

Evolution of Elemental Abundances in Planetary Nebulae

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Abstract—We study the evolution of elemental abundances in an ensemble of Galactic planetary nebulae as a function of the masses of the central stars (M_{cs}) and their progenitors (M_{ini}). We derive the dependences of the C, N, Ne, Cl, Ar, and S abundances on M_{cs} and M_{ini} for a large sample of nebulae. We calculate the theoretical elemental abundances in nebulae under the assumption of complete mixing of the progenitor's matter ejected at different stages of its evolution. The theoretical dependences of the C and N abundances on M_{ini} have been found to correspond to the observed ones. At the same time, the observed mean O abundance is approximately half its theoretical value. The Ne, Cl, Ar, and S abundances monotonically increase with increasing mass of the progenitor star, which reflects an increase in the mean abundances of heavy elements during the chemical evolution of the Galaxy. We have derived the relation between the abundances of the elements under consideration in planetary nebulae and the masses of their central stars. This relation is used to construct the mass function for the nuclei of planetary nebulae.

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INTRODUCTION

Understanding the evolution of our Galaxy is the key to studying the evolution of other stellar systems. To study the evolution in the whole body of the Galaxy, it is appropriate to choose planetary nebulae (PNe). Since nebulae, as very bright objects, are seen at considerable distances from the Sun, the Galactic PNe known to date occupy a sizeable fraction of the body of the Galaxy (Perek and Kohoutek 1967).

The formation of PNe is one of the evolutionary stages of most Galactic stars, since the progenitors of the nebulae are intermediate-mass stars with main-sequence masses in the range $M \simeq 1M_{\odot}$ to $M \simeq 8M_{\odot}$. PNe exhibit significant differences not only in elemental abundances, but also in spatial distribution, kinematic properties, and, what is particularly interesting for observational studies of the evolution of Galactic stars, in masses of their central stars and the progenitors of the nebulae.

In view of the convenience of using PNe as objects for studying the chemical and dynamical evolution of the Galaxy, a large number of papers are devoted to such studies. However, in most cases, such studies are restricted to comparing the abundances of various

elements for various PN samples (see, e.g., Perinotto et al. 2004; Groenewegen and Marigo 2003; Matteucci 2003; and references therein).

At the same time, analyzing the dependence of elemental abundances in nebulae on the masses of their central stars or the progenitors of the nebulae is of greatest interest in testing the theory of the Galaxy's chemical evolution, in general, and the theory of the evolution of intermediate-mass stars, in particular. Studies of this kind are considerably more complex, because the masses of the progenitors for specific nebulae are known poorly. We can only note the paper by Stasińska et al. (1997), who analyzed the dependence of the CNO abundances on the masses of the central stars of PNe.

Nevertheless, since the masses of the nuclei have been estimated for a large number of PNe and since there is a well-known and distinct relationship between the masses of the PN nucleus and its progenitor (see, e.g. Binney and Merfield 1998; Matteucci 2003), it is quite realistic to study the abundance variations in a nebula as a function of the mass of its progenitor. Such a study is the subject of this paper.

The ensemble of PNe being analyzed is described in the second section of the paper. We give the sources

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of data on the masses of the nuclei and chemical composition of the nebulae. We consider the dependence of elemental abundances on the masses of the central stars and derive and analyze the dependences of the C, N, O, Ne, Ar, Cl, and S abundances on the masses of the PN progenitors. Subsequently, we describe the construction of the mass function for the ensemble of Galactic planetary nebulae. Some conclusions drawn from our study are presented in Conclusions.

THE SYSTEM OF GALACTIC PLANETARY NEBULAE

The results of this paper are based on our analysis of a sample of Galactic planetary nebulae with known chemical composition. A list containing parameters for more than 300 nebulae with the He, C, N, O, Ne, Ar, Cl, and S abundances from various sources and masses of their central stars is accessible at the website of the Astronomical Institute of St. Petersburg State University (Milanova and Kholtygin 2005).

The elemental abundances in the nebulae were taken from the papers the references to which can be found at the above site. In addition, we used data from Costa et al. (1996), Maciel and Köppen (1994), Maciel and Quireza (1999), Perinotto et al. (2004), Stanghellini et al. (1994, 1995), and Escudero et al. (2004).

When different abundance determinations were available for a specific nebula, we used the data obtained by analyzing CCD observations. Only if such observations were unavailable, we used the results of our analysis of photoelectric observations. For several nebulae (NGC 3242, NGC 6720, NGC 7009, NGC 7662, and NGC 7027), there is a large number of abundance determinations for the elements under consideration by different authors. For these nebulae, we used the elemental abundances averaged over different determinations if the abundance difference was within 0.1–0.2 dex. The remaining determinations were rejected.

