

Radiative transitions and photoprocesses

2.1 Transition probabilities, oscillator strengths and mean lifetimes

The relations between the transition probabilities and oscillator strengths are determined by Eqs. (1.1 — 1.4). When only one of the values (transition probability) or (oscillator strength) are given in the table under consideration the remain can be calculated via relations (1.2)-(1.3) as soon as the type of the transition is known. Using the experimental energy differences Δe in last mention relations is preferable (see discussion in Rudzikas et al. (1990)).

Neglecting the induced transitions the spontaneous transition probabilities A_{ki} are connected with the mean lifetimes of level k via relation:

$$\tau_k = \left(\sum_{i < k} A_{ki} \right)^{-1}. \quad (2.1)$$

The probability that an atom in the state k emits a line $k \rightarrow i$ is called the branching ratio and it can be expressed by

$$B_{ki} = \tau_k A_{ki}. \quad (2.2)$$

Due to limited volume of our catalogue we must exclude large amount of transition probabilities and oscillator strengths data. Such kind of data are given only for HI (Table 1) and HeI (Table 2). The much more complete data for HI can be found in the review paper by Omidvar (1983) and for He in paper by Theodosiou (1987). The references on the numerous transition probabilities and oscillator strength compilations and tables can be found in the Appendix A and also may be taken from the atomic data catalogue and banks described in the Appendix B.

The largest database for spectral lines of atoms and two-atomic molecules has been calculated and composed by Kurucz. His current list includes data for more than 58 millions spectral lines, more than 42 millions of them belonging to atoms and their ions (Kurucz (1992, 1995)). A

somewhat smaller former set of data by Kurucz is available on CD-ROMs.

A large international action named "Opacity Project" has been undertaken under the leadership of Seaton. The results have been compiled as the TOPBASE atomic database. The exact data for spectral lines and photoionization rates, including the autoionizational resonances have been compiled for all ions of 14 elements, namely for H, He, C, N, O, Ne, Na, Mg, Al, Si, S, Ar, Ca and Fe. Some of the results have been published in a special issue of "Revista Mexicana de Astronomía y Astrofísica" Vol. 23, 1992, devoted to calculation of astrophysical opacities (see Seaton et al. (1992), Cunto & Mendoza (1992) and other papers in the issue). A lot of data concerning with absorption lines with $\lambda > 912\text{\AA}$ compiled by Morton (1991). Even more vast list of the lines including those in the region $\lambda > 228\text{\AA}$ have been presented by Verner et al. (1994).

For obtaining the values A_{ki} and f_{ik} for hydrogenic ions with $Z > 1$ we can use the next scaling relations (see., e.g., Rudzikas et al. (1990)):

$$A_{ki}^Z = Z^4 A_{ki}^{\text{HI}},$$

and

$$f_{ki}^Z = f_{ki}^{\text{HI}}.$$

The last relation means that the oscillator strengths are the same for all hydrogenic ions. These relations are exact only in the framework of the non-relativistic approximation. For ions with $Z \gg 1$ the relativistic corrections should be taken into account. Recent study of the problem (see Rudzikas et al. (1990) for details) showed that the relativistic corrections are significant only for highly stripped H-like ions ($Z > 50$) and for all astrophysically important hydrogenic ions one can use the non-relativistic scaling relations.

2.2 Photoionization

Photoionization is the main mechanism of ionization of atoms in low-density plasma. The ions of type X^i absorb the photons having energies $h\nu \geq n\nu_0(X^i)$, i.e. the energies which exceed the threshold value of releasing the electrons which usually belong to the external shell

$$X^i(\alpha nlLS) + h\nu \rightarrow X^{i+1}(\alpha L'S') + e,$$

where n is the main quantum number, l is the azimuthal quantum number of photoelectrons, L and S are respectively the total orbital and spin moment of ion X^i and α is an unspecified set of all other quantum numbers which fix the state of atomic residue. The number of acts of photoionization in unit volume per unit time is

$$\dot{N} = n(X^i) \int_{\nu_0(X^i)}^{\infty} \sigma_{nc}(\nu, X^i) 4\pi J_\nu \frac{d\nu}{h\nu}, \quad (2.1)$$

where J_ν is the mean intensity of the ionizing radiation (both the stellar and the diffuse one) in the given point of the ionized medium and σ_{nc} is the effective cross-section of photoionization of ion X^i from level n .

