Radiative Transfer and Spectra in Stochastic Atmospheres

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Various cosmic objects – stars, active galactic nuclei, accretion discs, etc., suffer the stochastic variations of temperature, large and small scales gas motions, magnetic fields, number densities of atoms and molecules. These stochastic variations give rise to changes of absorption factors, Doppler widths of lines and so on. The existence of numerous reasons for fluctuations lead to a Gaussian distribution of fluctuations. The observed spectra represent quantities averaged over the time and space. The common model explanations do not include the effect of fluctuations. In many cases, the consideration of fluctuations improves the agreement between theoretical explanations and observed values.

1 The radiative transfer equation in stochastic atmosphere

In a stochastic atmosphere the absorption factor has a fluctuating component: $\alpha = \langle\alpha\rangle + \alpha' \equiv \alpha^{(0)} + \alpha'$, $\langle\alpha'\rangle = 0$. The change of radiation intensity along the path s is determined by the equation

$$dI(\mathbf{n}, s) = -[\alpha^{(0)}(s) + \alpha'(s)]\, I(\mathbf{n}, s)ds. \tag{1}$$

The solution of this equation is

$$I(\mathbf{n}, s) = I(\mathbf{n}, 0)\exp\left[-\int_0^s ds'(\alpha^{(0)}(s') + \alpha'(s'))\right] \equiv I(\mathbf{n}, 0)\exp\left(-(\tau^{(0)} + \tau')\right). \tag{2}$$

The average of this expression, adopting for Gaussian probability distribution for fluctuations, gives

$$\langle I(\mathbf{n}, s)\rangle = I(\mathbf{n}, 0)\exp\left[-\left(\tau^{(0)} - \frac{1}{2}\langle\tau'^2\rangle\right)\right] \equiv I(\mathbf{n}, 0)\exp\left(-\tau_{eff}(s)\right). \tag{3}$$

It is seen from this expression that the averaged intensity $\langle I(\mathbf{n}, s)\rangle$ in stochastic medium decreases with the distance weaker than when accounting for the mean absorption factor $\alpha^{(0)}$.

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V. Grinin et al. (eds) Radiation mechanisms of astrophysical objects. Yerevan: Edit Print, 2017, pp. 93–95.

The radiative transfer equation for $\langle I(\mathbf{n},s)\rangle$ has the form

$$\begin{aligned}\frac{d\langle I\rangle}{ds} = &- \left[\alpha^{(0)}(s) - \langle\alpha'(s)\tau'(s)\rangle\right]\langle I\rangle + \left[\alpha^{(0)}_{sc}(s) - \langle\alpha'_{sc}(s)\tau'_{sc}(s)\rangle\right] \\ &\times \int d\mathbf{n}'\,\kappa(\mathbf{n}\cdot\mathbf{n}')\langle I(\mathbf{n}',s)\rangle + \langle S\rangle.\end{aligned} \tag{4}$$

Here $\kappa(\mathbf{n}\cdot\mathbf{n}')$ is the phase function, $\langle S(\mathbf{n},s)\rangle$ is the averaged source function. Note that $\alpha(s) = \alpha_{sc}(s) + \alpha_{abs}(s)$. The detailed derivation of radiative transfer equations for all Stokes parameters in turbulent magnetized atmosphere is presented in [1].

2 Influence of Doppler width fluctuations on the center of absorption lines

The centers of absorption lines have a Gaussian form, i.e. the broadening is determined by Doppler's mechanism. Fluctuations of the thermal $u_{th} = u^{(0)}_{th} + u'_{th}$ and turbulent $u_{turb} = u^{(0)}_{turb} + u'_{turb}$ velocities give rise to fluctuations of the Doppler width

$$\Delta\lambda_D = \Delta^{(0)}_D + \Delta\lambda'_D, \quad \langle\Delta\lambda'_D\rangle = 0. \tag{5}$$