The elemental abundances can be refined considerably by taking into account the electron temperature fluctuations in a nebula and the significant difference between the accuracies of determining the fluxes

in weak and strong lines compared to $H\beta$ (Kholtygin 1998a, 1998b, 2000). For this reason, we used data from the above papers for the C and O abundances.

The PN central-star masses taken from different sources can differ greatly. We used the combined list of masses from Mal'kov (1997) and Górný et al. (1997) to remove the error due to the inhomogeneity of the sample of PN central-star masses, because the central-star masses for the nebulae present in both lists are close. Table 1 gives the correlation coefficients between the two scales of PN central-star masses.

Analyzing the list from Milanova and Kholtygin (2005), we determined the mean distances from the Galactic plane, the mean central-star masses, and the standard deviations of these parameters as a function of the PN type according to the classification by Peimbert (1978). The results are given in Table 2. Analysis of this table shows that the masses of the PN nuclei decrease with increasing distance from the Galactic plane, which corresponds to the transition from young PN progenitors to older objects.

For comparison, Table 2 gives the mean heights above the Galactic plane from Maciel and Dutra (1992). Comparison of our results with the calculations of these authors shows that the mean distances of PNe of different types from the Galactic plane that we derived closely agree with the data from Maciel and Dutra (1992) within the limits of one standard deviation σ .

THE DEPENDENCE OF ELEMENTAL ABUNDANCES IN NEBULAE ON THE MASSES OF THEIR CENTRAL STARS

During the evolution of an intermediate-mass star, its outer layers are enriched with chemical elements (Iben and Renzini 1983). When a planetary nebula is formed, the matter of the outer stellar layers becomes the nebula's matter. In this case, the elemental abundances in the nebula correspond to the chemical composition of the star at the ejection time of the stellar shells at the post-AGB stage.

The elemental abundances in a post-AGB star are determined mainly by its age and initial chemical composition. Therefore, the chemical composition of a nebula carries information both about the chemical composition of the interstellar medium at the nebula's formation time and about the rate of nuclear reactions in the star's interiors. The mean chemical composition of the interstellar medium depends mainly on the total evolution time of the star, which, in turn, is determined by the mass of the nebula's progenitor M_{ini} .

Thus, we conclude that the elemental abundances in PNe depend mainly on M_{ini} or on the mass of the

Table 1. Correlation coefficients for the masses of PN nuclei

| Scale | LL | LG | GL | GG |
|---|------|------|------|------|
| Correlation coefficient, Górný et al. (1997)/Mal'kov (1997) | 0.80 | 0.79 | 0.79 | 0.79 |

Note. L and G are the masses on linear and logarithmic scales, respectively. The masses of the PN central stars in both lists were taken from Mal'kov (1997).

Table 2. Mean PN distances from the Galactic plane for the types of nebulae according to the classification by Peimbert (1978), $\langle|z|\rangle$, and mean masses of the PN central stars, $\overline{M_{cs}}$

| Type | Data from Maciel and Dutra (1992) | | | This paper | | | | |
|------|-----------------------------------|--------------------------|-----------------|------------|--------------------------|-----------------|---------------------|------------|
| | n | $\langle z \rangle$, pc | σ_z , pc | n | $\langle z \rangle$, pc | σ_z , pc | $\overline{M_{cs}}$ | σ_M |
| I | 53 | 0.15 | 0.13 | 42 | 0.27 | 0.33 | 0.71 | 0.16 |
| IIa | 32 | 0.28 | 0.21 | 37 | 0.43 | 0.32 | 0.62 | 0.03 |
| IIb | 28 | 0.42 | 0.36 | 24 | 0.68 | 0.56 | 0.60 | 0.02 |
| III | 33 | 0.66 | 0.64 | 23 | 1.04 | 0.60 | 0.61 | 0.03 |

Note. n is the number of stars of a given type, σ_z and σ_M are the standard deviations of $\langle|z|\rangle$ and $\overline{M_{cs}}$, respectively.

nebula's central star M_{cs} , because there is a relation between the initial and final stellar masses (see below). A preliminary analysis shows that, in several cases, the dependence of elemental abundances $N(X) = N(X, M_{ini})$ cannot be fitted by any smooth curve because of both the stochastic nature of this dependence and considerable errors in the masses of the PN nuclei and in the chemical composition of the nebulae themselves.

