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Inhomogeneous planetary nebulae: carbon and oxygen abundances

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Abstract. We reconsider the problem of the difference between the abundances of carbon and oxygen in galactic planetary nebulae (PN) derived from the intensities of the recombination and collisionally excited lines. This discrepancy can be explained by an inhomogeneity of the PNe and an overestimation of the weak line intensities. The formulae for calculation of the nebular line intensities in presence of both temperature and density fluctuations are given. The intensities of the forbidden [OIII] lines, the CII, CIII and CIV recombination lines and the CIII] λ 1909 UV intercombination doublet for different values of the mean electron temperature T_0 in PNe and the *rms* temperature variation $\boldsymbol{t^2}$, are calculated. Results of these calculations are used to find the values of T_0 and $\boldsymbol{t^2}$ which allow to provide the best fit of the observed and calculated line intensities (taking into account the observational errors). In most cases, the obtained values of T_0 appear to be significantly smaller than ordinarily used for the abundance determinations T_e([OIII]), while $\boldsymbol{t^2} < 0.16$. The carbon and oxygen abundances for more than 70 PNe are calculated. For these PNe average chemical abundances are evaluated separately for nebulae of type I, II and III. For the first, we found C/H=$6.67 \cdot 10^{-4}$ cm^{-3} and O/H=$5.74 \cdot 10^{-4}4$ cm^{-3}. For the second they are C/H=$8.94 \cdot 10^{-4}$ cm^{-3} and O/H=$6.36 \cdot 10^{-4}$ cm^{-3}. For the third we obtained C/H=$3.94 \cdot 10^{-4}$ cm^{-3} and O/H=$4.79 \cdot 10^{-4}$ cm^{-3}. Results of the fitting of the line intensities for the NIII λ4640 and NIV] λ1486 lines are also given.

Key words: planetary nebulae: general ISM: abundances

1. Introduction

It is well known that the carbon, nitrogen and oxygen abundances in planetary nebulae derived from optical recombination lines (*recombination* abundances) strongly differ from those determined using collisionally excited UV, IR and optical lines (*collision abundances*) (see, for example, Kaler 1981; Kholtygin & Feklistova 1992, 1995). As it was first indicated by Peimbert (1967) the presence of small amplitude temperature fluctuations inside PN can be responsible for at least a part of this discrepancy (see also Peimbert & Costero (1969) and Rubin (1969) for details). Thus, this discordance is an evidence that PNe are strongly inhomogeneous.

The effect of observational errors has been used by Rola & Stasinska (1994, hereafter RS) to explain the discrepancy of the recombination and collision abundances for a large number of PNe. Recently it was demonstrated that the intensity of the weak recombination lines in the PN spectra can be strongly overestimated in the case of a signal/noise ratio S/N ≤ 6 (Rola & Pelat 1994). A detailed analysis of the carbon abundance discordances (e.g., Peimbert et al. 1995) has shown that the small temperature fluctuations and observational errors can be considered as the most important sources of the discrepancies. In the present paper we investigate the total effect of temperature inhomogeneity and line intensity errors on the nebular parameters derived as a result of the fitting the observed and calculated line intensities.

A classical treatment of the small amplitude temperature fluctuations is outlined in Sect. 2. In the next section we study the joint effect of the density and temperature fluctuations. The considered model and the procedure used to fit the calculated and observed line intensities are discussed in Sect. 4. The carbon and oxygen abundances determined in the present paper are given and discussed in Sect. 5. Then, in Sect. 6 the fitting of the calculated and observed nitrogen line intensities is considered. The effect of the different probability distribution functions for observed line intensities on the derived nebular parameters is studied in Sect. 7. Some conclusions are drawn in the last section.

2. Intensity of lines in the presence of small amplitude temperature fluctuations

Throughout this paper we shall consider the excited by electron collisions the intercombination and forbidden lines as well as the recombination lines of carbon, nitrogen and oxygen ions. For these lines, all PNe are transparent and thus the total energy emitted by the PN in a recombination or collisionally excited line $k \rightarrow i$ of the ion X is

$$E_{\rm ki} = h\,\nu_{\rm ki} \int\limits_V n_{\rm F}\, r_{\rm ki}^{\rm eff}\, dV\ . \qquad (1)$$

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Here ν_{ki} is the frequency of the line; V is the total volume of the region emitting in the line; $n_{\rm F} = n({\rm X_F})$ is the number density of the ion $\rm X_F$, which is responsible for the formation of the line. For collision lines $\rm X_F \equiv X$, but for recombination ones $\rm X_F \equiv X^+$. The coefficient $r^{\rm eff}_{\rm ki}$ is known as the effective line formation coefficient (see Rudzicas et al. 1990). In the case of recombination lines, $r^{\rm eff}_{\rm ki} = n_e\, \alpha^{\rm eff}_{\rm ki}$, where $n_{\rm e}$ is the electron number density and $\alpha^{\rm eff}_{\rm ki}$ is the effective recombination coefficient of the line. For collision lines, $r^{\rm eff}_{ki} = n_e\, {\rm q}^{\rm eff}_{ki}$. Here ${\rm q}^{\rm eff}_{ki}$ is the effective coefficient of the collision excitation for the line $k \to i$, determined in such a way that $(4\pi)^{-1}\, n_e\, n({\rm X})\, {\rm q}^{\rm eff}_{\rm ki}$ is the emission coefficient of the line.

The considered values of E_{ki} can be easily converted into the fluxes F_{ki} corrected for the interstellar extinction. The ratio of the total energies emitted by PNe in the lines is equivalent to that of line fluxes. So we use the first values everywhere in the paper to calculate the ratios of the line fluxes. If the emission in the line is registered only for a part of the nebulae, the effect of the light scattering by the dust particles should be considered. Quantitatively the róle of the effect can be estimated from the deviations of the extinction constant c for the localized region of the nebulae from its mean value for all nebulae. This deviations are lower than 0.1 (e.g. Barker 1987,1991). This means that the light scattering by the dust particles does not change the line intensities more than by 20–25%. The most part of the effect can be taken into account by using the individual extinction constants for each separate region in the nebulae. Hereafter we shall only use the integrate line fluxes of the whole nebulae or those fluxes averaged over all positions inside the nebula where line intensities were measured. Because of this, the effect of the dust scattering can be neglected.

The coefficient $r^{\rm eff}_{\rm ki}$ varies slowly inside the emitting volume, thus, in the first approximation it can be replaced by its value $r^{\rm eff}_{\rm ki}(T_0, n^0_{\rm e})$ taken for the mean values of the temperature and density of the nebula. Therefore

$$E_{\rm ki} \approx E^0_{\rm ki} = h\,\nu_{\rm ki}\, r^{\rm eff}_{\rm ki}(T_0, n^0_{\rm e}) \int\limits_V n_{\rm F}\, dV \ = \ h\,\nu_{\rm ki}\, r^{\rm eff}_{\rm ki}(T_0, n^0_{\rm e})\, N_{\rm F}, \qquad (2)$$

where $N_{\rm F}$ is the total number of the ions $\rm X_F$. The value of T_0 has been determined by Peimbert (1967) and Peimbert et al. (1995):

$$T_0 = \int\limits_V T_e n_e\, n_{\rm F}\, dV \Big/ \int\limits_V n_e\, n_{\rm F}\, dV\ . \qquad (3)$$

At the same time, the intensities of the collisionally excited lines depend weakly on the value of $n^0_{\rm e}$, and the intensities of the recombination lines are independent of $n^0_{\rm e}$. This means that any reliable value of $n_{\rm e}$ found from the intensity ratio of density dependent lines can be used. Following Peimbert (1967) we consider the temperature fluctuations $\boldsymbol{t^2}$ as the *rms* deviations from the mean temperature

$$\boldsymbol{t^2} = \frac{\int (T_e - T_o)^2\, n_{\rm F}\, n_e\, dV}{T^2_o \int\, n_{\rm F}\, n_e\, dV}\,. \qquad (4)$$

In the linear approximation when the parameter $\boldsymbol{t^2}$ is small in comparison with the value of T_0, the total energy emitted in the line $k \to i$ is

$$E_{ki} = E^0_{ki}\,(1 + \mu\, \boldsymbol{t^2})\ , \qquad (5)$$

where the energy emitted by the nebula in the case of no $T_{\rm e}$ and $n_{\rm e}$ fluctuations (E^0_{ki}) is determined by the Eq. (2) and

$$\mu = \left[\frac{1}{2}\,\frac{{\rm d}^2\, r^{\rm eff}_{\rm ki}}{{\rm d}\,T^2_e}\,(r^{\rm eff}_{\rm ki})^{-1}\, T^2_e\right]_{T_{\rm e}=T_o,\, n_e=n^o_e}\ . \qquad (6)$$

The effective line formation coefficients can be approximated using the analytical fitting of the effective recombination and collision excitation coefficients (see e.g. Kholtygin & Feklistova 1992):

$$\alpha^{eff}_{ki}(T_{\rm e}) \ = \ \alpha^0_{ki}\,(T_{\rm e}/10^4\,{\rm K})^{\eta^r}\,,$$
$$q^{eff}_{ki}(T_{\rm e}) \ = \ q^0_{ki}\,(T_{\rm e}/10^4\,{\rm K})^{\eta^c}\,\exp(-E_k/kT_{\rm e}). \qquad (7)$$

Substituting expressions (7) into Eq. (6) we obtain

$$\mu = \mu^r = \eta^r\,(\eta^r - 1) \qquad (8)$$

for recombination lines and

$$\mu = \mu^c = \frac{1}{2}\,\left[\beta^2_k - 2\,\beta_k\,(1 - \eta^c) + \eta^c\,(\eta^c - 1)\right] \qquad (9)$$

for collisionally excited lines. Here $\beta_k = E_{\rm k}/k\,T_0$ is the excitation parameter, where $E_{\rm k}$ is the energy of the upper level of the transition considered. The general expression for the parameter μ for the lines generated both by photo and dielectronic recombination is given by Eq. (11) from Kholtygin & Feklistova (1992).

