V.V. Sobolev and Physics of Gaseous Nebulae

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First works by V.V. Sobolev were devoted to gaseous nebulae. He proposed a method for determining the nebula temperature based on the analysis of the energy balance of the electron gas. He also investigated the transfer of L_α and L_c radiation and the role of light pressure in their dynamics. The works by V.V. Sobolev were at the bases of constructing the ionization models of gaseous nebulae. The Sobolev method is widely used in solving the problems of radiation transfer in the nebulae. In our report we review the impact of the ideas and methods proposed by V.V. Sobolev on both the classic and contemporary researches in physics of gaseous nebulae.

1 Introduction

There were about 200 papers published in 1800 − 1941 (according to ADS) before V.V. Sobolev started to study gaseous nebulae. First planetary nebulae, the Dumbbell Nebula M 27, was discovered by Ch. Messier in 1764 (see, e.g., [10]). W. Herschel proposed in 1791 that PNe derive their energy from a nearby star.

The first step towards an understanding of the nature of PNe was made by W. Huggins in 1864 with his spectroscopic observation of NGC 6543 [5]. He saw a single bright line in the spectra. Subsequent observations, with better resolution, showed that this bright line was actually the famous N_1+N_2 doublet.

Studying H_β, Menzel (1926) suggested that all the stellar outputs beyond the Lyman limit (912 Å) should be utilized to ionize the hydrogen atom [11]. In 1927 Zanstra supposed that H-lines are the result of the recombination of ionized hydrogen [20]. In 1928 Bowen identified the 8 strongest nebular lines as being due to metastable states of NII, OII and OIII [3].

In 1929 Perrine interpreted the broad (or even split) emission lines in PNe spectra as a result of an expansion of PNe [13] (not rotation as had been supposed before). In 1933 Ambartsumian found that the mean electron temperature of PNe is about of 10000 K [2].

For determination of the parameters of central stars (CS) of PNe, the methods by Zanstra [21] and Stoy [18] were proposed. In 1938 Whipple established that PNe ages are in the interval 10^3–10^5 years [19]. The physics of PNe was considered in the series of 18 papers written by Menzel and Aller in 1937–1945 (see [1]).

Here we review the papers by V.V. Sobolev which influenced on our understanding of the physics of planetary nebulae.

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2 Sobolev's works on physics of gaseous nebulae

V.V. Sobolev started to study the PNe when he was still a graduate student by V.A. Ambartsumian.

Here we list the early papers by Sobolev which were not included in ADS:

1) *V.V. Sobolev,* "Determination of electron temperatures of planetary nebulae and improvement of the method of nebulium to determine the temperature of their central stars," Trudy Astron. Obs. Leningr. Univ., **12**, 3–16, 1941;
2) *V.V. Sobolev,* "Light pressure in the expanding nebula," Astron. Zh., **21**, 143–148, 1944;
3) *V.A. Ambartsumian, E.R. Mustel, A.B. Severnyi, V.V. Sobolev,* Teoreticheskaya Astrophysika. Moscow: GITTL, 1952;
4) *V.V. Sobolev,* "Physics of planetary nebulae," Voprosy Kosmogonii, **VI**, 112–155, 1958.

Results of the paper [1] were included in the PhD thesis of V.V. Sobolev.

A short review of other papers by V.V. Sobolev devoted to physics of PNe is as follows. The diffusion of L_α radiation in nebulae and stellar envelopes was studied in [15]. The problem of the brightness of a spherical nebula was considered in [16]. The scattering of radiation of the central star in a spherical nebula was investigated by Kolesov and Sobolev in [7, 8].

Many problems devoted to physics of the nebulae were generalized in the famous Sobolev's Course in Theoretical Astrophysics [17].

3 Energy balance in gaseous nebulae

In his paper [14] V.V. Sobolev used the law of energy conservation in the following form:

$$n_1 \alpha \varepsilon = n_\mathrm{e} n^+ \sum_{i=1}^{\infty} \varepsilon_i + \mathsf{E} + n_1 n_\mathrm{e} \left(\gamma h\nu_0 + \sum_{i=2}^{\infty} D_i h\nu_{1i} \right). \tag{1}$$

In the left part of the equation the energy gained by electrons due to photoionization of the hydrogen atom is given, in the right part the energy losses are enumerated. Here n_1 is the population of the first atom level, α is the rate of ionization of the neutral hydrogen atom due to the radiation of the central star of the PN, and ε is the mean energy obtained by photoelectron after photoionization.

