Polarization of Resonance Lines in the Case of Polarized Primary Sources of Radiation

A.V. Dementyev[1]

E-mail: *a.dementiev@spbu.ru*

Transfer of polarized radiation in a spectral line in a non-magnetic semi-infinite plane-parallel atmosphere is considered. Complete frequency redistribution is assumed. It is supposed that primary sources of the radiation distributed in the atmosphere are partially polarized. The dependence on the optical depth of these sources is described by the product of a polynomial in the exponent. The problem is to find the radiation emergent from the atmosphere. The general theory of $\widehat{\mathbf{I}}$-matrices is applied to this problem. It turns out that the solution of the problem with any of the primary sources of this type is reduced to the solution of the so-called standard problem, and the subsequent simple numerical integration.

We consider multiple resonance scattering of radiation in a spectral line that takes place in a semi-infinite plane-parallel atmosphere without a magnetic field. Due to the symmetry, the radiation field can be described by two Stokes parameters I and Q. Therefore, the scattering is completely described by the two-component Stokes vector $\mathbf{i}(\tau, x, \mu) = (I, Q)^T$; its arguments are the usual optical depth averaged over line τ, the dimensionless frequency measured from the center of the line x, and the cosine of the zenith angle μ. Also, complete frequency redistribution is assumed. There are polarized primary sources of radiation embedded in the atmosphere; we suggest they are given by the vector function

$$\mathbf{s}_k(\tau) = \tau^k \, e^{-\tau/z_0} \, \mathbf{s}_0, \quad k = 0, 1, 2, \ldots, \tag{1}$$

where $z_0 \in (0, \infty)$ is a parameter, and $\mathbf{s}_0$ is a known constant vector.

In the works [1, 2] the theory of $\widehat{\mathbf{I}}$-matrices was developed. This theory makes possible to generalize a number of well-known results of the standard scalar theory of line formation to the problems when polarization of the radiation is taken into account. The scalar version of the problem considered here was studied in [3]. All the details regarding the theory of $\widehat{\mathbf{I}}$-matrices, for example, the relation of the matrix transfer equation to the vector one, as well as designations used here can be found in [1, 2].

By definition, the Stokes matrix $\widehat{\mathcal{I}}(\tau, z)$ is a solution of the matrix transfer equation

$$z \, \frac{\partial \widehat{\mathcal{I}}(\tau, z)}{\partial \tau} = \widehat{\mathcal{I}}(\tau, z) - \widehat{\mathbf{S}}(\tau). \tag{2}$$

[1] Saint Petersburg State University, Russia

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Here $z \equiv \mu/\phi(x)$, $\phi(x)$ is the line absorption profile, and the matrix source function $\widehat{\mathbf{S}}(\tau)$ is given by

$$\widehat{\mathbf{S}}(\tau) = \widehat{\mathbf{S}}_*(\tau) + \int_{-\infty}^{+\infty} dz'\, \widehat{\mathbf{G}}(z')\, \widehat{\mathcal{I}}(\tau, z'), \tag{3}$$

$$\widehat{\mathbf{S}}_*(\tau) = \mathrm{diag}(s_I^*(\tau),\, s_Q^*(\tau)), \tag{4}$$

where $s_I^*(\tau)$ and $s_Q^*(\tau)$ are the components of the vector source function of the *scattered* radiation, $\widehat{\mathbf{G}}$ is directly related to the phase matrix of resonance scattering.

The problem with $\widehat{\mathbf{S}}_* \equiv \mathrm{diag}(\sqrt{1-\lambda}, \sqrt{1-0.7W\lambda})$ is called standard (λ is the albedo of single scattering, W is the depolarization parameter). It was analyzed in detail and solved numerically in the works [1, 2] and, also, in [4] where absorption in the continuum was taken into account. In particular, it was shown that the Stokes matrix at $\tau = 0$ for the standard problem can be found from the solution of the matrix generalization of the integral Ambartsumian–Chandrasekhar equation. We denote the solution of this equation by $\widehat{\mathbf{I}}(z)$ (it is the $\widehat{\mathbf{I}}$-matrix).

For the source function matrix $\widehat{\mathbf{S}}(\tau)$ of the problem under consideration it is not difficult to write an integral equation similar to the equation for the analogous scalar source function, when the polarization of the radiation is not taken into account. Application of Sobolev's resolvent method in the case of the atmosphere with an exponential distribution of primary sources (i.e., when $k = 0$ in Eq. (1)) provides the following Stokes matrix for the emergent diffuse radiation:

$$\widehat{\mathcal{I}}(0, z) = \widehat{\mathbf{I}}(z) \left[\frac{z_0^2\, \widehat{\mathbf{I}}^T(z_0)}{z_0 + z} \int_{-\infty}^{\infty} \frac{\widehat{\mathbf{F}}(z')\, dz'}{z_0 + z'} - z_0 \int_0^{\infty} \frac{z'\, \widehat{\mathbf{I}}^T(z')\, \widehat{\mathbf{F}}(z')\, dz'}{(z_0 - z')(z' + z)} \right], \tag{5}$$

where $\widehat{\mathbf{F}}$ is expressed through elements of the matrix $\widehat{\mathbf{G}}$.

In general case, if $k > 0$, it is not difficult to show that the Stokes matrix satisfies the recurrence formula

$$\widehat{\mathcal{I}}_k(0, z) = z_0^2 \frac{\partial}{\partial z_0} \widehat{\mathcal{I}}_{k-1}(0, z). \tag{6}$$

Thus, in the case of primary sources (1), the Stokes matrix of the emergent radiation for any of such sources is expressed in terms of the solution of standard problem $\widehat{\mathcal{I}}(z)$ via the equations (5) and (6).

References

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