An analysis of Eqs.(8) and (9) indicates that the small amplitude temperature fluctuations of PNe do not change significantly the intensities of the recombination lines but can lead to an enhancement of those for the collisionly excited lines.

3. Small amplitude temperature and density fluctuations

The temperature fluctuations suspected in some PNe cannot be completely explained by a regular electron temperature dependence on the distance to the central stars of the nebulae (see, for example, Gruenwald & Viegas 1995). A significant part of the temperature fluctuations probably originates from small-scale temperature fluctuations, which can be caused by small-scale density fluctuations (see Kholtygin & Feklistova (1992) for details). Therefore the integrate effect of the density and temperature fluctuations is worth to be considered. Let us suppose that these fluctuations are small relative to the values of $T_{\rm e}$ and $n_{\rm e}$ and determine the mean electron temperature and number density for the considered ion, forming the line:

$$\langle T_{\rm e}\rangle_{\rm F} = \overline{T}_{\rm e} = \int\limits_V T_{\rm e} n_{\rm F} dV \Big/ \int\limits_V n_{\rm F}\, dV = \int\limits_V T_{\rm e}\, n_{\rm F} dV/N_{\rm F} \qquad (10)$$

and

$$\langle n_e \rangle_F = \overline{n}_e = \int_V n_e n_F dV \Big/ \int_V n_F dV = \int_V n_e n_F \, dV / N_F \,. \quad (11)$$

Let us determine the following parameters of the *rms* fluctuations of T_e and n_e:

$$\boldsymbol{\tau^2} = \frac{\int_V (T_e - \overline{T}_e)^2 \, n_F \, dV}{\overline{T}_e^2 \, N_F} \,, \quad (12)$$

$$\boldsymbol{\tau\eta} = \frac{\int_V (T_e - \overline{T}_e)(n_e - \overline{n}_e) \, n_F \, dV}{\overline{T}_e \, \overline{n}_e N_F} \,, \quad (13)$$

$$\boldsymbol{\eta^2} = \frac{\int_V (n_e - \overline{n}_e)^2 \, n_F \, dV}{\overline{n}_e^2 \, N_F} \,. \quad (14)$$

In the linear approximation for small amplitude fluctuations of T_e and n_e, the total line intensity can be presented in the form:

$$E_{ki} = E_{ki}^0 \, \left(1 + \mu_{tt}\boldsymbol{\tau^2} + \mu_{tn}\boldsymbol{\tau\eta} + \mu_{nn}\boldsymbol{\eta^2}\right) \quad (15)$$

The parameters μ_{tt}, μ_{tn} and μ_{nn} are determined by the following expressions:

$$\mu_{tt} = \left[\frac{1}{2}\frac{\partial^2 r_{ki}^{\rm eff}}{\partial T_e^2} \, (r_{ki}^{\rm eff})^{-1} \, T_e^2\right]_{T_e=\overline{T}_e;\, n_e=\overline{n}_e} \,, \quad (16)$$

$$\mu_{tn} = \left[\frac{\partial^2 r_{ki}^{\rm eff}}{\partial T_e \partial n_e} \, (r_{ki}^{\rm eff})^{-1} \, T_e \, n_e\right]_{T_e=\overline{T}_e;\, n_e=\overline{n}_e} \,, \quad (17)$$

$$\mu_{nn} = \left[\frac{1}{2}\frac{\partial^2 r_{ki}^{\rm eff}}{\partial n_e^2} \, (r_{ki}^{\rm eff})^{-1} \, n_e^2\right]_{T_e=\overline{T}_e;\, n_e=\overline{n}_e} \,. \quad (18)$$

A similar treatment of the problem of small-amplitude density and temperature fluctuations but only for some special cases has been given by Vidal (1979). Instead of n_e we can use its logarithm, as it has been suggested by Rubin (1989). It does not change the form of Eqs. (13)–(14) and (16)–(18), but the value of n_e must be replaced by $\log n_e$.

From equations (3), (10) and (11) we have

$$T_0 = \overline{T}_e(1 + \boldsymbol{\tau\eta}). \quad (19)$$

Simple estimates show that for small amplitude T_e and n_e fluctuations $| \; \boldsymbol{\tau\eta} \; |\leq| \; \boldsymbol{\tau^2} \; |$. This means that for any ion its mean temperature ($\overline{T}_e$) is very close to the mean electronic temperature of the nebula T_0.

The connection between the parameters $\boldsymbol{t^2}$ and $\boldsymbol{\tau^2}$ can be found from Eqs. (4) and (12)-(14)

$$\boldsymbol{t^2} = \frac{\boldsymbol{\tau^2} - (\boldsymbol{\tau\eta})^2 + \boldsymbol{\tau^2\eta}}{(1 + \boldsymbol{\tau\eta})^2} \approx \boldsymbol{\tau^2} \,, \quad (20)$$

where

$$\boldsymbol{\tau^2\eta} = \frac{\int_V (T_e - \overline{T}_e)^2 (n_e - \overline{n}_e) \, n_F \, dV}{T_e^2 \, n_e N_F} \approx (\boldsymbol{\tau\eta})^2 \,. \quad (21)$$

The calculated values of the coefficients μ_{tt}, μ_{tn} and μ_{nn} for selected lines of O and C ions are given in Fig. 1 in dependence on n_e (a-c) and T_e (d-g). The atomic data used for the calculation of these coefficients were taken from the catalogue Golovatyj et al. (1997). The difference between the coefficient μ_{nn} for the considered values of T_e = 12000 K and 14000 K is too small and cannot be shown in the Figs. 1 (a-c). The coefficient μ_{nn} for lines [OIII] λ 1663 and λ 4363 is extremely small at $n_e \leq 10^3$ cm^{-3}, so it is not plotted in Fig. 1d-e. Fig. 1g shows the behavior of the values μ_{tt} for the line CIII λ 1907 in the low density approximation, when its dependence on n_e is negligible. This coefficient for the recombination lines is also plotted in Fig. 1g. It is small in comparison with its value for line CIII λ 1907 and weakly depends on the electron temperature. The slight decrease of the value of μ_{tt} when T_e increases for the recombination line CIII λ 4650 can be explained by the large contribution of the dielectronic recombination to the value of the total effective recombination coefficients for this line.

4. Model and fitting procedure for the line intensities

4.1. The model

Our calculations demonstrate that for the electron densities and temperatures typical of PNe, the coefficients μ_{tn} and μ_{nn} are well below the μ_{tt} (see Fig. 1). For this reason we can consider nothing but the temperature fluctuations as the most important ones. In this case formulas (5) and (15) give similar results as $\mu \equiv \mu_{tt}$ and $\boldsymbol{\tau^2}$ is close to $\boldsymbol{t^2}$. Therefore, from now on we use the formalism of the Sect. 2.

We use an empirical model of a PN to reproduce the observed line intensities of the OIII, CII, CIII and CIV lines. In this model a nebula is described by its mean electron temperature T_0, mean electron number density n_e^0 and rms temperature fluctuation $\boldsymbol{t^2}$. The relative carbon and oxygen abundances are assumed to be constant in the whole volume of the nebula. The lines under consideration are the forbidden lines [OIII] $\lambda\lambda$1663, 4363, 4959 + 5007 (hereafter [OIII] λ5007), the intercombination doublet CIII] $\lambda\lambda$1907, 1909 (hereafter referred to as CIII] λ1907) and the recombination lines CII λ4267, CIII] λ4647, 4650 (hereinafter CIII λ4650) and CIII λ4658 which are the most intensive lines of the carbon ions in the spectra of PNe. In the case of a multiplet we consider the sum of the intensities of all its components as the multiplet intensity.

