Behavior of Perturbations in an Accretion Flow on to a Black Hole

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We investigate the behavior of small acoustic perturbations in the spherical adiabatic relativistic accretion flow on to a non-rotating black hole. The Das model of the accretion [1, 2] is a general relativistic generalization of the classical spherical Bondi accretion. We consider a general relativistic linear wave equation for small acoustic perturbations and fulfill the mode analysis of solutions. We find numerically that perturbations remain finite in amplitude on the event horizon due to the effects of the general relativity in contrast to predictions of the non-relativistic model based on the Bondi accretion approximation [3]. This circumstance downranges the possibilities for detection of black holes.

1 Motivation

It is known that converging flows are often subject to hydrodynamic instabilities. For example, small acoustic perturbations have to increase without limit in the spherical adiabatic accretion flow on to a non-rotating black hole [3]. The predictions of the referenced article are based on the non-relativistic Bondi model of the adiabatic spherical accretion on to a point gravitating mass. However, the more realistic model of accretion taking into account the relativistic nature of flow near the event horizon should take proper account of the finiteness of the accretor's radius (the Schwarzschild radius r_g) and the velocity limit (the light speed c). The aim of the present work is to find the influence of relativistic effects on the efficiency of amplification of small acoustic perturbations in the spherical accretion flow.

2 Model

We examine the spatial stability of spherical adiabatic flow of a non-self-gravitating non-viscous homogeneous matter on to a black hole from infinity. The model of accretion is described by the general relativistic hydrodynamic equations: equation of fluid motion

$$u^\nu \nabla_{;\nu} u^\mu = -\frac{1}{n}\left(g^{\mu\nu} + \frac{u^\mu u^\nu}{c^2}\right)\nabla_{;\nu} p, \tag{1}$$

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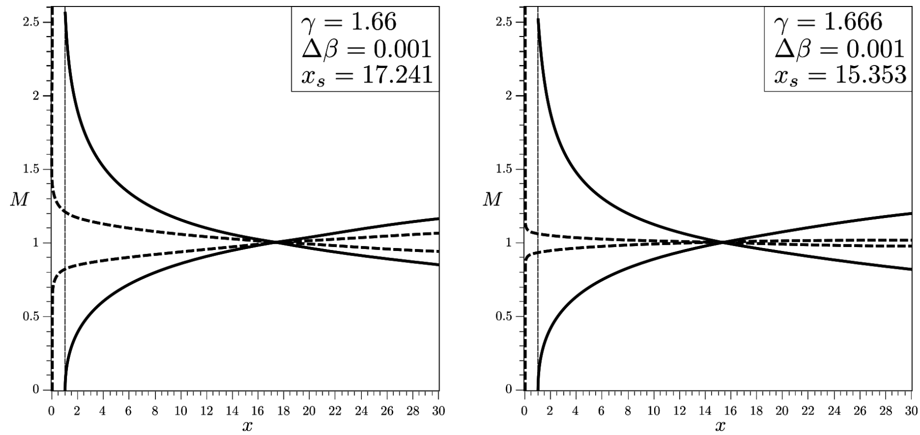

Figure 1: The Mach number M as the function of the dimensionless radial coordinate x (red lines). The subsonic (lower curve) and the supersonic (upper curve) separatrices of a relativistic solution (solid line) and a classic solution (dashed line).

and continuity equation

$$\nabla_{;\mu}(nu^{\mu}) = 0. \tag{2}$$

Here c is a speed of light, $g^{\mu\nu}$ a contravector of metric tensor, u^{μ} a geometrical contravector 4-velocity

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}, \tag{3}$$

n is a specific relativistic density, and the pressure

$$p = Kn^{\gamma}. \tag{4}$$

A specific enthalpy is

$$h = mc^2 + \frac{\gamma}{\gamma - 1} K n^{\gamma-1}. \tag{5}$$

The equation of state (4) allows us to formulate an equation for the speed of sound c_s

$$c_s^2 = c^2 \frac{K\gamma n^{\gamma-1}}{mc^2 + K\frac{\gamma}{\gamma-1} n^{\gamma-1}}. \tag{6}$$

Here K is the gaseous constant, m the rest mass of matter, γ the adiabatic constant. In the case of the steady state spherically symmetric flow the system (1)–(2), (6) is reduced to the algebraic system of two integrals of motion, the mass flux conservation [1, 2]