In the conditions of astrophysical low-density plasma (gaseous nebulae) due to large radiation dilution and low gas densities the electrons of atoms are mainly in the ground state. Consequently, for computations primarily the photoionization cross-sections from the ground state are needed. Formation of free electrons due to the photoionization processes takes place from the outermost atomic layer. However, there are special cases, say, in modelling of radiative transfer in the vicinity of quasars and active galactic nuclei which are the intensive X-ray sources, where the photoionization from inner electron shells is important. This holds, for instance, for NI, OIII and SII. Determination of the cross-sections of photoionization σ_{nc} for all atoms, the lines of which are observed in nebular spectra, has been a topic for numerous experimental and theoretical investigations. A detailed list of such studies has been given in papers by Davidson & Netzer (1979), Mendoza (1983), Stasinska (1984) and Verner et al. (1993).

At the present time the values of σ_{1c} are known for most of atoms and ions, the lines of which are observed in the spectra of nebulae. The values can usually be given with about 10% accuracy (excluding the contribution of the resonances) by the following formula convenient for the photoionization and recombination rate computations:

$$\sigma_{1c}(X^i) = \sigma_0 \left[ax^{-s} + bx^{-s-1} + cx^{-s-2} \right] \text{cm}^2, \quad (2.2)$$

where $a+b+c = 1$. Here s is the approximation parameter in the law, describing the dependence of σ_{1c} on frequency ν or on the energy near the ionization threshold which is denoted by subindex "0" and $x = \nu/\nu_0(X^i) = E/E_0(X^i)$. The data on cross-sections of photoionization from the ground and excited states, which have been compiled by us from different published papers, for atoms and ions of elements from H to Si, for S and Ar are given in Table 3, where only the most reliable data were used. The most complete lists of photoionization cross-sections for atoms and ions having the nuclear charge number $Z \leq 30$ are given in the paper by Reilman & Manson (1979) and Verner et al. (1993), where are brought the values of σ_{1c} , found in the Hartree-Slater approximation for energies $5 \text{ eV} \leq E \leq 5 \text{ keV}$. Unhappily in the first paper the dependence $\sigma(\nu)$ has been given in the form of a table. Nevertheless, in Table 3 there are given the values of approximation parameters, found based on the results by Reiman & Manson (1979) in the case if in the other papers the corresponding data were lacking. Though the lines of Li, Be and B in nebular spectra have not been detected yet, their inclusion in Table 3 can turn out to be useful. References to the calculations of the photoionization cross sections for individual atoms and ions can be found in Appendix A.

2.2.1 Photoionization from K and L shells

The common formulae have been derived for an idealized model picture for the photoionization cross sections from K and L shells. These cross sections can be simply expressed using the

functions

$$f_n(\nu) = \frac{2^7 \pi \sigma_T}{\alpha^3 Z^2} \left(\frac{I_n}{h\nu} \right)^4$$

and

$$\Phi_n(\nu) = \frac{\exp[-4\eta^{2-n} \text{ctn}^{-1}(\eta/n)]}{1 - \exp(-2\pi\eta)},$$

where σ_T is the Thompson scattering cross section, $\eta^2 = I_n/(h\nu - I_n)$ and I_n is the ionization energy from the shell with principal quantum number n . For the K-shell electrons we can write (Lang (1974), Akhiezer & Berestetsky (1969))

$$\sigma_{1s} = f_1(\nu) \cdot \Phi_1(\nu),$$

for the L-shell electrons in the $2s$ state

$$\sigma_{2s} = 8 \left(1 + 3 \frac{I_2}{h\nu} \right) f_2(\nu) \cdot \Phi_2(\nu),$$

and in the $2p$ state by

$$\sigma_{2p} = 16 \frac{I_2}{h\nu} \left(3 + 8 \frac{I_2}{h\nu} \right) f_2(\nu) \cdot \Phi_2(\nu).$$

Daltabuit & Cox (1972) have represented the effective cross-sections of photoionization from K-shell of H, He, C, N, O and Ne as a special case of (2.2) in the form

$$\sigma_{1c}(X^i) = \sigma_0 \left[ax^{-s} + (1-a)x^{-s-1} \right] \text{cm}^2. \quad (2.3)$$

This approximation is applicable in the region of ionization threshold and at moderate energies. The values of approximation parameters for these atoms and some of their ions (in units 10^{-18}cm^2) are given in Table 4.