In general each ion X^{n+} has to be described by its own values of $T_0(X^{n+})$ and $\boldsymbol{t^2}(X^{n+})$. However, the numerous calculations have shown (e.g. Harrington et al. 1982) that the nebular temperatures averaged over the distribution of the CIII and OIII ions in the nebular volume (see Eq. 3) are very close. These values do not differ more than by 100-200 K for most of the PNe. Recent calculations (Gruenwald 1997) based on the modern photoionization code (Gruenwald & Viegas 1995) demonstrates that this result does not depend on the model parameters and holds in a very wide range of the effective temperatures and luminosities of the central stars as well as the mean nebular density and the element abundances.

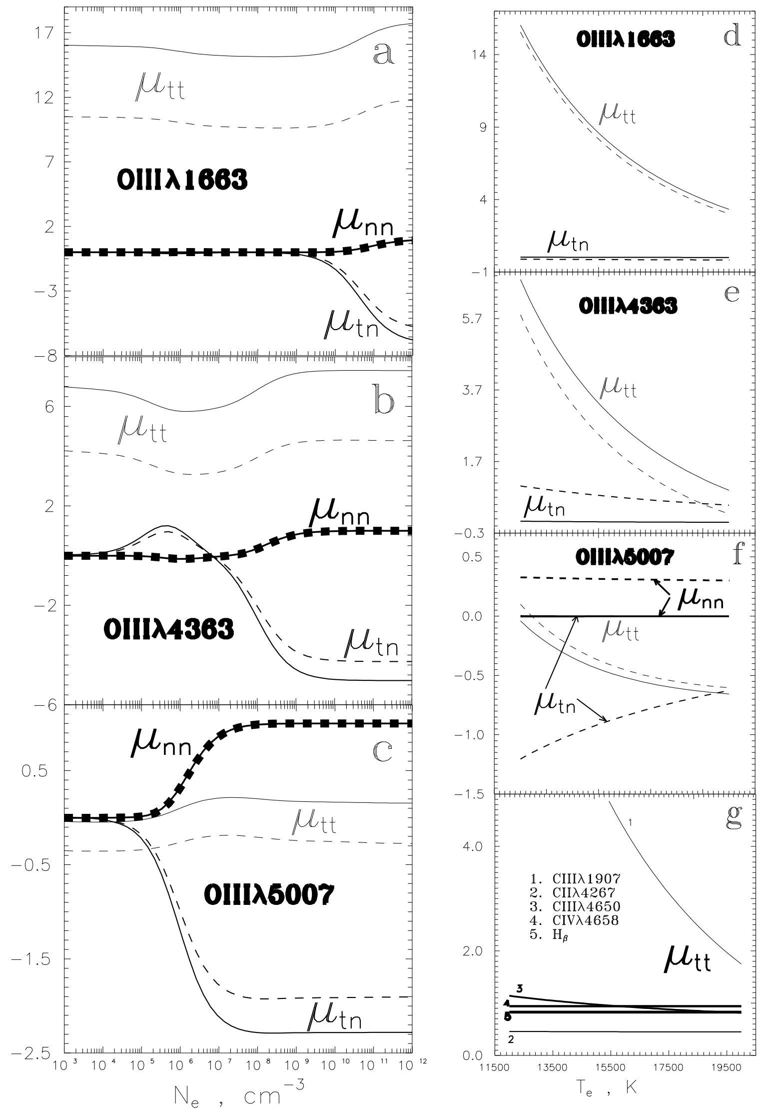

Fig. 1a–g. Parameters μ_{tt}, μ_{tn}, and μ_{nn}, for lines of ions OIII and CII-CIV as a function of n_e. (**a-c**) and T_e (**d-g**). **a-c** T_e = 12000 K (solid line) and 14000 K (dotted line). **d-f** $n_e = 10^3$ cm^{-3} (solid line) and 10^6 cm^{-3} (dotted line)

Recently Gruenwald & Viegas (1995) have calculated the values of the parameter $\boldsymbol{t^2}$ for different ions in the framework of the homogeneous photoionization models in dependence of the effective temperatures of the central star as a function of the distance to the central stars. These calculations show that the temperature fluctuations can differ for singly and doubly ionized atoms especially at the large distances from the central star where the singly ionized and neutral species are mainly located.

In the same time the values of the parameter $\boldsymbol{t^2}$ for the different ions averaged over the whole nebular do not differ considerably. In the cited paper the results of the calculations for ion C^{2+} were not given but, basing on the similarity of photoionization cross sections for these ions and their ionization potentials, we can conclude that the values of the parameter $\boldsymbol{t^2}$ for these ions can also be supposed to be very close and thus we can use the same values of T_0 and $\boldsymbol{t^2}$ for both CIII and OIII. Moreover the main contribution to the parameter $\boldsymbol{t^2}$ probably originates from the small-scale temperature fluctuations which seem to be not very different for the different parts of the nebulae (see Sect. 5.5.3).

The CIII λ 4650 and CIV λ 4658 lines are formed in the CIV and CV ionization zones. The values of T_0 for these zones can differ from their values for CIII and OIII by more than 200 K. Nevertheless, we can use for the CIV and CV zones the same values of T_0 and $\boldsymbol{t^2}$ because of the weak dependence of the considered CIII and CIV recombination line intensities on these parameters. Moreover one should bear in mind that the CIV and CV ions are mainly located in the inner parts of the nebulae where the variations of the electron temperature are small (Golovatyj & Mal'kov 1991).

Finally we list the parameters of the model: $T_0, \boldsymbol{t^2}, n_e^0$, C/H, O/H, where C/H and O/H denote the relative C and O abundances: N(C)/N(H) and N(O)/N(H).

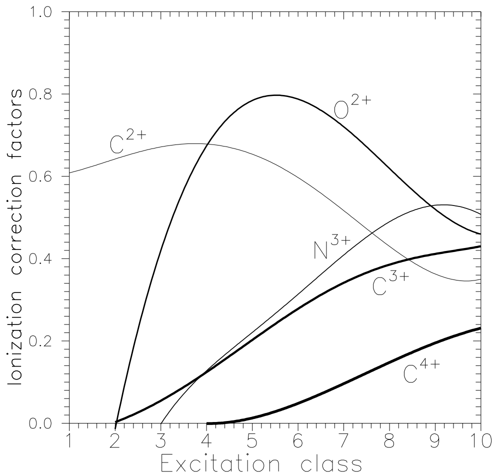

Fig. 2. The smoothed-out dependence of the mean ionization correction factors for C, N and O ions versus excitation class of a nebula

4.1.1. Ionization correction factors

The abundances of carbon ions can be obtained from the total abundances of C :

$$N(C^{m+}) = ICF(C^{m+}) \times N(C)\,. \tag{22}$$

where ICF(C^{m+}) is the ionization correction factor for the C^{m+} ion. Similar relations can be written for oxygen and nitrogen ions. Usually the ionization correction factors are obtained from the results of photoionization model calculations. These results show the regular dependence of the ionization correction factors averaged over a sample of the PNe of a single excitation class E_x on the values of E_x (see Fig.2) where the excitation class of the nebulae is determined by the relative intensities of the lines of the lowly and highly ionized ions in the nebular spectra (Aller 1956). The data for this figure are obtained from numerous photoionization model calculations by Aller & Czyzak (1983) and used by us to calculate the ionic abundances from the corresponding atomic abundances. These ionization correction factors were calculated for PNe without small-scale temperature fluctuations. Nevertheless we can probably use these data, as simple estimations show that such fluctuations do not change the ionization structure of the nebula significantly.

Fig. 2 shows that the CIII/C and OIII/O ratios for the nebulae of high and intermediate excitation ($E_x \geq 4$) never deviate from their mean value (≈0.6) by more than 30-40%. So the errors in the total carbon and oxygen abundances obtained from even very inexact values of E_x hardly exceed this value.

4.1.2. Atomic data and computer codes for line intensity calculations

The atomic data for the C, N and O ions were taken from the catalogue Golovatyj et al. (1997). The data of the catalogue were also used to obtain the effective recombination coefficients for the CII, CIII, CIV and NIII lines. The contribution of both the photo and dielectronic recombination in the total effective recombination coefficients has been taken into account. The intensities of the forbidden [OIII] lines were calculated with our own computer code for the 6-level OIII ion. The effective collision excitation coefficients for the intercombination doublets CIII λ 1907 and NIV λ 1486 were presented as

$$q_{ki}^{eff} = q_{ki}^{0} \left(\frac{T_{\rm e}}{10^4\,K} \right)^{\eta^c} \exp\left[\frac{T_c}{T_{\rm e}} \right], \tag{23}$$

where parameters $q^0_{ki} = 8.50\,10^{-8}$, $\eta^c = -0.53$ and $T_c = 75430$ K for CIII λ 1907 have been calculated to fit the expression for the abundance ratio N(C^{++})/N(H^{++}) given by Peimbert et al. (1995) in their Eq. (1). We have also calculated: $q^0_{ki} = 7.33\,10^{-8}$, $\eta^c = -0.59$ and $T_c = 96924$ K for NIV λ1486 (in the low-density approximation) from atomic data given by Golovatyj et al. (1997).