Based on our analysis of the derived dependences, $N(X, M_{ini})$ or $N(X, M_{cs})$, we may suggest that it can be fitted by a piecewise linear continuous function. To construct the sought-for fit, the mass range of progenitors (PN central stars) is divided into several (typically one or two) intervals in such a way that each interval contains at least ten data points. In each of the intervals, the dependences of elemental abundances on the PN central-star mass were fitted by linear functions:

$$N = N(M_{cs}) = N(X, M_{cs}) = a + b \times M_{cs}. \quad (1)$$

Here, a and b are the numerical constants to be determined by least squares from the condition for the squares of the deviations, $\sum_{i=0}^n (N_i^* - N(X, M_{cs}^i))^2$,

where the sum is over all of the available abundance determinations of a given element N_i^* for nebula i , being at a minimum.

As an illustration, Fig. 1 shows our piecewise linear fit to the dependence of the carbon and nitrogen abundances on the central-star mass. To fit the dependence $N(M_{cs})$, the entire mass range of PN nuclei was divided into two intervals with a boundary value of $0.62M_{\odot}$. The steep abundance gradient $dN(X)/dM_{cs}$ for PN central-star masses $M_{cs} < 0.62M_{\odot}$ reflects a sharp rise in the formation rate of low-mass stars at initial evolutionary stages of the Galaxy, in agreement with predictions of most models for the Galaxy's chemical evolution (see, e.g., Matteucci 2003). For oxygen, we failed to obtain a statistically significant dependence of its abundance

on the mass of the PN nucleus for any division of the total mass range of nuclei.

The dependence $N(M_{cs})$ for elements heavier than C, N, and O, the abundances of which change only slightly during the evolution of the nebula's progenitor star, is considerably simpler and can be fitted by one formula (1) for the entire mass range of PN nuclei, as shown in Fig. 2. Table 3 gives the parameters a and b that we derived for C, N, Ne, Ar, and Cl.

Since the currently known elemental abundances in PNe and masses of PN nuclei are subject to considerable errors, of great importance is the question of how significant our empirical dependences $N(M_{cs})$ are. We used the following approach to solve this question. We specified a low significance level, $\alpha \ll 1$. For a given size N of the sample of uncorrelated random variables, we determined such r_0 that the probability $P_N(|r| \geq r_0)$ that the correlation coefficient $|r| \geq |r_0|$ would not exceed α . If the correlation coefficient for the abundances and masses of PN nuclei under consideration exceeds r_0 , then it may be considered statistically significant at the significance level α . The derived correlation coefficients and the corresponding α are also given in Table 3. Our analysis using $P_N(|r| \geq r_0)$ (see, e.g., Taylor 1982) shows that the dependences $N(M_{cs})$ found are significant at a level of $\alpha = 0.05-0.001$ for all of the elements considered, except oxygen and sulfur as well as nitrogen and carbon for low-mass stars.

Because of the large spread in elemental abundances in our PN sample, it is important to clarify the nature of the deviations of the abundances N_i^* of a particular element in nebula i from those obtained using fit (1). To solve this question, we hypothesized that the difference $\varepsilon = N_i^* - N(X, M_{cs})_i$ for each element i was a normally distributed random variable with a zero mean and the same variance σ^2 for all values of M_{cs} . We tested the validity of this hypothesis using the χ^2 test (see, e.g., Tyurin and Makarov 2003). The elements for which the above

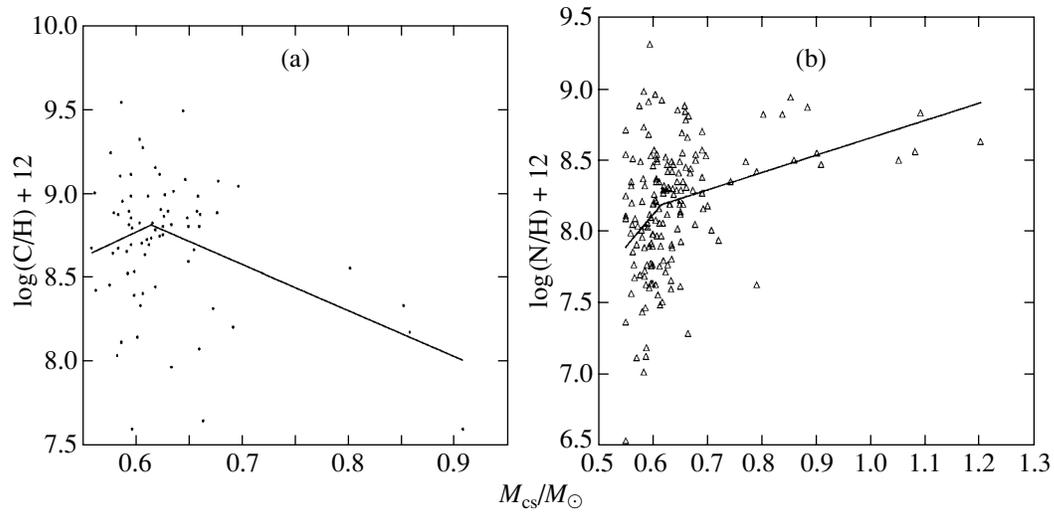


Fig. 1. Fit (1) to the dependence of the carbon and nitrogen abundances on the central-star mass (solid lines). The dots and triangles represent the (a) C and (b) N abundances, respectively, for individual nebulae.

