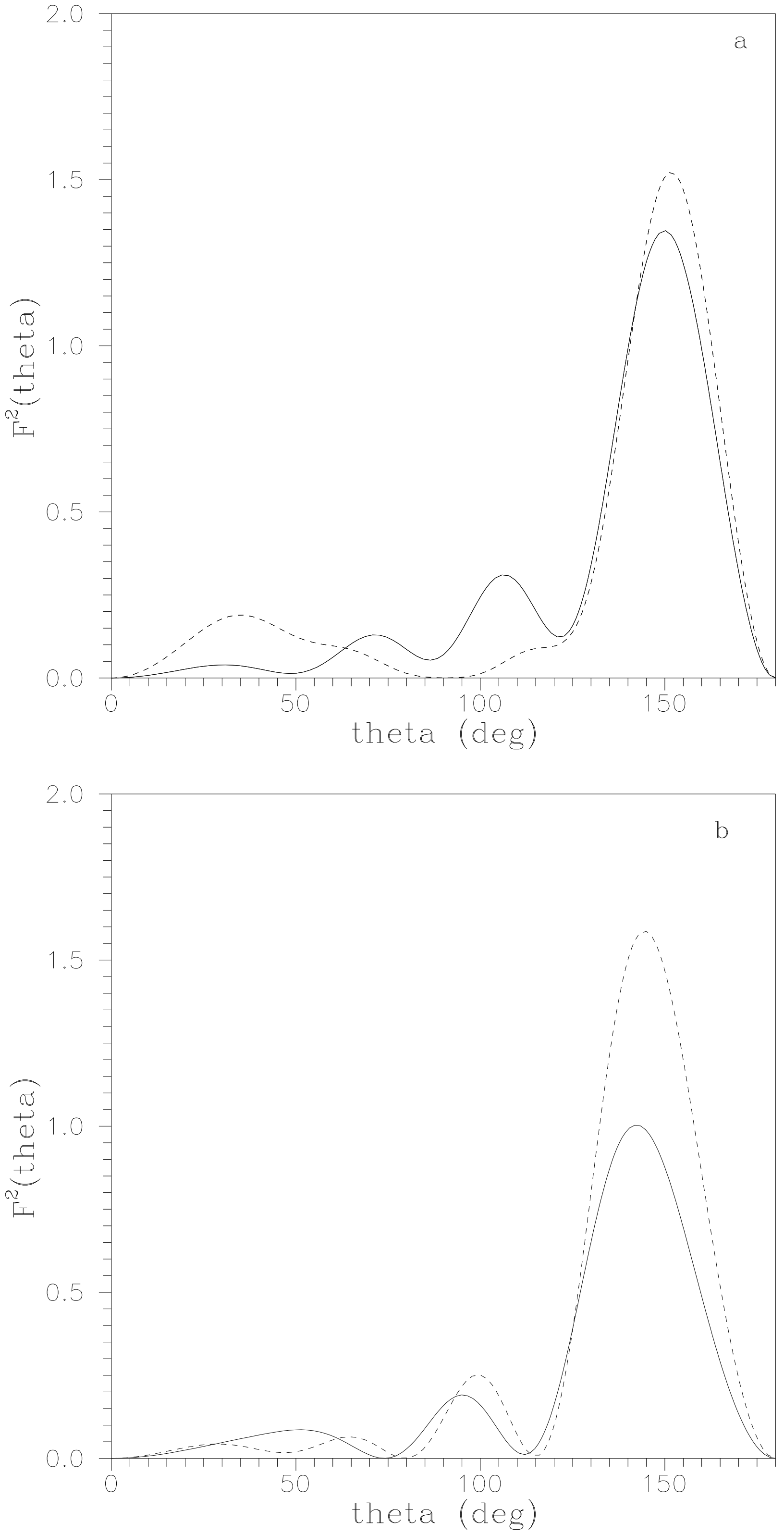

Fig. 1. **a** and **b**

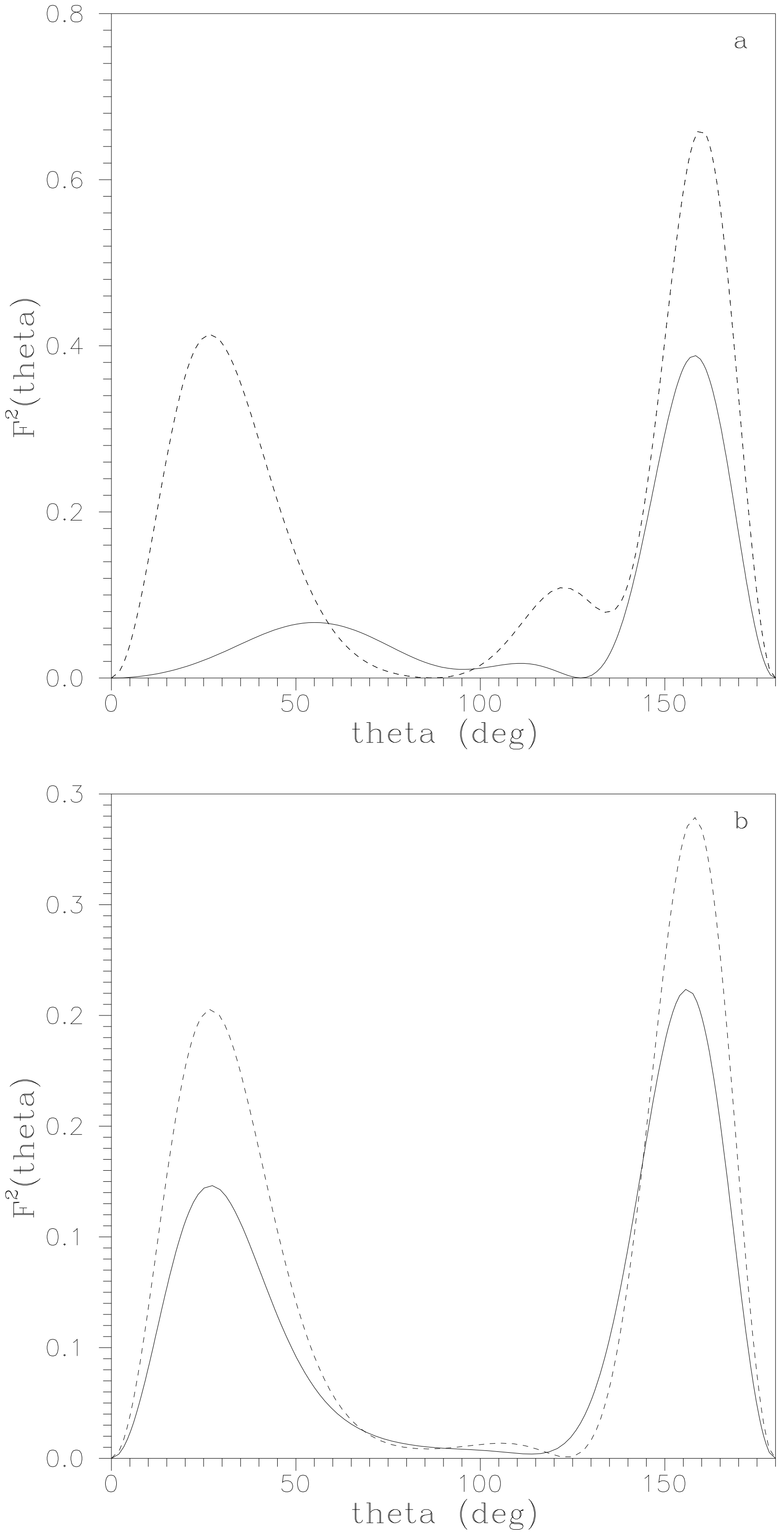
0.8
0.6
0.4
0.2
0.0
F^2(theta)
a
0
50
100
150
theta (deg)
0.3
0.3
0.2
0.2
0.1
0.1
0.0
F^2(theta)
b
0
50
100
150
theta (deg)