4.2. Fitting procedure for the line intensities

An usual way to determine such nebular parameters as electron temperature and density, fractional ionic abundances and the total element abundances is some fitting iterative procedure aimed to obtain an optimal agreement between observed and calculated relative line intensities. As usually we employ the name "line intensities" for the "relative line fluxes". Evidently, the result of such fitting strongly depends both on the fitting procedure itself and on precision of the data. The general procedure of the spectral line diagnostics has been considered by Judge et al. (1997). Here we present a simpler method specially proposed for the diagnostics of the nebular plasma using the relative line intensities. Assume that we have a set of observed line intensities referred to the whole nebula or to a part of the nebula: $I_1^{\rm obs}, I_2^{obs}, I_3^{\rm obs}, ...I_K^{\rm obs}$, where K is the total number of the measured line intensities. For convenience we suppose that all intensities are already corrected for the interstellar extinction.

Let us suppose that each line can be described by its own line intensity probability distribution function (PDF) determined in such a way that $P(I)\,dI$ is the probability for line I to have an intensity I in an interval $(I, I + dI)$. As a good approximation we can assume different line intensities measurements to be independent, so the total line intensity probability distribution function is the product of individual ones: $P(I_1, I_2, ...I_K) = P(I_1)P(I_2)...P(I_K)$. Let us assume that the nebular model can be described by a set of parameters $[\Pi] = \Pi_1, \Pi_2, ...\Pi_M$. Here the values of Π_i can denote for example T_0, $\boldsymbol{t^2}$ and the relative carbon, nitrogen and oxygen abundances.

To find the optimal values of the parameters we use the *maximal PDF principle*, assuming that optimal set of the parameters $[\Pi]^{\rm opt}$ is characterized by the maximal value of the PDF for all considered line intensities. This means that

$$P(I_1([\Pi]^{\rm opt}), I_2([\Pi]^{\rm opt}), ...I_K([\Pi]^{\rm opt}) = max\,.$$

It is often more convenient to use the probability $Q_m(I) = P(\mid I - I^{\rm obs} \mid \geq \mid I^{\rm calc} - I^{\rm obs} \mid)$, instead of $P_m(I)\,dI$. Here $I^{\rm obs}$ is the observed and $I^{\rm calc}$ is the calculated line intensities, whereas

I is the real (*true*) intensity. The expression $P(F)$ denotes the probability of condition F to be true.

In this case the total probability $Q(I_1, I_2, ...I_K) = Q(I_1)Q(I_2)...Q(I_K)$ and the *maximal PDF principle* can be expressed in the form:

$$Q(I_1([\Pi]^{\rm opt}), I_2([\Pi]^{\rm opt}), ...I_K([\Pi]^{\rm opt}) = max\,. \quad (24)$$

For the sake of simplicity we suppose that in most cases we can use the normal law for the line intensity probability distribution function:

$$P^{\rm N}(I) = \frac{1}{\sqrt{2\pi}\,\sigma^{\rm N}} \exp\left[-\frac{1}{2}\left(\frac{I^{obs} - I}{\sigma^{\rm N}}\right)^2\right], \quad (25)$$

where the standard deviation $\sigma^{\rm N}$ depends on the observed intensity $I^{\rm obs}$. The index N reminds that the normal PDF is used.

For weak narrow lines the log.-normal distribution considered by Rola & Pelat (1994):

$$P^{\rm LN}(J) = \frac{(I^{obs}/I)}{\sqrt{2\pi}\,\sigma^{\rm LN}} \exp\left[-\frac{1}{2}\left(\frac{J^{obs} - \mu^{\rm LN} - J}{\sigma^{\rm LN}}\right)^2\right] \quad (26)$$

is more suitable. Here $J = \ln(I)$, $J^{obs} = \ln(I^{obs})$. The values referred to log.-normal distribution are marked with index LN.

The values of parameters $\sigma^{\rm LN}$ and $\mu^{\rm LN}$ depend on the signal to noise (S/N) ratio for considered lines and for S/N ≤ 6 they are given by Rola & Pelat (1994) in their Table 6. For S/N > 6 the relations $\sigma^{\rm LN} \approx ({\rm S/N})^{-1}$ and $\mu^{\rm LN} \approx ({\rm S/N})^{-2}$ can be used (see Rola & Pelat (1994) for details).

To obtain the optimal nebular parameters we need to find their values which provide the maximum value of the expression (24). We solved this problem by the standard descent methods. The initial values of the parameters T_0, $n_{\rm e}^0$ are taken from the results of others authors or can be put equal to their typical values for PNe. As usually we used as initial values of the electron temperatures the values taken from RS. For the temperature fluctuations the initial condition $\boldsymbol{t^2} = 0$ was used.

Results of our fitting show that the calculated values of the optimal parameters practically do no depend on their initial values.

Figs. 3-4 demonstrate, for example, how the optimal value of the $n_{\rm e}$, $T_{\rm e}$ and $\boldsymbol{t^2}$ can be found.

5. Results: carbon and oxygen abundances

5.1. Sources of the line intensities

As the main source of data we have used the line intensities from Table 1 given by RS with small corrections of Peimbert et al. (1995). The intensities of CIII λ 4650 and CIV λ 4658 lines are taken from Aller & Czyzak (1979). For BD+30° 3639, IC 351 and NGC 6886 we use recent data by Aller & Hyung (1995), Feibelman et al. (1996) and Hyung et al. (1996). The intensities of the UV lines in the spectra of some PNe: NGC 650, NGC 1535, NGC 2440, NGC 7027 and Hu 2-1 are taken from Henry et al. (1996) and Kwitter et al. (1996). The nebula IC 4997 has strongly variable line intensities. We fit its line intensities obtained in 1990 and in 1991 (Hyung et al. 1994) separately.

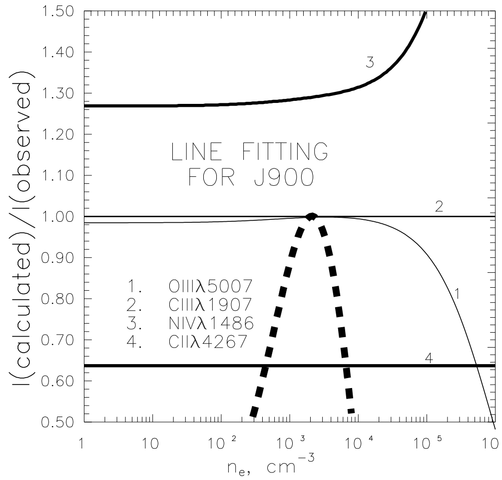

Fig. 3. $n_{\rm e}$ fitting for the nebula J 900. Calculated line intensities (solid lines) are normalized to the observed ones. The dashed line represents the probability distribution function normalized to its maximal value

Table 1. Fitted line intensities for nebulae IC 4997 (in 1991) and J 900

Ion	λ, Å	I^{obs}	I^{calc}	$(I^{obs}-I^{calc})/\sigma(I^{obs})$	$\sigma(I^{obs})$
IC 4997					
OIII	1663	80.30	79.97	0.16	2.05
OIII	4363	117.38	117.65	0.11	2.48
OIII	5007	523.34	524.35	0.00	5.23
CII	4267	0.22	0.25	0.30	0.11
CIII	1907	333.00	333.00	0.00	4.17
CIII	4650	0.45	0.34	0.72	0.15
CIV	4658	0.00	0.01		
J 900					
OIII	1663	40.0	36.7	1.37	2.4
OIII	4363	12.6	16.9	3.30	1.3
OIII	5007	1440	1433	0.47	14.4
CII	4267	0.83	0.47	1.04	0.4
CIII	1907	1640	1638	0.13	15.4
CIII	4650	0.80	1.39	1.74	0.3
CIV	4658	0.68	0.44	0.77	0.3

5.2. The $n_{\rm e}$ data

The important problem of the fitting procedure is what mean $n_{\rm e}$ values should be accepted. In most cases we have used the paper by Stanghellini and Kaler (1989) (their [ClIII] or [ArIV] data) as the most complete source of the $n_{\rm e}$ values. For nebulae which are absent in this paper we have applied the [SII] $\lambda\lambda 6717/6731$ electron densities from RS. We have not found any $n_{\rm e}$ data for NGC 5873, 5882, IC 2553, M 3-1, He 2-7 and SwSt 1 in the literature. For these objects we accepted the typical for PNe value $n_{\rm e} = 10^3\,{\rm cm}^{-3}$.