The value n_e is the electron number density, n^+ is the number density of ionized hydrogen atoms, ε_i is the part of energy lost by electrons at recombination to level i, E is the energy spent to emission in OIII lines N_1 ($\lambda\,5007\,\mathrm{Å}$) and N_2 ($\lambda\,5007\,\mathrm{Å}$, see Fig. 1, left panel), $h\nu_0$ is the energy of the hydrogen atom ionization from the ground level, γ is the rate of the H atom ionization by electron collisions, and D_i is the rate of energy lost by electrons due to collision excitation from

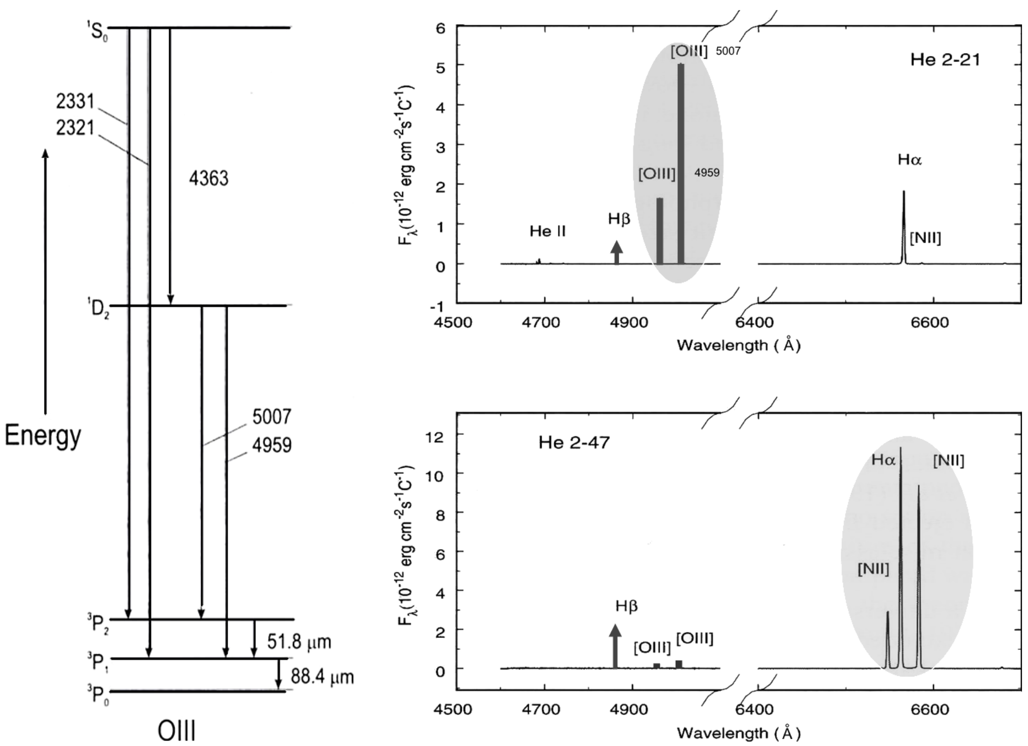

Figure 1: Left panel: ground and metastable OIII levels and corresponding forbidden lines. Right panel: spectra of the high and low excitation nebulae (based on Fig. 12.3 in [9]).

the ground level to level i (in units of $h\nu_1$ being the energy of excitation of the level i).

The left part of Eq. (1) was calculated by Sobolev, supposing that the emission of the central star (CS) of the PNe can be presented as the black-body emission with the temperature emission and for $\propto \nu^{-3}$ dependence of the photoionization cross sections of the H atoms with the ground level $i = 1$.

The same approximation for a dependence of the photoionization cross sections of the H atoms for an arbitrary level i was used to calculate the first term in the right part of Eq. (1).

In the time when Sobolev wrote his paper the values of effective cross sections for excitation of the upper levels of ion O^{2+} were unknown, so he estimated the value of E via the line intensity ratio $I(N_2)/I(H_\beta)$ supposing that the ratio $I(N_1)/I(N_2) = 3$. Then

$$\mathsf{E} = \int E dV = 4\frac{I(N_2)}{I(H_\beta)} A_{42} h\nu_{42} \int n_4 dV, \tag{2}$$

where A_{42} is the transition probability for line H_β, $h\nu_{42}$ is the energy of the transition $4 \to 2$ and n_4 is the population of the level $n = 4$ of hydrogen. It means that $\mathsf{E} \propto I(N_2)/I(H_\beta)$.

Table 1: Calculations by Sobolev in a comparison with the data of other authors

Nebular parameters						Contribution		
Name	$\frac{I(\mathrm{N}_2)}{I(\mathrm{H}_\beta)}$	$\frac{T_*^{VVS}}{10^3\mathrm{K}}$	$\frac{T_*}{10^3\mathrm{K}}$	$\frac{T_\mathrm{e}^{VVS}}{10^3\mathrm{K}}$	$\frac{T_\mathrm{e}}{10^3\mathrm{K}}$	Recombination	N_1+N_2	Excitation HI
IC 4593	1.7	25	28^a	13	6.0^c	0.25	0.60	0.15
NGC 6543	1.6	41	47^a	16	7.9^c	0.20	0.30	0.50
NGC 6572	2.4	48	66^a	18	8.9^c	0.15	0.40	0.45
NGC 6826	2.0	29	33^a	13	8.3^c	0.20	0.60	0.20
NGC 7009	3.1	45	98^b	15	6.5^c	0.15	0.55	0.30
NGC 7662	3.7	76	118^b	19	12.3^c	0.10	0.30	0.60

Notes: [a]Kaler, Jacobi (1991), [b]Capriotti, Kovach (1968), [c]Milanova, Kholtygin (2009).