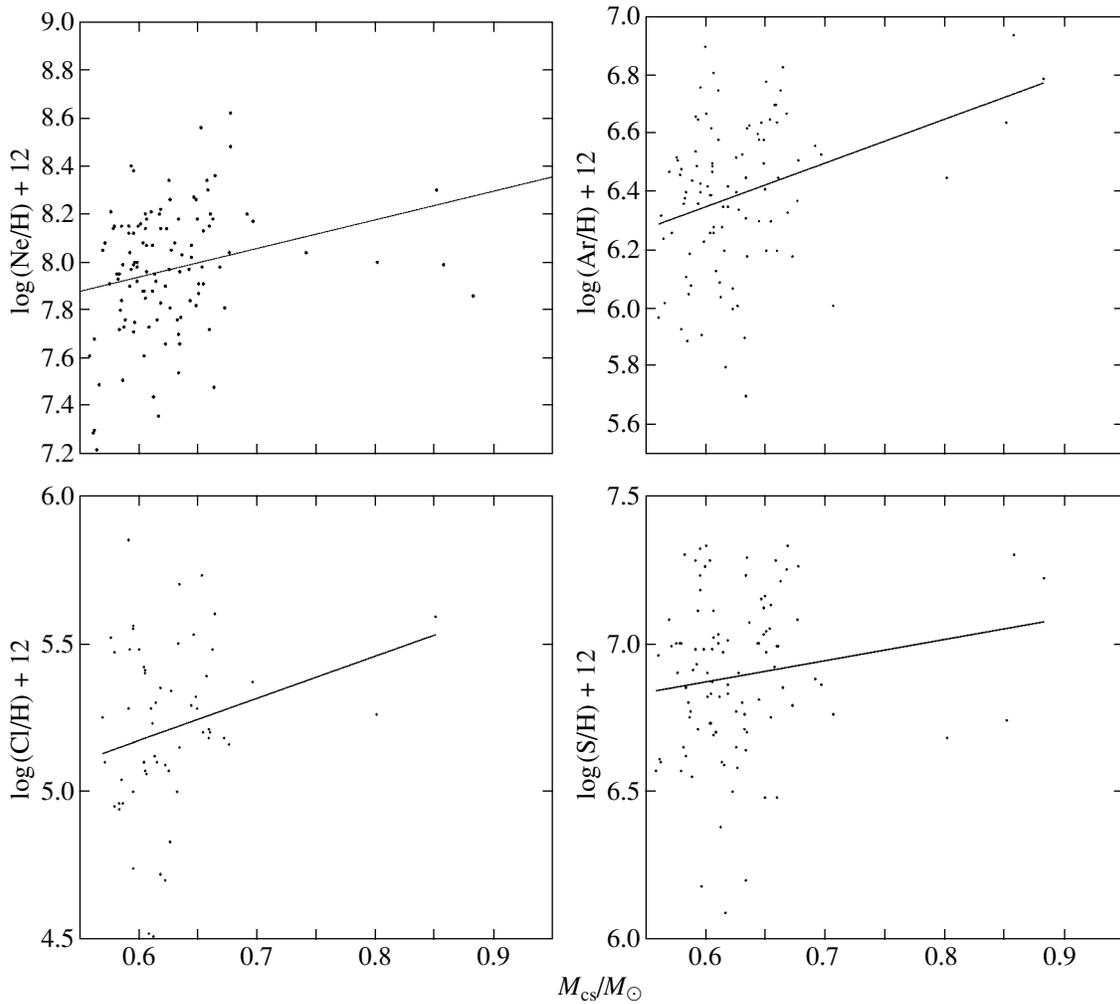


Fig. 2. Fit (1) to the dependence of the Ne, Ar, Cl, and S abundances on the central-star mass (solid lines). The dots represent the abundances for individual nebulae.

hypothesis is valid at a level of $\alpha = 0.01$ are marked in Table 4.

The calculated means $\bar{\varepsilon}$ and variances $\sigma^2(\bar{\varepsilon})$ of the random variable ε for each element are given in Table 4 (columns 3 and 4). The values of $|\bar{\varepsilon}|$ do not exceed 10^{-14} , which confirms the hypothesis that ε is a normally distributed random variable. The mean abundance error does not exceed 0.3 dex and increases to 0.4 dex only for C and N. For O, we found no statistically significant correlation between the nebula's oxygen abundance and the mass of its nucleus.