For most of the nebulae with $n_{\rm e} \leq 2 \cdot 10^3\,{\rm cm}^{-3}$ the fitted parameters appeared to be very weakly dependent on the accepted value of $n_{\rm e}^0$. In this case we accept this value to be equal to the electron density given in the above cited sources and do not vary this value during the line fitting procedure. Only in the

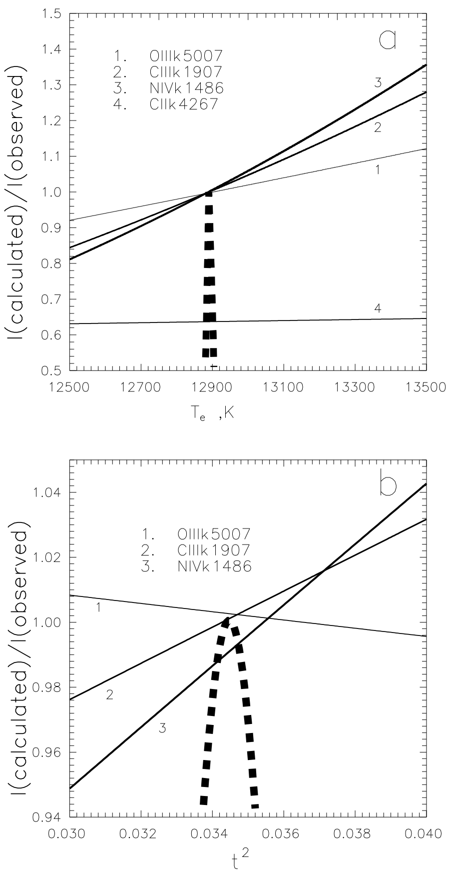

Fig. 4a and b. The same as Fig. 3, but for **a** T_e fit and **b** rms temperature fluctuations fit

case of the high density nebulae ($n_e > 2 \cdot 10^3$ cm^{-3}) the n_e dependence of the fitted parameters should be taken into account. Fig. 3 shows the dependence of the calculated line intensities on n_e value. Evidently, this dependence can be used for fitting the n_e values themselves by means of the fitting procedure described above. The resulting electron densities are presented in the Table 2 (see Sect. 5.4 for the detailed explanation of the table). It must be emphasized that in most cases these densities appeared to be larger than those obtained usually. This fact can be explained by the higher electron number density in the OIII and CIII zones.

5.3. Results of C and O lines fitting

Both intensities and the ratios of the collisionally excited lines are strongly dependent on the T_e and the $\boldsymbol{t^2}$ values. In the vicinity of its maximum the PDF value depends sharply on the electron temperature and the *rms* temperature fluctuations (see Fig. 4).

Results of our calculations demonstrate strong dependence of the optimal nebular parameters on the accepted PDFs for observed line intensities and their standard deviations. Throughout the paper we use the normal PDF with the expression for standard deviation given by RS:

$$\sigma = \sigma(\mathrm{I}_\lambda) = 0.01\,\sqrt{I_\lambda\, I([OIII]\lambda\,5007)}\,, \qquad (27)$$

where intensities are expressed in the usual scale (I(Hβ) =100). Results for more elaborated log.-normal PDF are presented in Sect. 7. Table 1 illustrates the typical quality of the line intensity fitting.

5.4. C and O abundances

We have fitted the line intensities for more than 70 PNe including all of those which were considered by RS except NGC 2242, IC 418 and M1-14. For some nebulae, as NGC 7662 we used the line intensities averaged over all positions given by RS94. Results of the fit are presented in the Table 2. We applied for all PNe the *maximal PDF principle* in the form of Eq. (24). In the first two columns of the table we give the name of the nebula and its excitation class E_x taken in most cases from Aller & Czyzak (1979). If the E_x value is not given in the cited paper we estimate it using the criteria given by Aller (1956) in the modification proposed by Gurzadyan (1969).

The fitted values of the parameters T_0 and $\boldsymbol{t^2}$ together with the optimal CIII, OIII, C and O abundances, which have been obtained from the above described line intensities fitting procedure, are listed in columns 3-4 and 6-9 of Table 2. In column 5 the accepted values of n_e (see Sect. 5.2) are given. In the case when successful fit of the observed line intensities can be attained only if we include the n_e fit in the common fitting procedure, we list in column 5 the fitted value of n_e.

The abundances of carbon derived from intensities of pure recombination lines of CII, CIII and CIV ions (excluding the intensity of UV lines CIII λ 1907 from the fitting procedure) are given in the column 10. As would be expected these *recombination* carbon abundances appeared to be close to those obtained by Nikitin et. al. (1994) also from carbon recombination line intensities but without taking into account the small-scale T_e fluctuations. Column 11 gives the ratio of calculated and observed line intensities for CII λ 4267. The ratio of our *optimal* C and O abundances can be found in the column 12. Some comments are given in the last column of the table.

Overall we did not fit the intensities of the lines of the nitrogen ions, so the nitrogen abundances are not presented in the table 2 and will be given by Kholtygin (1997a). Some previous results can be found in Sect. 6 of the present paper.

5.5. Discussion of the results

In the framework of the foregoing line intensity fitting procedure the evaluated abundances are *optimal* in the sense that the differences between the observed and the calculated intensities both for recombination and collision excited lines are minimal (in units of the standard deviation which is different for different lines). It means that there is no problem of discordance between the *recombination* and *collision* abundances of carbon. The errors in the observed line intensity, as clearly indicates the relation (27), are significant in the comparison of the intensity itself for the weak recombination lines ($I/I(H_\beta) \leq 1.0$), so the differences between observed and calculated (determined by the

Table 2. Optimal nebular parameters and C and O abundances

Object	E_x	T_0	t^2	Ne	{C^{2+}/H}	{O^{2+}/H}	{C/H} (10^{-4})	{O/H}	{C/H}$_{rec}$	I(4267) calc/obs	C/O	comm.
1	2	3	4	5	6	7	8	9	10	11	12	13
NGC 40	2	8053	0.026	2.09E3	5.42	0.14	8.74	1.76:	8.09	0.98	5.0	
NGC 650	8p	11218	0	4.27E3	13.79	2.84	30.6	4.89	19.0	1.60	6.3	
NGC 1535	7	7825	0.16	1.05E3	5.97	9.28	12.71	14.51	12.8	0.99	0.88	
NGC 2022	10	9977	0.17	1.48E3	3.28	2.55	9.65	6.03	31.0	0.31	1.8	
NGC 2346	7	10516	0.105	2.95E3	1.09	3.04	2.33	4.75	-	-	0.49	
NGC 2371-2	9	8427	0.084	2.09E5	7.77	8.56	20.45	16.47	21.83	0.93	1.24	
NGC 2392	8p	11541	0.131	2.00E3	0.66	3.32	1.48	5.73	10.7	0.14	0.26	m
NGC 2440	9	13399	0.000	6.02E3	3.09	2.16	8.14	4.16	10.7	0.76	1.96	
NGC 2440		13131	0.002	5.83E4	3.45	2.44	9.07	4.69	10.8	0.84	1.93	n
NGC 2818	9	10646	0.159	3.51E2	2.61	3.71	6.88	7.13	13.31	0.52	0.97	
NGC 2867	7	10592	0.000	1.95E3	9.84	4.16	20.93	6.50	20.8	1.00	3.22	
NGC 3132	6	11024	0.046	4.27E2	1.66	1.83	2.72	2.10	-	-	1.30	
NGC 3211	6	13008	0.000	2.09E3	2.85	1.70	6.34	2.94	-	-	2.16	
NGC 3242	6	10443	0.052	3.55E3	2.11	4.31	4.50	6.73	13.1	0.34	0.67	m
NGC 3918	6	13500	0.001	5.01E4	1.79	1.67	4.70	3.21	10.9	0.41	1.46	
NGC 3918		12790	0.015	4.58E4	2.23	2.06	5.86	3.97	11.7	0.50	1.48	n
NGC 4361	6	18926	0.145	6.76E3	0.10	0.25	0.29	0.51	-	-	0.56	
NGC 5315	5	7873	0.100	1.95E4	1.27	4.60	2.02	5.22	-	-	0.39	
NGC 5873	7	6780	0.003	1.00E3:	10.38	20.58	22.09	32.15	22.02	-	0.69	:
NGC 5882	9	9349	0.000	1.00E3:	5.45:	4.82	14.33:	9.27	14.32	1.00	1.55:	
NGC 6153	7	6601	0.113	8.16E3	7.44	15.35	15.84	23.99	75.23	0.21	0.66	
NGC 6210	5	8318	0.000	3.02E3	2.39	7.54	3.79	8.57	9.10	0.41	0.44	
NGC 6302	9	14420	0.090	1.32E4	0.83	2.10	2.18	4.04	4.06	0.54	0.54	
NGC 6309	8	6400	0.000	1.78E3	8.14:	27.94	18.08:	48.17	17.85	1.00	0.38:	
NGC 6543	5	8761	0.000	4.68E3	9.51	3.90	15.09	4.4	11.90	1.25	3.41	
NGC 6572	6	9389	0.000	1.20E4	5.17	5.67	8.48	6.52	7.1	1.14	1.30	
NGC 6644	7	11500	0.000	2.29E4	3.17	3.46	6.75	5.41	9.62	0.69	1.25	
NGC 6720	6	10320	0.042	1.35E3	3.12	3.72	5.12	4.28	15.35	0.33	1.20	m
NGC 6741	8	11978	0.058	7.76E3	3.29	3.11	7.32	5.37	13.61	0.54	1.36	
NGC 6778	5	7922	0.000	1.62E3	7.63:	4.49	12.11	5.10	11.93	1.00	2.37:	
NGC 6790	6	11412	0.000	2.88E4	3.31:	3.93	5.43:	4.52	5.41	1.00	1.20:	
NGC 6803	6	9587	0.000	7.41E3	5.41:	4.99	8.86:	5.73	8.86	1.00	1.55:	
NGC 6818	6	12412	0.006	1.07E3	4.38	3.13	11.53	6.03	12.10	0.95	1.91	
NGC 6818		12387	0.006	1.37E4	4.46	3.17	11.74	6.11	11.73	0.97	1.92	n
NGC 6826	5	8103	0.043	3.16E3	3.58	5.68	5.67	6.45	9.70	0.58	0.88	m
NGC 6833	5	14619	0.000	3.16E7	0.99	36.50	1.58	41.48	1.55	1.00	0.04	n
NGC 6853	7	10073	0.040	4.37E3	4.01	3.29	8.53	5.14	19.39	0.44	1.66	m
NGC 6884	6	10807	0.000	9.77E3	5.11:	3.79	8.37:	4.36	8.37	1.00	1.92:	
NGC 6886	8	11914	0.025	6.00E3	5.85	3.55	13.00	6.13	10.17	1.28	2.12	
NGC 6891	5	9242	0.000	2.95E3	5.47	4.30	8.68	4.89	8.67	1.00	1.78	
NGC 6905	7	13819	0.081	7.08E2	0.38	1.27	0.81	1.99	-	-	0.41	
NGC 7009	6	7419	0.142	4.37E3	2.74	11.43	4.49	13.14	1 5.89	0.28	0.34	m
NGC 7026	6	9081	0.000	6.31E3	8.93:	4.82	14.64	5.54	14.59	1.00	2.64:	
NGC 7027	10	11505	0.063	9.33E4	5.87	3.78	17.26	7.87	17.10	1.00	2.19	
NGC 7027		11124	0.082	7.34E4	6.41	4.13	18.84	8.61	17.57	1.07	2.19	n
NGC 7662	8	11161	0.051	3.55E3	3.51	3.36	7.80	5.80	8.23	0.95	1.35	m
IC 351	7	10038	0.109	1.05E3	3.83	4.03	8.15	6.30	16.83	0.48	1.29	
IC 1297	9	9308	0.041	2.96E3	3.13	6.38	8.25	12.27	19.0	0.43	1.49	
IC 1747	6	10116	0.000	2.75E3	5.46	4.64	8.95	5.33	12.35	0.72	1.68	
IC 2003	8	11066	0.027	2.95E3	3.21	2.88	7.14	4.96	10.63	0.67	1.44	
IC 2149	5	9749	0.048	2.00E3	0.92	1.81	1.46	2.06	13.62	0.11	0.71	
IC 2165	8	13024	0.000	1.05E5	5.52	1.85	12.27	3.19	13.67	0.90	3.85	n
IC 2165	8	13376	0.000	2.82E3	4.70	1.52	10.44	2.63	13.49	0.77	3.97	
IC 2501	5	10404	0.000	1.00E3:	2.40	1.86	3.80	2.11	-	-	1.80	
IC 2553	9	10161	0.000	1.00E3:	0.63	4.56	27.96	8.77	27.95	1.00	3.19	