The photoionization cross section for the atoms of H and for the hydrogenic ions from the states with the principal quantum number n can be written in the form

$$\sigma_n = \frac{2^4 e^2 I_z^2}{3\sqrt{3} mch^2 n^5 \nu^3} \cdot g_n(\nu) \quad (2.4)$$

Here $g_n(\nu)$ is the Gaunt correction factor to the Kramers approximation and I_z is the ionization energy of ion with charge number Z from its ground state.

A simple approximation formula to the complicated exact formulae found by Karzas & Latter (1961) and Goldwire (1968) for the Gaunt factor of H and hydrogenic ions has been found by Sapor & Kuusik (1974). It has high precision (more than 1%) for wide energy range for all states with $n > 2$ and only for $n = 1$ it reaches 3% in a narrow energy interval. The formula has the form:

$$g_n = g_n^I / [1 + 0.02494 (2.4 - \frac{0.014 x^y}{1 + 0.01 x^y}) x^{5/6}], \quad (2.5)$$

where g_n^I is the Gaunt factor in the well-known first approximation found by Menzel & Pekeris (1935)

$$g_n^I = 1 - 0.1728 x^{1/3} (\frac{2}{n^2 x} - 1).$$

In these formulae $x = h\nu/I_z$ and

$$y = 0.43 + 0.6 \log(x + 10).$$

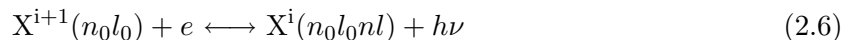
At large energies σ_n is proportional to $\nu^{-3.5}$.

In determination of photoionization cross sections for different atoms it is necessary to take into account not only the direct photoionization, but also ionization from intermediary autoionization states. The autoionization states are dielectronic excited states with excitation energy exceeding the ionization energy of outermost single electron from ground state of the atomic particle. However, the energy of each excited electron is less than the ionization energy. From the autoionization state an electron can transit to lower bound states with photon emission (this process is called radiative stabilization) or there can take place autoionizational stabilization, where one electron is deliberated from atom but the other goes to some bound state. Due to presence of autoionization states there appear autoionizational resonances of photoionization

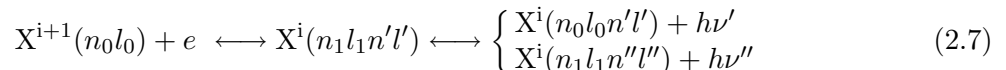
cross sections. This circumstance must be taken into account in modelling of ionization state and structure of gaseous nebulae and other low-density astrophysical objects. If we ignore the autoionization phenomena then we can get the results but with 20–30% precision. The autoionizational resonances have been studied in detail in the framework of the above-mentioned "Opacity Project" (see Seaton et al. (1992))

2.3 Photorecombination

Free electron can recombine on a level of ion X^{i+1} forming thus ion X^i . The process of recombination can be the radiative, the dielectronic or the triple one. The last process is usually negligible in conditions of low-density astrophysical plasma. Photorecombination rate can be easily obtained using the condition of its detailed equilibrium (see, for example, Sobolev (1985)) with the photoionization rates in the case of complete thermodynamical equilibrium. The processes of radiative and dielectronic recombination and their inverse processes can be described by the following schemes (see, for example, Nikitin et al. (1988)) for photorecombination



and for dielectronic recombination



where the quantum numbers of type nl specify the ion states. The electron capture by neutral atoms leads to formation of negative ions. Such processes, however, are negligible for nebulae.

In photorecombination processes the electrons are captured on the discrete levels nl with emission of photons having energies $h\nu_{ic} = E + I_i$ where E is the energy of recombining electron and I_i is the ionization potential of the level $i = nl\gamma$, where γ denotes the set of complimentary to nl quantum numbers of the state i . The radiative cascade transitions between the discrete

levels following recombination generate the recombinational spectral lines.

The most important quantity determining the ionization degree of the atom is the sum of the recombination rates onto all atomic states (total recombination rate):

$$\alpha^{\text{rad}} = \alpha^{\text{A}} = \sum_{n=1}^{\infty} \alpha_n(T_e)$$

and the same value excluding the recombinations onto the ground state:

$$\alpha^{\text{B}} = \sum_{n=2}^{\infty} \alpha_n(T_e) = \alpha^{\text{rad}} - \alpha_1(T_e).$$

Here $\alpha_n(T_e)$ is the recombination rate onto level n .