Formula (1) can be used to estimate the mass of a nebula's central star with a known abundance $N(X)$ of element X :

$$M_{cs} = M_{cs}(N(X)) = \frac{N(X) - a}{b}, \quad (2)$$

where a and b are the constants determined for Eq. (1).

For a group of k nebulae with similar elemental abundances, the mean error in the central-star mass decreases by a factor of \sqrt{k} . This error can be reduced further using the abundances of several elements to estimate the central-star mass M_{cs} . The masses of PN nuclei obtained in this way are unaffected by the errors in the PN distances. Since the abundances of chemical elements can be derived with a relatively high accuracy, the PN central-star masses calculated using Eq. (2) can be used to estimate the PN distances themselves.

EVOLUTION OF THE MEAN ELEMENTAL ABUNDANCES IN AN ENSEMBLE OF PLANETARY NEBULAE

Based on a synthetic model for the evolution of PNe (van den Hoek and Groenewegen 1997; Groenewegen and Marigo 2003), we determined the predicted elemental abundances in PNe by assuming that the matter of the progenitor star ejected at different stages of its evolution was completely mixed in the nebula as well as the elemental abundances in the interstellar medium at the formation time of the nebula's progenitor star. These dependences are shown in Fig. 3 (the dotted and dashed lines, respectively). The predicted abundances are compared with our derived mean C, N, and O abundances for a sample of Galactic PNe with known initial masses of their central stars in four intervals of initial masses—0.9–1.5, 1.5–2.0, 2.0–3.0, and 3.0–6.0 M_{\odot} . For each of the four selected intervals, we obtained the mean C, N, and O abundances in a given interval and the corresponding standard deviations shown in Fig. 3. The derived mean values and standard deviations are referred to the mean masses for each of the intervals of PN central-star masses under consideration.

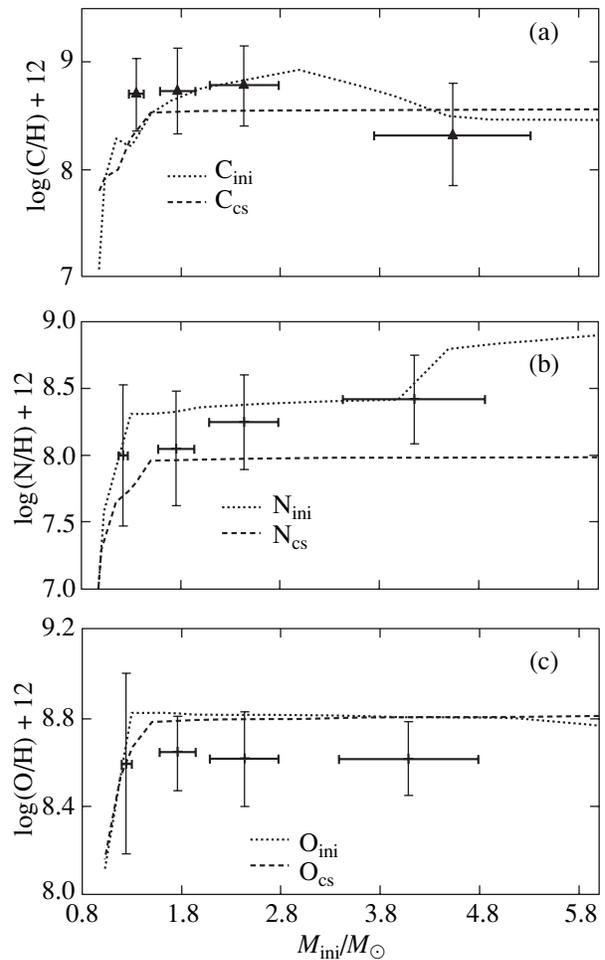


Fig. 3. CNO abundances in PNe vs. masses of the PN progenitor stars: for (a) carbon, (b) nitrogen, and (c) oxygen. The dotted lines represent the initial abundances of the elements under consideration; the dashed lines represent the predicted abundances in the ejected envelope of an AGB star (van den Hoek and Groenewegen 1997). The triangles (for C) and crosses (for N and O) indicate the mean abundances of the corresponding elements for the selected intervals of progenitor-star masses. The error bars are shown for the abundances and masses of the progenitor stars in the intervals under consideration.

We used our empirical initial mass–final mass (the mass of the nebula's central star M_{cs}) relation presented in Fig. 4 to determine the initial masses of PN progenitors. This relation can be fitted by the formula

$$M_{ini}/M_{\odot} = 7.08 + 10.55 \log(M_{cs}/M_{\odot}), \quad (3)$$

where M_{ini} is the initial mass of the nebula's progenitor star and M_{cs} is the mass of the nebula's central star. The following sources were used to derive this relation: Blöcker (1995), B95; Iben (1998), Ib98; Vassilidias and Wood (1993), VW93; and Weidemann (1987), W87.