Table 2. (continued)

Object	E_x	T_0	t^2	Ne	$\{C^{2+}/H\}$	$\{O^{2+}/H\}$	{C/H} (10^{-4})	{O/H}	$\{C/H\}_{rec}$	I(4267) (calc/obs)	C/O	comm
1	2	3	4	5	6	7	8	9	10	11	12	13
IC 3568	5	7978	0.159	3.09E3	4.08	5.59	6.48	6.36	4.57	1.30	1.02	
IC 3568	5	9171	0.038	9.79E4	3.29	4.15	5.23	4.71	4.52	1.14	1.11	n
IC 4634	6	9797	0.000	4.07E3	1.34:	3.46	2.20:	3.98	2.20	1.00	0.55:	
IC 4846	5	9940	0.000	8.51E3	-	4.15	-	4.72	-	-	-	
IC 4997	5	10476	0.059	5.55E6	2.79	10.95	4.43	12.15	3.63	1.21	0.36	n, a
IC 4997	5	10742	0.062	6.18E6	2.79	10.44	4.43	12.15	3.86	1.15	0.36	n, b
IC 5117	6	11856	0.000	5.75E4	4.44:	3.30	7.28:	3.79	7.27	1.00	1.92:	
IC 5217	6	11352	0.000	4.17E3	2.36:	2.81	3.86:	3.23	3.84	1.00	1.20:	
BD+30^o...	1	8800	0.000	1.30E4	3.58:	0.04	5.78:	1.36	5.75	1.00	4.24:	
CN 3-1	2	22443	0.000	1.02E4	2.62:	0.01	4.22:	0.10	4.09	1.00	43.3:	
HB 12	4	26500	0.063	3.02E3	1.78:	0.16	2.41:	0.23	2.39	1.00	10.7:	
He 2-7	8	12667	0.000	1.00E3:	8.18:	1.92	18.19:	3.31	18.18	1.00	5.5:	
Hu 1-2	10	12775	0.153	3.24E3	1.16	1.50	3.42	3.13	9.59	0.36	1.09	
Hu 2-1	4	7520	0.074	2.57E3	4.47	4.09	6.04	5.84	6.01	1.00	1.04	
J 320	5	10986	0.052	2.21E3	4.38	3.37	6.95	3.83	3.45	1.99	1.82	
J 900	5	12890	0.034	2.69E3	5.88	1.83	12.52	2.85	18.35	0.68	4.39	
M 1-74	2	9513	0.000	2.69E4	6.10:	4.83	9.83:	-	9.76	1.00	0.16:	
M 1-80	6	12444	0.000	1.20E3	16.66:	2.45	27.32:	2.81	27.21	1.00	9.72:	
M 3-1	5	12315	0.000	1.00E3:	14.42:	1.31	22.89:	1.49	22.88	1.00	15.3:	
M 3-27	5	12476	0.000	4.58E6	2.68	5.21	4.25	5.92	-	-	0.72	n
Me 2-1	8	13202	0.000	1.48E3	2.64	2.08	5.87	3.59	10.87	0.54	1.63	
Me 2-1		13161	0.000	1.50E4	2.69	2.12	5.98	3.66	10.92	0.55	1.63	n
SwSt 1	8	13962	0.090	1.00E3:	0.11	0.05	0.18	1.83	3.34	0.05	0.10	
Vy 1-2	6	10689	0.000	2.04E3	3.27:	3.50	5.36:	4.03	5.36	1.00	1.33:	
mean (all types)							7.43	5.89				
mean (Type I)							6.67	5.74				
mean (Type II)							8.94	6.36				
mean (Type III)							3.94	4.79				

p in 2nd column marks the nebulae with pequliarities in the spectra; **:** in columns 6, 8 and 12 means that I(1909) is unknown, so the carbon abundances can be overestimated, **:** in the 5th column marks the n_e values which was assumed to be equal to their typical value 10^3 cm^{-3}. Comments in the last column: m - line intensities were averaged over all given by RS positions n - n_e was fitted, the years of observations for IC 4997 are marked by letters a (1990) and b (1991).

optimal abundance of the considered element) line intensity can be large for the recombination line.

5.5.1. Intensity of CII $\lambda\,4267$ and carbon abundances

The inspection of the Table 2 shows that the calculated intensities of this line in the majority of cases are far below the observed ones. It means evidently that the observed line intensities are often overestimated. This conclusion is in reasonable good agreement with the main result by Rola & Pelat (1994) (see also Kaler 1981, 1986).

The dependence of the above-mentioned ratio on the observed CII $\lambda\,4267$ line intensity is plotted in Fig. 5 (a). This figure shows that the smaller is the observed intensity, the smaller is this ratio. This fact means that if we use to obtain the carbon abundances the intensities of the recombination lines only, the derived abundances in most cases will be overestimated. These (possibly overestimated) carbon abundances are marked by colon in Table 2. Nevertheless we think that using of some empirical procedure for the line intensity correction (e.g. Kaler 1981) is valid in average, but can give erroneous results for the individual objects, since the real (*true*) intensity is scattered over a wide range which is determined by the value of the standard deviation for CII $\lambda\,4267$ line intensity.