For hydrogen and hydrogenic ions about one half of all recombination acts proceed straight into the ground state of ion X^{i+1} , and the rest of them recombine onto the excited states.

The total number of recombinations in unit volume per unit time is

$$\dot{N}_{\text{r}} = n(X^{i+1}) n_e \alpha^{\text{rad}}(X^{i+1}), \quad \text{cm}^3/\text{s}. \quad (2.8)$$

The total recombination rate is often approximated by the expression

$$\alpha^{\text{rad}}(X^{i+1}) = A_{\text{rad}} [T_e/10^4 \text{ K}]^{\chi_{\text{rad}}}. \quad (2.9)$$

The numerical values of recombination rates α^{rad} for a large number of ions have been found by Aldrovandi & Pequignot (1973, 1976), Woods et al. (1981) and Shull & Van Steenberg (1982). The values of parameters A_{rad} and χ_{rad} for all ions of elements from C to Ni taken from the last mentioned paper are given in Table 5. The lacking numerical values of these parameters for ions of Ar, Ca and Ni in the last paper have been found by interpolation of the values in the corresponding isoelectronic sequences using the scaling relations as given in item 2.5. For studying an ionization balance in cold low density plasma ($T \ll 1e4 \text{ K}$ (for example HI regions of interstellar medium)) the values of radiative recombination rates for such temperatures are

needed. In this temperature rang the slightly different from Eq.2.9 expression

$$\alpha^{\text{rad}}(\text{X}^{i+1}) = A_{100} [T_e/100 \text{ K}]^{\chi_{\text{rad}}} \quad (2.9)$$

can be in operation. Fit parameters A_{100} and χ_{rad} for radiative recombinations rates of some singly ionized atoms with ionization potentials (< 13.6 eV) taken from Péquignot & Aldrovandi (1986) are given in the next table.

Fit parameters for radiative recombinations rates of
singly ionized atoms with low ionization potentials

in the interval 10 – 1000 K					
Atom	A_{100}	χ_{rad}	Atom	A_{100}	χ_{rad}
C*	8.29-12	0.621	Cl*	8.10-12	0.607
Li*	9.60-12	0.606	K	5.54-12	0.683
Na	5.82-12	0.682	Ca	5.58-12	0.683
Mg	5.87-12	0.681	Ca+	2.79-11	0.647
Al*	1.54-11	0.567	Ti	5.50-12	0.684
Si*	8.42-12	0.617	Mn	5.45-12	0.686
P	6.98-12	0.645	Fe	5.45-12	0.686
S *	1.05-11	0.593	Ni	5.56-12	0.681

All fits are better than 3% in 20-500 K and than 6% in 10-1000K.

Fits better than 2.5% in 10-1000 K are marked by an asterisk.

The recombination rates α^{rad} for the hydrogenic ions have been approximated by Seaton (1959) in the form

$$\alpha^{\text{rad}}(T_e) = 5.197 \cdot 10^{-14} Z \beta^{1/2} S^A, \quad (2.10)$$

where Z is the nuclear charge, $\beta = I/kT_e = 157890Z^2/T_e$, where I is the atomic ionization potential and $S^A = 0.4288 + 0.5 \ln \beta + 0.469\beta^{-1/3}$. The error of the approximation (2.10) do not exceed about 3% if $T_e \leq 10^6 Z^2$ and do not exceed about 30% if $T_e \leq 5 \cdot 10^6 Z^2$.

The approximate formulae for α^{rad} and α^{B} , for non-hydrogenic ions have been derived by

Tarter (1971). They have the same form as the Eq.(2.10), but

$$S^A = 0.431 + 0.501 \ln \beta + 0.460\beta^{-1/3}, \quad (2.11)$$

and for the recombination rate α^B the expression S^A must be replaced by

$$S^B = -0.493 + 0.504 \ln \beta + 0.857\beta^{-1/3}. \quad (2.12)$$

The error of the fit formula, Eq.(2.10), for non-hydrogenic ions is estimated to be about 3% at characteristic temperatures of gaseous nebulae. Similar calculations of photorecombination rates for Fe ions have been carried out by Woods et al. (1981). The error of the numerical values obtained by them has been estimated to be about 10%.

An extensive compilation of recombination rates on the levels of hydrogenic, He - like and Li - like ions has been given in a paper by Arnaud & Rothenflug (1985) where the values of α^{rad} for some ions of these sequences have been improved and presented in the form Eq.(2.9).