Of considerable importance is the question of

Table 3. Coefficients a and b derived for dependence (1), the correlation coefficients r for the abundance–mass relation, and the corresponding significance levels

| Element | a | b | Mass range, M_{CS} | r | α | Significance of correlation |
|---------|-------|-------|-----------------------------|-------|----------|-----------------------------|
| C | 6.98 | 2.99 | 0.559–0.615 | –0.02 | 0.93 | – |
| | 10.51 | –2.75 | 0.615–0.908 | –0.52 | 0.001 | + |
| N | 5.38 | 4.56 | 0.55–0.615 | 0.11 | 0.32 | – |
| | 7.43 | 1.22 | 0.615–1.2 | 0.35 | 0.001 | + |
| Ne | 7.22 | 1.19 | 0.55–0.95 | 0.25 | 0.01 | + |
| Cl | 4.32 | 1.42 | 0.55–0.9 | 0.23 | 0.05 | + |
| Ar | 5.45 | 1.51 | 0.55–0.9 | 0.33 | 0.001 | + |

Table 4. Results of using the χ^2 test to test the hypothesis that the random variable has a normal distribution

| Element | Number of objects | $\bar{\varepsilon}$ | $\sigma^2(\bar{\varepsilon})$ | Normality of ε distribution |
|---------|-------------------|------------------------|-------------------------------|---|
| C | 74 | 1.69×10^{-15} | 0.14 | + |
| N | 168 | -7.4×10^{-17} | 0.17 | + |
| O | 166 | – | – | – |
| Ne | 114 | 2.06×10^{-15} | 0.06 | + |
| Cl | 60 | 1.05×10^{-15} | 0.08 | + |
| Ar | 98 | 3.62×10^{-16} | 0.06 | + |

Note. ε are the errors in the abundance of a given element, “+” and “–” mean that the hypothesis is accepted and rejected, respectively.

the mean error in the masses of PN nuclei and the masses of their progenitor stars. To answer this question, we used the following approximation. We assumed that the PN central-star masses determined by Mal’kov (1997) and Górny et al. (1997) were random variables with the same means and a variance $D(M_{\text{CS}})$ dependent on the mass of the PN nucleus. In this case, the nucleus mass difference for the nebula common to both lists will be a random variable with a zero mean and the variance $D(M_{\text{CS}})$.

This approach yields standard deviations $\sigma \approx 0.01$ for nucleus masses $M_{\text{CS}} \approx 0.6M_{\odot}$ and $\sigma \approx 0.04$ for $M_{\text{CS}} \approx 0.8M_{\odot}$. Formula (3) can be used to pass from the errors in the central-star masses to the errors in M_{ini} . The errors in M_{ini} obtained in this way (at a level of one standard deviation) are shown in Fig. 3.

As we see from Fig. 3, to a first approximation, the theoretical dependence of the C and N abundances on M_{CS} match those derived in this paper to within one standard deviation. At the same time, the mean O abundance we found is considerably (a factor of ~ 2)

lower than its theoretical value. As the progenitor-star mass increases from 1 to 6–8 M_{\odot} , the mean C and N abundances increase by a factor of 3–4, while the O abundance changes only slightly.

The Ne and Cl abundances are plotted against the initial progenitor-star masses in Fig. 5. An increase in the abundance with progenitor-star mass is obvious. The Ne and Cl abundances change only slightly during the evolution of intermediate-mass stars (Blöcker 1995; Iben 1998). Thus, the abundances of these elements in a nebula correspond to those at the formation time of the nebula’s progenitor star. This, in turn, implies that the Ne and Cl abundances in a PNe are determined by the composition of the interstellar medium at the formation site of the progenitor star at the time of its formation.

The time elapsed since the formation of the PN progenitor star until the formation of the nebula itself is equal to the sum of the times the star stays at the pre-main-sequence ($\tau_{\text{Pre-MS}}$), main-sequence

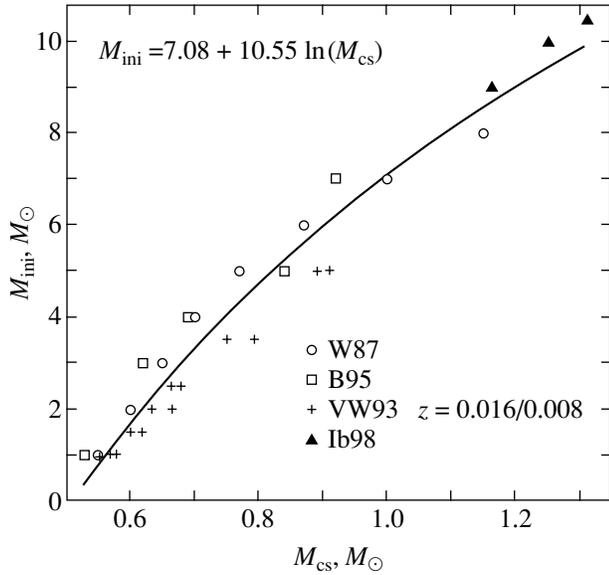


Fig. 4. Final mass–initial mass relation for the central stars of PNe.