Taking into consideration the small-scale T_e fluctuations inside the nebula we diminish the difference between the *recombination* and *collision* carbon abundances (and, evidently between the calculated and observed CII $\lambda\,4267$ line intensities). This follows from Fig. 5 (b) presenting the ratios of the *recombination* to *optimal* carbon abundances versus the latter. The general form of the dependence is similar to that given by Kholtygin & Feklistova (1992) (Fig. 1 in their paper), but the ratios themselves are 2-3 times smaller due to the effect of T_e fluctuations. Both dependencies (given in this paper and by Kholtygin & Feklistova 1992) reflect predominantly the effect of an overestimation of the CII $\lambda\,4267$ line intensity.

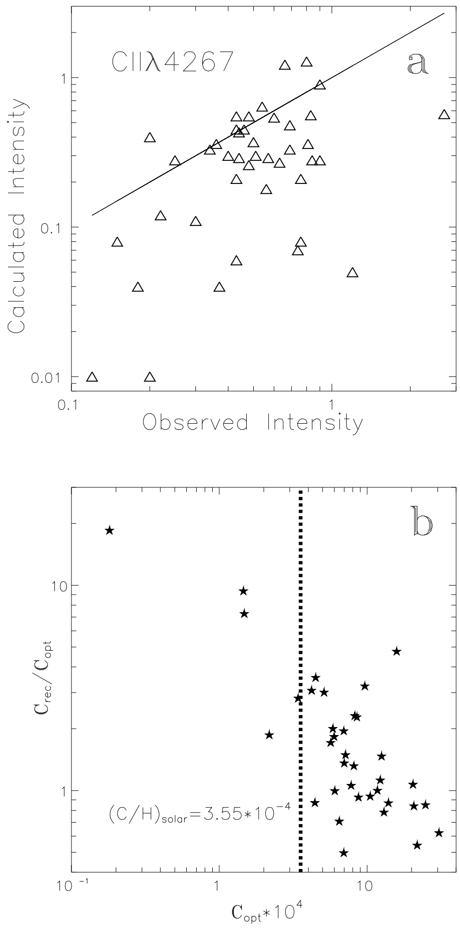

Fig. 5. a The calculated intensities ($I^{\rm calc}$) for the CII λ 4267 recombination line versus observed intensities ($I^{\rm obs}$) of this line (in logarithmic scale). The solid line is the dependence $I^{\rm obs} = I^{\rm calc}$. **b** The ratio (in logarithmic scale) of *recombination* and *optimal* abundances versus the last ones

The *optimal* abundances obtained in the present paper are in most cases intermediate between the *recombination* and the *collision* ones. As we see from the Fig. 5 the main part of the difference between the *recombination* and the *collision* abundances can be explained by the errors in the CII λ 4267 line intensity. This means that in the framework of the accepted fitting procedure the carbon problem does not exist. Instead we have a problem of the low accuracy of the measurements of weak recombination lines in the spectra of PNe.

5.5.2. Comparison with the results of other authors

The mean nebular electron temperatures T_0 for the PNe with the $T_{\rm e}$ fluctuations are often significantly smaller than those for the homogeneous nebulae. This is evident from a comparison of the values of T_0 in column 3 of Table 2 with the T(O^{++}) values given by Peimbert et al. (1995) and RS (in their Table 2). Our values of T_0 in most cases lie in the range from T(C^{++}) and T(O^{++}) from Peimbert et al. (1995) and $T_e(C^{2+})$ and T_e[O III] from Kaler (1986). In most cases, our CIII/H abundance ratios are lower than those given by Peimbert et al. (1995).

The *optimal* carbon and oxygen abundances obtained in the present paper are generally larger than those derived by RS because of accounting for the $T_{\rm e}$ fluctuations. At the same time the ratio C/O depends very weakly on the values of $\boldsymbol{t^2}$ in agreement with the conclusion by Zuckerman and Aller (1986). The ratios C/O found in the present paper are on average close to those given by Kaler (1981), Zuckerman and Aller (1986) and by Perinotto (1991). The C/O ratios of the present paper are in good agreement with these ratios from RS. The C/O ratios in RS tend to be slightly lower than in the present paper. These distinctions are not connected with the difference in the atomic data (which do not exceed 5–10%) and can be explained by the approximate atomic model for CIII used in the present paper. Using the more elaborate atomic model for this ion tends to minimize the differences (see Kholtygin 1997b).

A comparison of our *optimal* abundances of C and O with abundances given in the most complete compilation by Perinotto (1991) shows that for the majority of the nebulae the difference of abundances does not exceed a factor of 1.5 –2 (the *optimal* abundances are slightly higher due to the effect of the temperature fluctuations). Nevertheless in some cases our abundances differ up to ten times from those given by Perinotto. These differences have two main reasons. The first one is the effect of $T_{\rm e}$ fluctuations (for the nebulae NGC 1535, 2371-2, 3132, 3211, 4361 and IC 4997) and the second is very low mean nebular temperature for NGC 5873, 6153 and 6309 obtained by us as a result of line fitting for the optical OIII lines. The reason why the electron temperatures for these nebulae are low is the very low ratio of OIII line intensities I(4363)/I(5007)<0.01. Similar low temperatures for these objects have been derived by RS.

The discordance is especially large for three peculiar objects. Two of them (CN 3-1, and Hb 12) appear to be too hot for PNe ($T_{\rm e} > 22000\,K$). Almost the same temperatures for these objects have been obtained by RS. This conclusion is based only on the intensities of the optical lines and could be wrong if the UV line intensities would be taken into account. The most unusual is the result of our line fit for the nebula NGC 6833. This nebula has a peculiar line intensity ratio I(4363)/I(5007) $\approx$ 1. We are able to reproduce this ratio only with the very large value of $n_{\rm e}$ = 3.16 10^7 cm^{-3}.

5.5.3. Mean abundances and abundances of groups of PNe

We have found the mean abundances for all PNe considered in the present paper, excluding the PNe with unknown intensities of the line CIII λ 1907 and the peculiar objects NGC 5873, 6153 and 6309. These mean abundances are given in Table 2. Despite of the significant differences for individual PNe between the *optimal* abundances derived in the present paper and obtained by more traditional methods, the mean abundances are in better agreement. The mean carbon and oxygen abundances for all PNe are only about 1.3 times larger than those given by Perinotto (1991). In our opinion such a difference reflects the effect of temperature fluctuations. The mean *optimal* carbon abundance is 2 times larger than the solar one ($[C/H]_\odot$ $=3.55\cdot 10^{-4}$), whereas the mean *optimal* ratio O/H is close to that for the Sun ($7.41\cdot 10^{-4}$, Grevesse et al. 1996).

We also evaluated the mean abundances for the different types of the nebulae accordingly with the classification system by Peimbert (1978). The types of the PNe were taken from Maciel & Chiappini (1994). These mean abundances are given in the last rows of Table 2. The mean values for types I and II are close to their values for all sample of the PNe. Carbon and oxygen abundances are a bit higher (factors 1.34 and 1.11) in type II. For the metal underabundant type III PNe the obtained mean abundances reduce to approximately half of their values for types II and III. Faundez-Abans & Maciel (1987) divided the type II PNe into IIa and IIb subtypes. We found that the mean abundances for IIa and IIb subtypes do not differ by more than 8% from their values for all PNe of type II. For all types of PNe the ratio O/H appeared to be smaller than its solar value.

5.5.4. Reality of the small-scale T_e fluctuations

Table 2 shows that the obtained (averaged over the whole volume of the nebula) values of the parameter t^2 in many cases far exceed its maximal values (0.03) for the nebulae without small-scale temperature fluctuations (Gruenwald & Viegas 1995). It means that the significant part of the contribution to the total value of t^2 may originate from small-scale T_e fluctuations.

Moreover, the parameter t^2 weakly depends on the size of the volume of the nebulae emitting in the lines under consideration (for example, in the case of the nebula NGC 2392). The value of t^2 obtained for 3 bright regions inside the nebula (positions 1, 2, 4 observed by Barker 1991) varies from 0.08 to 0.1, whereas the mean value for all nebulae is 0.13. The difference between these values can be explained by large-scale T_e fluctuations, but the values of t^2 for the individual positions probably reflect the contribution from small-scale fluctuations. The values of t^2 for the regions of moderate and low brightness (positions 3, 5 and 6) are rather small: $t^2 \leq 0.01$.

Most of the nebulae with large values of t^2 are bipolar or have a double or broken ring structure in accordance with their morphological types given by Zuckerman & Aller (1986) and Górny et al. (1997). It probably reflects the correlation between complex inner nebular structure and temperature fluctuations pointed out by Kholtygin & Feklistova (1995), who have investigated the differences between *recombinational* and *collisional* abundances of C, N and O ions for a large sample of the PNe and found that the maximal discordance (up to 20 – 40 times) occurs for bipolar and irregular PNe.