Important significance for calculating the line emission intensities in the spectra of gaseous nebulae have the recombination rates to the different levels of the most abundant species H^+ , He^0 and He^+ . In Table 6 the values of the recombination rates for lower states of He^+ have been given.

2.4 Photoheating and recombination cooling

This section is based on the paper by Oskinova and Kholtygin (1996). The data for hydrogenic ions (HI and HeII) which provide the main energy gains and losses for the astrophysical plasma are tabulated only here .

2.4.1 Photoheating rates

The mean energy gained by electrons per 1 cm^3 and per second due to photoionization of an atom (ion) species from level i is specified by the mean intensity of ionizing radiation $\bar{J}_n u$ at photon frequency ν and by photoionization cross section for this level $\sigma_i^{\text{phi}}(\nu)$:

$$n_i \Gamma_{ic} = n_i \int_{\nu_i^0}^{\infty} \sigma_i^{\text{phi}}(\nu) \cdot \frac{4\pi \bar{J}_\nu(T_*)}{h\nu} \cdot (h\nu - h\nu_i^0) d\nu. \quad (2.13)$$

Here Γ_{ic} is the heating rate, index c is hold for designation of the continuum state, n_i is the level i occupation number, ν_i^0 is the treshold value of the frequency for ionization from level i .

Intensity of the ionizing radiation at given point is determined both by the frequency distribution of the source of radiation and by the optical distances τ_ν of the screening medium. As a model we consider a point source of the ionizing radiation in the spherically-symmetric gaseous envelope. This model is good both for the gaseous nebulae and for stellar envelopes. The Planck function is usually a good approximation for radiation of the astrophysical sources. Taking into account the dilution and extinction of radiation in the medium we have

$$\bar{J}_\nu = \bar{J}_\nu(T_*) = W B_\nu(T_*) e^{-\tau_\nu}, \quad (2.14)$$

where W is the dilution coefficient and τ_ν the optical distance in the frequency ν to the ionizing source. The dilution coefficient

$$W = \frac{1}{2} \left(1 - \sqrt{1 - \left(\frac{R_*}{R} \right)^2} \right). \quad (2.15)$$

Here R is the distance to the ionizing source and R_* is the radius of the ionizing source. The optical distance τ_ν is connected with the value τ_i^0 of this quantify at the treshold frequency $\nu = \nu_i^0$ via relation

$$\tau_\nu = \tau_i^0 \frac{\sigma_i^{\text{phi}}(\nu)}{\sigma_i^0} = \tau_i^0 f_i(\nu), \quad (2.16)$$

where $\sigma_i^0 = \sigma_i^{\text{phi}}(\nu_i^0)$ and

$$\tau_i^0 = \int_{R_*}^R \sigma_i^0 n_i(R) dR. \quad (2.17)$$

The energy of photoelectron E can be expressed in dimensionless threshold units $u = (h\nu - I_i)/I_i = E/I_i$, where $h\nu$ is the photon energy, $E = mv^2/2$ - the photoelectron energy and I_i - the ionization potential from the level i . Frequency ν of ionizing photon can be written in threshold units as $\nu = \nu_i^0(1 + u)$. Substituting expression (2.14) into Eq.(2.13) and using the threshold units, one obtains

$$\Gamma_{ic} = W \cdot G_{ic} = W \frac{1}{8\pi^2} \cdot \frac{c\alpha^3}{a_0^3} \left[\frac{I_i}{Ry} \right]^3 \cdot I_i \cdot \mathcal{J}(\beta_i^*, \tau_i^0). \quad (2.18)$$

Here $\beta_i^* = I_i/kT_*$, and

$$\mathcal{J}(\beta_i^*, \tau_i^0) = \int_0^\infty \frac{u(1+u)^2 \sigma_i^{\text{phi}}(\nu) e^{-\tau_i^0 f_i(u)}}{e^{\beta_i^* (u+1)} - 1} du. \quad (2.19)$$

Here $\alpha = 1/137.036$ is the fine structure constant, $a_0 = 5.2918 \cdot 10^{-9}$ - the Bohr radius and c is the velocity of light.