Fig. 2. **a** and **b**

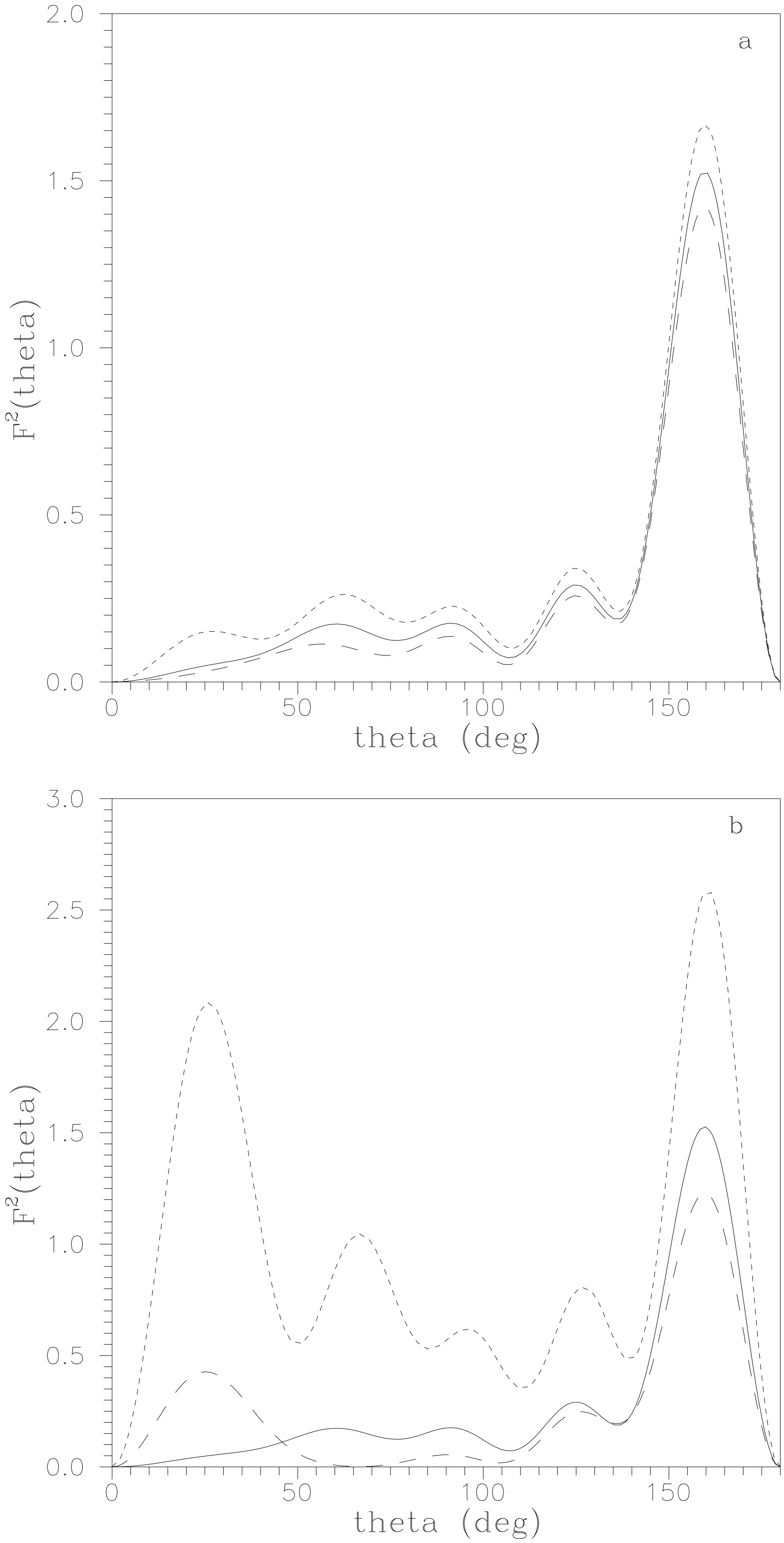

Fig. 3. **a** and **b**