6. Fitting of the line intensities for nitrogen ions

Nitrogen is commonly less abundant in planetary nebulae than C and O. Like it is for carbon and oxygen, the nitrogen abundances derived from the intensities of recombination lines usually exceed those derived from collisionally excited lines (e.g. Kholtygin & Feklistova 1992). The most strong nitrogen recombination lines in spectra of PNe are the lines of the multiplet NIII $\lambda\lambda 4634, 4640, 4634$, hereinafter referred to as NIII $\lambda 4640$. We use the intensity of this multiplet for estimating the nitrogen abundances in PNe in the framework of the fitting procedure described in Sect. 4. A comparison of the values of T_e and t^2 averaged over the whole N^{3+} ionization zone for the ionization models of high excitation PNe ($E_x > 5$) with those for CIII zone (e.g. Harrington et al. 1982) shows that these values are close. This is why we can use the same T_0 and t^2 parameters for fitting the OIII, CIII and NIII lines for high excitation nebulae.

Table 3. Fitted nitrogen line intensities for the nebula J 900

Ion	λ, Å	I^{obs}	I^{calc}	$(I^{obs}$-$I^{calc})/\sigma(I^{obs})$	$\sigma(I^{obs})$
NIII	4640	1.33	1.25	0.18	0.44
NIV	1486	105	103.9	0.28	3.89

For the sake of better reliability of the derived nitrogen abundances we also add in the list of the lines whose intensities are fitted the lines of collisionally excited intercombination doublet NIV $\lambda 1486$. The intensity of that doublet in spectra of high excitation nebulae is rather high, so the errors in its measurement are much smaller than those for multiplet NIII $\lambda 4640$. The result of the line fitting for the nebula J 900 with inclusion of NIII and NIV lines is given in Table 3. The ionization correction factor for N^{3+} is plotted in Fig. 2. The intensities of the NIII and NIV multiplets were taken from Aller & Czyzak (1979, 1983).

7. Impact of different probability distribution functions on the determination of the nebular parameters

Results of our calculations clearly show a strong dependence of the fitted optimal parameters on the accepted type of the probability distribution function and the value of standard deviation for the given probability distribution function. Table 4 demonstrates how the change of the standard deviation law $\sigma(I)$ influences the fitted parameters of the nebulae NGC 6720. The line intensities were taken for position 2 in the nebulae (Barker 1987). The last row of the table gives the absolute value of the difference of calculated and observed intensities of the lines [OIII] λ 1663 , [OIII] λ 5007 and CII λ 4267 in units of the observed intensity.

From the table one can see that the assumption of the standard deviation to be proportional to the intensity, leads to the incredibly low values of the mean electron temperature T_0 and abundance ratios C/H and O/H. Moreover in this case the absolute value of the differences between calculated and observed line intensities exceeds 5 standard deviations (determined by Eq.27) for [OIII] λ 1663 and 24 standard deviations for [OIII] λ 5007 . This means that such assumption should be wrong, but the approximation of RS for the standard deviation law (Eq. (27)) evidently is much more realistic.

The data in this table clearly indicate that one has to investigate in detail the PDF for the observed line intensities to be sure that obtained nebular parameters are real.

Finally we have tried to investigate the influence of the log.-normal law for the probability distribution functions of the observed line intensities on the results of the line fitting. Unfortu-

Table 4. Optimal nebular parameters for NGC 6720-2 for different standard deviation laws

Parameter	σ from Eq.(27)	$\sigma = \alpha\, I^{obs}$		
		α = 0.05	α = 0.1	α = 0.3
T_e, K	11350	7750	7900	7900
t^2	0.032	0.16	0.14	0.14
(C/H) $\times 10^4$	2.8	20.0	20.0	20.0
(O/H) $\times 10^4$	3.9	16.2	15.2	15.2
$(I^{obs} - I^{calc})/(I^{obs})$				
[OIII] λ 1663	0.12	0.44	0.44	0.45
[OIII] λ 5007	0.00	0.24	0.24	0.12
CIII λ 4267	0.90	0.35	0.35	0.44

Table 5. Results of nebular parameters fitting for J 900 in dependence on the probability distribution function

Parameter	PDF from Eq.(25)	PDF from Eq.(26)		
		N=0.1	N=0.3	N=0.6
T_e, K	12844	12820	12100	11730
t^2	0.034	0.040	0.064	0.078
(C/H) $\times 10^4$	12.2	12.7	14.6	16.0
(N/H) $\times 10^4$	3.8	2.7	4.5	5.1
(O/H) $\times 10^4$	3.4	3.8	4.1	4.6
$(I^{obs} - I^{calc})/(I^{obs})$				
[OIII] λ 1663	0.05	0.08	0.08	0.08
[OIII] λ 5007	0.00	0.00	0.00	0.00
CIII λ 4267	0.32	0.34	0.23	0.17

nately the real noise (N) level is often poorly known. It depends on the brightness of the nebula and spectral detectors used. As an illustration of the effect we fit the line intensity for the nebula J 900 using a log.-normal PDF. We accept the three relatively realistic noise levels based on our line intensities analysis (N(4000 Å– 5000 Å) = 0.1; 0.3; 0.6 in the scale I(H_β)=100). A detailed investigation of the errors in the observed line intensities has shown that for moderate and strong lines (S/N $\geq$ 10) it is better to apply the standard normal law (Eq. (25)) with the standard deviation given by Eq. (27). Results of fitting for the nebula J 900 are presented in Table 5. The fitted parameters appear to be dependent on the noise level. For larger values of noise the quality of the fitting for line CII λ 4267 becomes better. Besides that, as the comparison of columns 1-4 of Table 5 clearly demonstrates, these parameters depend significantly on the type of the PDF (normal or log.-normal) for weak lines. From the data of the table, we can also conclude that errors in the relative C,N and O abundances derived from the line intensity fitting can exceed 50%.

8. Conclusions

We have investigated the effect of small temperature and density fluctuations on the line intensities in the spectra of PN basing on the Peimbert (1967, 1995) treating of the problem and obtained the expressions for calculation of the nebular line intensities in the presence of such fluctuations. Our calculations demonstrate that for the electron densities and temperatures typical of PNe only temperature fluctuations affect strongly on the line intensities and analysing the nebular spectra we can consider nothing but the temperature fluctuations as the most important ones.

We found that the observed intensities of the recombination and collisionally excited lines of C and O ions can be successfully fitted in the framework of a simple model of the planetary nebula with single values of the mean electron temperature T_0, mean electron number density n_e^0 as well as with a single value for the temperature fluctuations t^2. We have calculated the intensities of the forbidden [OIII] lines, the CII, CIII and CIV recombination lines and the CIII] λ 1909 UV intercombination doublet for different values of the mean electron temperature T_0 in PNe and the *rms* temperature variation t^2.

This model was used to determine the nebular parameters, as well as the abundances of C and O for over 70 PNe. To find them we looked for the maximum of the probability distribution function describing the deviations of the calculated intensities for all considered lines from the observed intensities. In most cases, the obtained values of T_0 appear to be significantly smaller than usually used for the abundance determinations T_e([OIII]), while $t^2 < 0.16$.

The *optimal* carbon and oxygen abundances obtained in the present paper seem to be very close to those derived by RS being generally slightly larger than the latter because of accounting for the T_e fluctuations. Our abundances appeared to be close (on average) to those derived by Perinotto (1991) and Zuckerman & Aller (1986), however the individual discrepancies can be significant.

The investigation of the problem of discordance between the recombination and collision abundances of carbon ions in the framework of the accepted line intensity fitting method has shown that the fitted abundances are *optimal* in the sense that the differences between the observed and the calculated intensities both for recombination and collision excited lines are minimal (in units of the standard deviation which is different for different lines).

The following conclusions can be drawn as a result of our analysis:

1. Without taking into account the temperature fluctuations and observational errors the nebular parameters hardly can be evaluated realistically.

2. The values of the parameters of PNe derived as a result of the line intensities fitting depend on the type of the probability distribution function for the observed line intensities. The assumption that for low signal/noise ratios the probability distribution function follows the log.-normal distribution (Rola & Pelat 1994) can change significantly the results of the fitting.

3. For PNe under investigation the average chemical abundances were evaluated separately for nebulae of the types I-III. For the first, we found C/H=$6.67 \cdot 10^{-4}\,\mathrm{cm}^{-3}$ and O/H=$5.74 \cdot 10^{-4}\,\mathrm{cm}^{-3}$. For the type II nebulae we determine C/H=$8.94 \cdot 10^{-4}\,\mathrm{cm}^{-3}$ and O/H=$6.36 \cdot 10^{-4}\,\mathrm{cm}^{-3}$. The metal underabundant nebulae of type III demonstrate the significantly lesser values of the mean abundances: C/H=$3.94 \cdot 10^{-4}\,\mathrm{cm}^{-3}$ and O/H=$4.79 \cdot 10^{-4}\,\mathrm{cm}^{-3}$.

4. In the frame of the accepted model the carbon problem is absent. Our results show that there is not a problem in the determination of the carbon (nitrogen or oxygen) abundances, but instead the existent discrepancies lie in the low accuracy of the weak lines intensity measurements and in the effect caused by small-scale temperature fluctuations. These conclusions are in good agreement with what was claimed before by RS.

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