In the case of photoheating and recombination cooling processes the level splitting of hydrogenic ions onto nl sublevels is not essential, so one can use the cross sections averaged over values l :

$$\sigma_n(\nu) = \frac{1}{n^2} \cdot \sum_l (2l+1) \sigma_{nl}(\nu).$$

Confining in our expressions with the second order Gaunt correction terms, the averaged photoionization cross sections from level n can be written in the form

$$\sigma_n(\nu) = \sigma_n^0 \frac{1}{(1+u)^3} \sum_{k=0}^2 \frac{B_k^{(n)}}{(1+u)^k}. \quad (2.20)$$

Here $\sigma_n^0 = n \cdot \mathcal{G}_n \cdot 7.930 \cdot 10^{-18} / Z^2 \text{cm}^2$ is the threshold value of the cross section from level n , where Z is the ion charge; $B_k^{(n)} = g_k^{(n)} / \mathcal{G}_n$ where the quantities $g_k^{(n)}$ are the coefficients of expansion of the Gaunt factor $g_n(\nu)$ for level n onto the powers of $1/(1+u)$, given by Johnson (1972). The sum $\mathcal{G}_n = g_0^{(n)} + g_1^{(n)} + g_2^{(n)}$. Substituting the expansion (2.20) into the Eq.(2.19)

we find

$$\mathcal{J}(\beta_n^*, \tau_n^0) = \sigma_0 \sum_{k=0}^2 B_k^{(n)} \left[\mathcal{Q}_k^f(\beta_n^*, \tau_n^0) - \mathcal{Q}_{k+1}^f(\beta_n^*, \tau_n^0) \right],$$

where

$$\mathcal{Q}_k^f(\beta^*, \tau) = \int_1^\infty \frac{e^{-\tau f(x)}}{x^k (e^{\beta^* x} - 1)} dx,$$

where $x = 1 + u$ and function $f(x)$ describes the frequency dependence of the photoionization cross section, given by Eq. (2.16).

Fig. 2.1 illustrates the dependence of HI heating rates on the level number n . Similar dependence holds for HeII. The calculated heating rates for the $n=1-4$ levels of HI and HeII at $\tau = 0$ are presented in Table 7.

For the case if $\tau \neq 0$ we should take into account the dependence of the values G_{nc} on τ . This dependence can be presented in the form

$$G_{nc}(\tau) = e^{-k_n \tau_n^0} G_{nc}(0), \quad (2.21)$$

where $G_{nc}(0)$ is the heating rate at $\tau_n^0 = 0$, and k_n is a slowly varying function of τ .

Values of parameter $k_1(\tau)$ for the ground ($n=1$) levels of HI and HeII are compiled in Table 8. Calculations by Kholtygin (1988) have demonstrated that the occupation numbers for $n > 2$ levels of HI and HeII are very small even for dense outflowing envelopes of the WR stars. As a result, the corresponding total optical depths of the envelopes are also small ($\tau_n^0 < 0.01$ for $n \geq 2$). These optical depths values are even smaller for envelopes (atmospheres) of other kinds of stars and gaseous nebulae. This means that one can use the heating rates presented in Table 7 for all $n > 2$ levels of HI and HeII.

Cooling by spontaneous recombinations

Mean energy loss by electrons per cm^3 and per second due to spontaneous electron recombinations with an ion X^+ onto the level i of atom (or ion) X is determined by photorecombination cross section onto level $\sigma_i^{\text{phr}}(\nu)$:

$$n_e L_i(T_e) = n_e \int_0^\infty \sigma_i^{\text{phr}}(v) v f(v) \frac{mv^2}{2} dv. \quad (2.22)$$

Here $L_i(T_e)$ is the partial cooling rate for recombination onto level i . Adopting for the electron velocity distribution the Maxwell function and using the threshold units, we can write:

$$L_i(T_e) = \frac{c \alpha^3}{2 \sqrt{\pi}} \cdot \frac{g_i}{g^+} \left[\frac{I_i}{Ry} \right]^{3/2} \beta_i^{3/2} I_i \mathcal{L}(\beta_i), \quad (2.23)$$

where

$$\mathcal{L}(\beta_i) = \int_0^\infty u(1+u)^2 e^{-\beta u} \sigma_i^{\text{phr}}(u) du.$$

Analytical presentation of the cross sections (2.20) gives

$$\mathcal{L}(\beta) = \sigma_0 e^{-\beta} \sum_k B_k [E_k(\beta) - E_{k+1}(\beta)].$$

Here $E_q(\beta)$ is the integral exponent (see for details Adramowitz & Stegun (1964)). The total recombinational cooling rate $L(T_e)$ is the sum of the partial rates (2.23). The calculated rates are tabulated in the Table 9.