The last term in Eq. (1) is determined by the value of n_1 which is the mean population of the ground level of hydrogen. For PNe, the ratio $n_1/n(\mathrm{H}) \ll 1$, where $n(\mathrm{H})$ is the full hydrogen number density. It means that the last term in Eq. (1) is proportional to n_1/n^+.

The final equation connecting the black-body temperature of the CS T_* and the mean electron temperature of the nebula T_e is as follows (Eq. 19 in [14]):

$$AT_* = BT_\mathrm{e} + C\,\frac{I(\mathrm{N}_2)}{I(\mathrm{H}_\beta)} + D\,\frac{n_1}{n^+}. \tag{3}$$

Here the ionization ratio n^+/n_1 can be determined using the equation of the ionization balance

$$\frac{n_\mathrm{e} n^+}{n_1} = W\sqrt{\frac{T_\mathrm{e}}{T_*}}\,\frac{(2\pi mkT_*)^{3/2}}{h^3}e^{-\frac{h\nu_0}{kT_*}}, \tag{4}$$

where W is the dilution factor, m is the mass of electron, k is the Boltzmann constant, h is the Planck constant, and $h\nu_0$ is the ionization potential for the ground level of H.

The coefficients A, B, C, D were calculated by Sobolev for $T_*/10^3\,\mathrm{K} \in [20, 80]$ and $T_\mathrm{e}/10^3\,\mathrm{K} \in [1, 50]$ (see Tables 1, 2 in [14]). He used these calculations to estimate the parameters T_e and $T*$ for selected PNe.

In Table 1 we list the values T_*^{VVS} and T_e^{VVS} obtained by Sobolev in [14] (columns 3 and 5) in a comparison with the data obtained in papers [4, 6, 12] (columns 4 and 6). In the columns 7–9 of Table 1 the contributions of different sources of cooling in the total cooling rate in Eq. (1) calculated by Sobolev are presented.

It is worth to note that the temperature of the central stars of the PNe obtained in the cited papers essentially exceeds the values given by Sobolev. Conversely, Sobolev's electronic temperatures for these PNe (in column 5) are significantly higher than the modern data by Milanova and Kholtygin [12] (column 6).

Table 2: Corrected cooling process contributions

Name	Contribution			$q(N_1+N_2)$
	Recombination	All collisions	Excitation HI	
IC 4593	0.13	0.87	–	0.66
NGC 6543	0.08	0.92	0.0002	0.32
NGC 6574	0.06	0.94	0.0003	0.31
NGC 6826	0.13	0.87	0.002	0.65
NGC 7009	0.03	0.97	–	0.24
NGC 7662	0.04	0.95	0.01	0.23

The main reason for this discrepancy in our opinion is underestimation of the energy loss due to excitation of the metastable levels of not only OIII but numerous ions of the other elements. If we look at Fig. 1 (right panel) we see that the fluxes of N_1+N_2 lines in the spectra of some nebulae are small in a comparison of the fluxes of other forbidden lines. It means that one has to add the new terms in the value E in Eq. (1).

The corrected energy balance equation (3) can be rewritten as

$$AT_* = BT_\mathrm{e} + E' + D\frac{n_1}{n^+}, \tag{5}$$

where E' is the corrected value of the energy losses due to excitation of *all* collisional transitions.

To estimate the value of E' we use the following procedure. Firstly, we take more exact than obtained by Sobolev values of T_* and T_e, which are given in the columns 4 and 6 of Table 1. Secondly, we calculate the coefficients A, B, D and the ratio n_1/n^+, using the data of Tables 1–4 in the paper [14] for these updated values of T_* and T_e.

Substituting those coefficients into Eq. (5), one can evaluate the full energy loss value E' and the ratio

$$q(\mathrm{N}_1 + \mathrm{N}_2) = C\left(\frac{I(\mathrm{N}_2)}{I(\mathrm{H}_\beta)}\right) \Big/ E',$$

the fraction of the energy excitation of N_1+N_2 lines in all collision energy losses E'.

In the columns 2–4 of Table 2 we give the corrected values of the relative contributions of the cooling processes for the same nebulae which are considered in Table 1. In the last column we present the ratio $q(N_1+N_2)$. Comparing Tables 1 and 2, we can conclude that the part of the energy losses which is spent to the excitation of hydrogen is negligible due to the lower electron temperatures than accepted in [14]. In the same time the recombination losses appear to be significantly less important than they were estimated by Sobolev. This result is in an agreement with Sobolev's main assumption that the collision processes give the most contribution to the energy loss by electron in the PNe.

4 Conclusions

Sobolev's works on physics of gaseous nebulae were among the first Foundation Stones of physics of planetary nebulae. Our review of Sobolev's work dedicated to physics of the nebulae showed that the main ideas proposed by Sobolev appear to be correct, but due to the poor knowledge of the atomic parameters in the time when Sobolev wrote his paper, the important corrections to the parameters of the PNe obtained by him have to be done.

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