(τ_{MS}), red-giant-branch (τ_{RGB}), asymptotic-giant-branch (τ_{AGB}), and post-asymptotic-giant-branch ($\tau_{\text{post-AGB}}$) evolutionary stages:

$$\tau_{\text{tot}} = \tau_{\text{Pre-MS}} + \tau_{\text{MS}} + \tau_{\text{RGB}} + \tau_{\text{AGB}} + \tau_{\text{post-AGB}}. \quad (4)$$

For intermediate-mass stars, their main-sequence lifetimes are considerably longer than the times they stay at other evolutionary stages.

To determine τ_{tot} , we will use its values tabulated in Binney and Merifield (2003). To calculate τ_{tot} at intermediate masses, let us divide the range of initial stellar masses from 0.8 to 9 M_{\odot} into six intervals and represent the dependence on the initial stellar mass in each interval as

$$\tau_{\text{tot}} = p + q/M^2, \quad 10^9 \text{ yr}. \quad (5)$$

The coefficients p and q are given in Table 5.

Substituting the mean stellar masses in the intervals of initial masses under consideration in this formula yields 7.2×10^9 , 5.2×10^9 , 4.2×10^9 , and 2.4×10^9 yr, respectively. Thus, we may conclude that the dependence of the Ne and Cl abundances on the PN progenitor-star masses we found reflects an increase in the mean heavy-element abundances during the chemical evolution of the Galaxy over a period of $\approx 7 \times 10^9$ yr until the current epoch.

THE MASS DISTRIBUTION FUNCTION FOR PN CENTRAL STARS

Based on the determinations of PN central-star masses presented in Milanova and Kholtygin (2005),

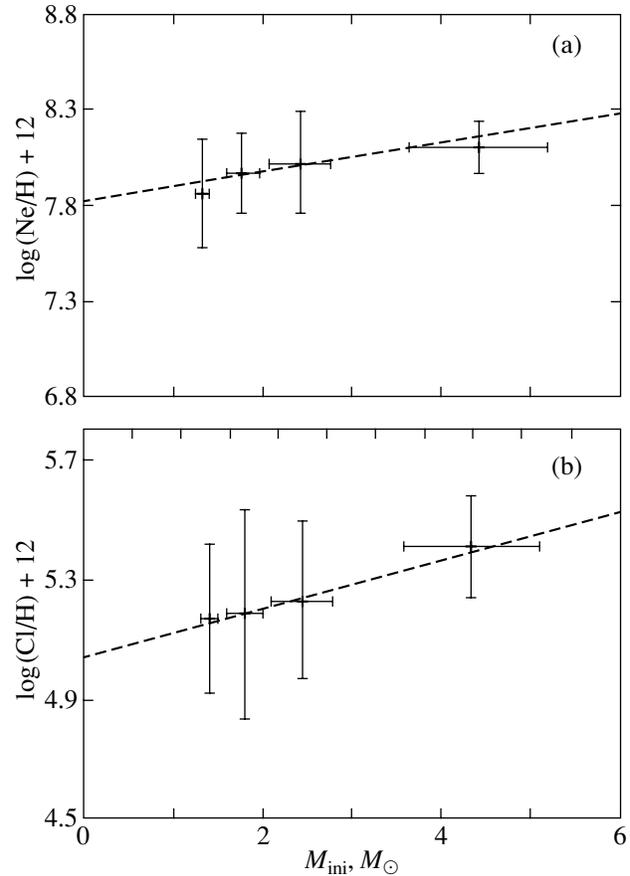


Fig. 5. Elemental abundances vs. central-star masses: for (a) neon and (b) chlorine. The dotted line represents a linear fit to this relation. The crosses mark the mean elemental abundances for the selected mass intervals of progenitor stars. The error bars are indicated for the abundances and masses of the progenitor stars in the selected intervals.

we constructed the PN central-star mass distribution function. Figure 6 presents a histogram of the density of the central-star mass distribution function, $f(M_{\text{cs}})$, in the mass interval 0.55–0.95 M_{\odot} . When constructing the histogram, we divided the entire mass range into 50 uniformly distributed bins and counted the number of nebulae $N(\bar{M}_i)$ with central-