$$yzx^2\sqrt{\frac{1-\frac{1}{x}}{1-y^2}} = \lambda, \tag{7}$$

and the Bernoulli integral

$$(1 + z^{\gamma-1})\sqrt{\frac{1-\frac{1}{x}}{1-y^2}} = \beta, \tag{8}$$

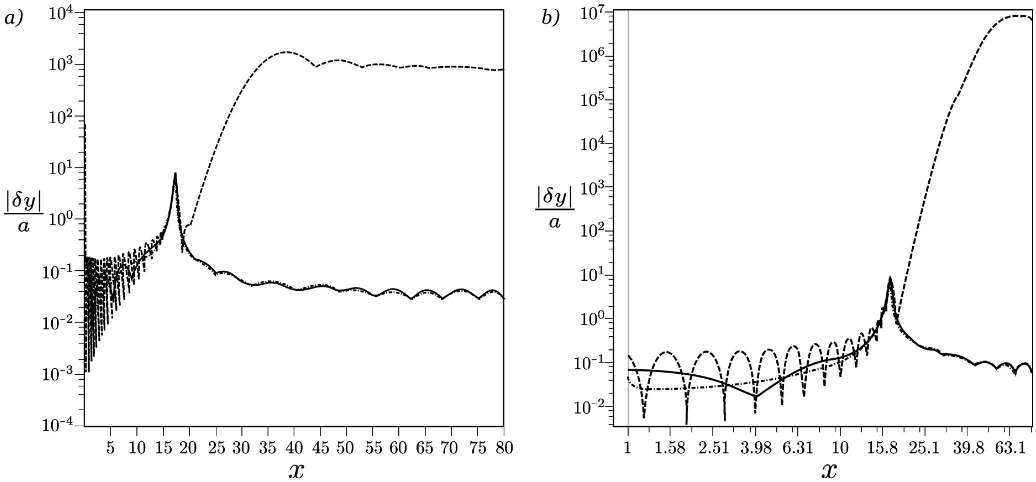

Figure 2: The ratio of the acoustic perturbations of radial velocity $|\delta y|$ for different azimuthal numbers l of spherical harmonics in the non-relativistic (a) and relativistic (b) cases. Dashed line with points is for $l = 0$, solid line for $l = 1$, and dashed line for $l = 10$. The frequency for all calculations $\omega = 0.01$, the adiabatic constant $\gamma = 1.66$, the energy $\Delta\beta = 0.001$.

and the equation for the non-dimensionalized sound speed a

$$a^2 = \frac{(\gamma - 1)z^{\gamma-1}}{1 + z^{\gamma-1}}. \tag{9}$$

Here λ and β are the mass flux and the Bernoulli constant, respectively. The non-dimensionalized flow variables are defined as follows:

$$x = \frac{r}{r_g}, \quad y = \frac{V_r}{c}, \quad z = \left(\frac{K\gamma}{mc^2(\gamma - 1)}\right)^{-\frac{1}{\gamma-1}} n, \quad a = \frac{c_s}{c}. \tag{10}$$

3 Solution

The perturbations are sought in the form

$$n = \delta n + n, \quad u^\mu = \delta u^\mu + u^\mu. \tag{11}$$

Potential of a 4-velocity is

$$u^\mu h = \nabla^{;\mu}\phi. \tag{12}$$

We find a solution in the form

$$\delta\phi = \delta\tilde{\phi}(x) Y_{lm}(\theta, \varphi) e^{-i\omega t}. \tag{13}$$

Cauchy-Lagrange integral is

$$\delta n = -\frac{u_{\sigma,0}}{K\gamma c^2 n_0^{\gamma-2}} \nabla^{;\sigma}\delta\phi. \tag{14}$$

Wave equation

$$\nabla_{;\mu}(d^{\mu\nu}\nabla_{,\nu}\delta\phi) = 0, \tag{15}$$

where d^{μ}_{ν} is a strange tensor

$$d^{\mu}_{\nu} = \frac{1}{K\gamma n_0^{\gamma-2}} \left(\frac{c_s^2}{c^2} g^{\mu}_{\nu} - \left(1 - \frac{c_s^2}{c^2}\right) \frac{u_0^{\mu} u_{\nu,0}}{c^2} \right). \tag{16}$$

A general relativistic wave equation

$$\delta\tilde{\phi}'' d^{rr} + \delta\tilde{\phi}' \left(\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \left(\sqrt{-g} d^{rr}\right) - 2i\omega d^{rt} \right) + \\ - \delta\tilde{\phi} \left(-\frac{\omega^2}{c^2} d^{tt} - i\frac{\omega}{c} \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x} \left(\sqrt{-g} d^{tr}\right) - d^{\theta\theta} l(l+1) \right) = 0. \tag{17}$$

A classical limit of reduced wave equation

$$\delta\tilde{\phi}'' \left(a^2 - y^2\right) + \delta\tilde{\phi}' \left(2\mathrm{i}\omega y - \left(y^2 + a^2\right) \frac{y'}{y} + 2y^2 \frac{a'}{a} \right) + \\ + \delta\tilde{\phi} \left(\omega^2 - 2\mathrm{i}\omega y \frac{a'}{a} - a^2 \frac{l(l+1)}{x^2} \right) = 0. \tag{18}$$

4 Results

We find that the acoustic perturbations are amplified significantly (many order of magnitudes) inside the sonic sphere in a spherical accretion flow, though remain finite compared to the case of the non-relativistic Bondi model.

References

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