Cooling by stimulated recombinations

For original Planck radiation, which is diluted and weakened by extinction (see Eq.(2.14)), the cooling rate due to the stimulated recombination onto level i

$$R_{ci}^{\text{st}} = W \cdot L_i^{\text{st}} = W \cdot \frac{c \alpha^3}{2 \sqrt{\pi}} \cdot \frac{g_i}{g^+} \cdot \left[\frac{I_i}{Ry} \right]^{3/2} \cdot \beta_i^{3/2} I_i e^\beta \sum_k B_k \cdot \mathcal{H}(\beta_i, \beta_i^*, \tau_i^0), \quad (2.24)$$

where

$$\mathcal{H}(\beta_i, \beta_i^*, \tau_i^0) = \int_0^\infty \frac{u(u+1)^2 \sigma_i^{\text{phi}}(u) e^{(-\beta_i u + \tau_i^0 f_i(u))}}{e^{\beta_i^*(u+1)} - 1} du.$$

Using the analytical expression of the photoionization cross sections (2.20) we have

$$\mathcal{H}(\beta, \beta^*, \tau_i^0) = \sigma_i^0 \sum_{k=0}^2 B_k^{(i)} \left(\mathcal{S}_k(\beta, \beta^*, \tau_i^0) - \mathcal{S}_{k+1}(\beta, \beta^*, \tau_i^0) \right),$$

where

$$\mathcal{S}_k(\beta, \beta^*, \tau) = e^\beta \int_1^\infty \frac{e^{-(\beta x + \tau f(x))}}{x^k (e^{\beta^* x} - 1)} dx.$$

The total stimulated photorecombination cooling rate $R^{\text{st}}(T_e, T_*)$ or $L^{\text{st}}(T_e, T_*)$ is the sum of all partial rates R_{ci}^{st} or L_{ci}^{st} , respectively. Calculations (Oskinova and Kholtygin (1996)) show that the total stimulated photorecombination cooling rates depend very weakly on the optical depth of the ionized plasma.

The total recombination cooling rate

Total cooling rates is the sum of the spontaneous recombination and stimulated recombination cooling rates given above, i.e.

$$L^{\text{tot}}(T_e) = L(T_e) + W \cdot L^{\text{st}}(T_e, T_*).$$

Table 9 incorporates the total cooling rates for both spontaneous and stimulated photorecombinations. From the table one sees the evident circumstance that the contribution of stimulated recombination into the total cooling rates is important only for regions close to the ionizing source ($W > 0.1$).

2.5 Scaling relations for photoionization and photorecombination rates

Some important scaling properties for recombination and ionization rates as well as for cooling and heating rates can be easily obtained for hydrogenic ions using the dependence of

their photoionization cross sections and level energies on the nuclear charge Z (see, Oskinova and Kholtygin (1996)). Then we have for photoionization rates

$$B_{ic} = B_{ic}(Z, T_e, \tau_i^0) = Z^4 \cdot B_{ic}(1, T_e/Z^2, \tau_i^0), \quad (2.25)$$

and for photoheating rates

$$G_{ic} = G_{ic}(Z, T_e, \tau_i^0) = Z^6 \cdot G_{ic}(1, T_e/Z^2, \tau_i^0). \quad (2.26)$$

Similar equalities hold also for photorecombination (spontaneous) rates

$$\alpha_i(T_e) = \alpha_i(Z, T_e) = Z \cdot \alpha_i(1, T_e/Z^2), \quad (2.27)$$

and for spontaneous recombination cooling rates

$$L_i(T_e) = L_i(Z, T_e) = Z^3 \cdot L_i(1, T_e/Z^2), \quad (2.28)$$

for stimulated photorecombination

$$\alpha_i^{\text{st}}(T_e, T_*) = \alpha_i^{\text{st}}(Z, T_e) = Z \cdot \alpha_i^{\text{st}}(1, T_e/Z^2, T_*/Z^2), \quad (2.29)$$

and for stimulated photorecombination cooling rates

$$L_i^{\text{st}}(T_e, T_*) = L_i^{\text{st}}(Z, T_e, T_*) = Z^3 \cdot L_i^{\text{st}}(1, T_e/Z^2, T_*/Z^2), \quad (2.30)$$

and taking into account Eqs. (2.27-2.30) similar equations hold for the total recombination and cooling rates.