Table 5. Coefficients p and q for age–mass relation (5)

| M_1 | M_2 | p | q |
|-------|-------|--------|-------|
| 0.9 | 1.5 | −4.68 | 17.28 |
| 1.5 | 2.0 | −0.886 | 8.743 |
| 2.0 | 3.0 | −0.32 | 6.48 |
| 3.0 | 4.0 | −0.057 | 4.114 |
| 4.0 | 6.0 | −0.034 | 3.744 |
| 6.0 | 9.0 | −0.002 | 2.592 |

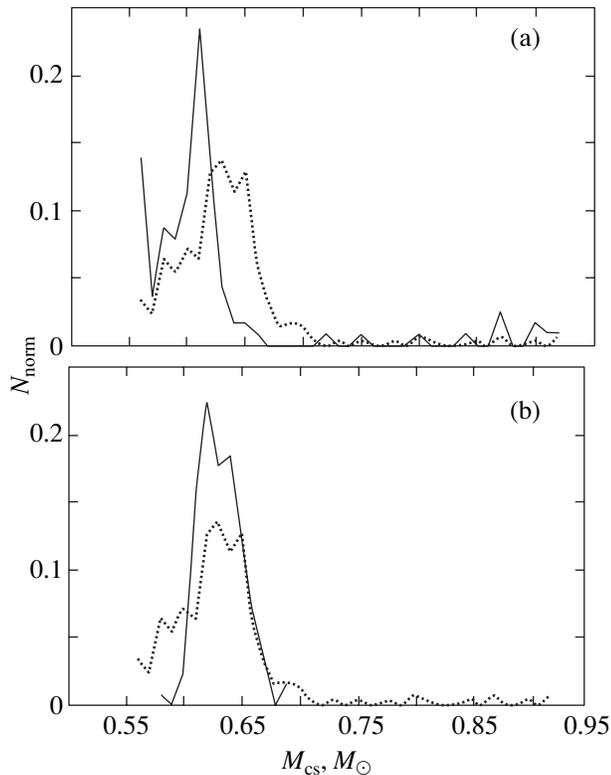


Fig. 6. (a) Comparison of the mass distribution function for PN central stars in the interval $0.55\text{--}0.95M_{\odot}$ derived in this paper (dotted curve) with the data from Stasińska et al. (1997) (solid curve) for nebulae with a shell mass of $0.1M_{\odot}$; (b) the same as panel (a), but the comparison is made with the distribution function of the predicted masses of PN central stars calculated using the formula suggested here (solid curve).

star masses in the bin $[M_i : M_{i+1}]$. The mean mass of the central stars with masses in a given bin was assumed to be $\bar{M}_i = (M_i + M_{i+1})/2$. The normalized mass function was determined from the relation

$$N_{\text{norm}} = N_{\text{norm}}(M_{\text{cs}}) = N(\bar{M}_i)/N_{\text{tot}}, \quad (6)$$

where N_{tot} is the total number of nebulae with known central-star masses.

To increase the statistical significance of the histogram values, we used Eq. (2) to estimate them for nebulae with unknown central-star masses.

Our distribution function for PN central-star masses in the mass interval $0.55\text{--}0.95M_{\odot}$ is compared with that from Stasińska et al. (1997) in Fig. 6a. A significant difference is seen between the density of the distribution function we derived and that found by Stasińska et al. (1997).

First, in contrast to $0.61M_{\odot}$ in Stasińska et al. (1997), our density of the distribution function reaches its maximum at $M_{\text{cs}} \approx 0.63M_{\odot}$. Second, the maximum of our density of the distribution function is considerably broader. The larger size of our PN

sample (by a factor of ~ 3) is most likely responsible for the discrepancy.

Figure 6b compares the mass function derived only from the central-star masses predicted using Eq. (2) with the total mass function. We see from the figure that the predicted mass distribution function is defined only in the mass interval $0.57\text{--}0.7M_{\odot}$. The narrow range of predicted masses is most likely explained by a lack of data on PNe with very high and low metal abundances. The maximum of the distribution function of the predicted PN central-star masses is reached at $M_{\text{cs}} \approx 0.62M_{\odot}$.

CONCLUSIONS

The following conclusions can be drawn from our studies.

(1) There is a statistically significant relation between the C, N, Ne, Cl, and Ar abundances in PNe and their central-star masses.

(2) The dependences of the C and N abundances in PNe on the masses of their progenitor stars agree with current synthetic models for the evolution of intermediate-mass stars. The mean O abundance in an ensemble of Galactic PNe is half its theoretical value.

(3) The empirical PN central-star mass distribution function reaches its maximum at $M_{\text{CS}} \approx 0.62\text{--}0.63M_{\odot}$.

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