The level of fluctuations η is determined by the ratio $\eta = |\Delta\lambda'_D|/\Delta\lambda^{(0)}_D$. The averaged value of absorption factor up to $\eta \le 0.3$ has the form

$$\begin{aligned}\langle\alpha_\lambda(x)\rangle = &\left\langle \frac{\alpha_0}{\Delta\lambda_D}\exp\left[-\left(\frac{\lambda-\lambda_0}{\Delta\lambda_D}\right)^2\right]\right\rangle \\ &\simeq \alpha^{(0)}(x)\exp\left(3x^2\eta^2\right)\left[(1+\eta^2)\cosh\left(2\eta x^2\right) - \eta\sinh\left(2\eta x^2\right)\right],\end{aligned} \tag{6}$$

where $x = (\lambda-\lambda_0)/\Delta\lambda^{(0)}_D$ and $\alpha^{(0)}(x) = (\alpha_0/\Delta\lambda^{(0)}_D)\exp(-x^2)$. It is seen from this expression that in the center of the line ($x = 0$)

$$\langle\alpha_\lambda(0)\rangle \simeq \alpha^{(0)}(0)\,(1+\eta^2). \tag{7}$$

Thus, the stochastic effect gives rise to additional increase of line's depth. Note that non-LTE models and also synthetic spectra often lead to an increase in the depth of the absorption line. Nevertheless, sometimes this increase is insufficient to explain the observed depth of the line. In these cases the consideration of stochastic effect may help.

3 The influence of fluctuations on spectra in the continuum

The convective and turbulent motions and the magnetic field evolution lead to temperature fluctuations in stellar atmospheres, active galactic nuclei and other cosmic objects. First, we demonstrate the nature of statistical effects by considering two realizations with temperatures $T_0 + T'$ and $T_0 - T'$. The mean value of Planck function (in Wien's limit) is equal to

$$\begin{aligned} \langle B_\lambda(T)\rangle &= \frac{1}{2}\,\frac{2hc^2}{\lambda^5}\left\{\exp\left[-\left(\frac{h\nu}{k(T_0+T')}\right)\right] + \exp\left[-\left(\frac{h\nu}{k(T_0-T')}\right)\right]\right\} \\ &\simeq B_\lambda(T_0)\cosh\left(\frac{h\nu}{kT_0}\,\frac{T'}{T_0}\right) \geq B_\lambda(T_0). \end{aligned} \tag{8}$$

Here we accepted $T' \ll T_0$. This simple example demonstrates that the averaged value $\langle B_\lambda(T_0)\rangle$ is larger than $B_\lambda(T_0)$. Analogously one can see that the averaged value of absorption factor $\langle \alpha_\lambda(T)\rangle$ may be either larger than $\alpha_\lambda(T_0)$ or smaller than this value, depending on specific form of function $\alpha_\lambda(T)$. The averaged radiation flux $\langle H_\lambda\rangle$ with allowance for temperature fluctuations is determined as

$$\langle H_\lambda\rangle = 2\pi\int_0^\infty d\langle\tau_\lambda\rangle\int_0^1 d\mu\,\exp\left[-\frac{\langle\tau_\lambda\rangle}{\mu}\right]\frac{\langle\alpha_\lambda(T)B_\lambda(T)\rangle}{\langle\alpha_\lambda(T)\rangle}, \tag{9}$$

where $d\langle\tau_\lambda\rangle = \langle\alpha_\lambda(T)\rangle ds$ determines the averaged optical length. It is interesting that in the Wien limit $\langle H_\lambda\rangle$ can be derived directly from observed spectrum, from the first and second derivatives over λ of the observed spectrum. The fluctuation effects are considered in the papers [2–4].

References

1. *N.A. Silant'ev*, Astron. Astrophys., **433**, 1117, 2005.
2. *N.A. Silant'ev, G.A. Alekseeva, V.V. Novikov*, Astrophys., **54**, 642, 2011.
3. *N.A. Silant'ev, G.A. Alekseeva, V.V. Novikov*, Astrophys. Space Sci., **342**, 433, 2012.
4. *N.A. Silant'ev, G.A. Alekseeva*, Astron. Astrophys., **479**, 207, 2008.