# Polarimetric Method for Measuring Black Hole Masses in Active Galactic Nuclei Based on Theory of V.V. Sobolev and S. Chandrasekhar

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The V.V. Sobolev and S. Chandrasekhar theory of polarized radiation presents the main basis for modern science of the polarized emission of cosmic objects including X-rays and gamma-rays. The accreting supermassive black holes (SMBH) in active galactic nuclei (AGN) are found in the center of modern astrophysics. The main problem is connected with determining the masses of these objects. The virial theorem accepted to a flattened configuration of a broad line region (BLR) in AGNs allows us to get a direct connection between the mass of SMBH and the inclination angle of the accretion flow. The inclination angle itself can be derived from the spectropolarimetric data of broad emission lines using the theory for generation of polarized radiation developed by Sobolev and Chandrasekhar. As a result, the new estimates of SMBH masses in AGN with measured polarization of BLR emission are presented. It is essential that the polarimetric data allow also to determine the value of the virial coefficient that is important for determining SMBH masses and to estimate the recoiling velocity of SMBH.

## 1 Introduction

AGNs are powered by accretion to a SMBHs, and the broad emission lines seen in Type I AGN are produced in the special region that is named as the broad line region (BLR). Unfortunately, the structure and kinematics of BLR remain unclear. Really, broad emission lines are emitted in the vicinity of SMBH in AGN, but this region is not resolved in interferometric observations. Nevertheless, the properties of the broad emission lines are used to estimate the mass of the central SMBH.

The commonly accepted method for estimating the SMBH mass is to use the virial theorem. It allows to get the following relation [1, 2]:

$$M_{BH} = f\frac{R_{BLR}V_{BLR}^2}{G}, \tag{1}$$

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where $M_{BH}$ is the mass of a black hole, $f$ is a virial parameter that defines the geometry, velocity field and orientation of BLR, $R_{BLR}$ is the radius of BLR and $V_{BLR}$ is the velocity dispersion that is usually measured as the full width of the emission line at a half of height in the radiation spectrum, i.e. FWHM. The BLR radius $R_{BLR}$ is usually determined by the reverberation method, i.e. with time delay between continuum and emission line variations.

Determination of the $f$ value is strongly depending on accepted BLR model. Labita et al. [3] found that the model of an isotope BLR fails to reproduce the observed line widths and shapes. They claimed that a disk model is preferred. A disk-like geometry of the BLR has been proposed by several authors [4]. Collin et al. [5] suggested that the disk may have a finite half thickness $H$, or a profile with $H$ increasing more than linearly with the disk radius. Other models propose the existence of warped disks [6].

The authors of [7] and [8] found that the hydrogen lines are emitted in a more flattened BLR configuration in comparison to the highly ionized lines. They estimated the ratios of $H/R$ in BLR for a number of AGNs and obtained the interval of $H/R \sim 0.07$–$0.5$. Also Pancoast et al. [9] found that the geometry of the BLR is generally a thick disk viewed close to face on. Eracleous and Halpern [10] have found that the inclination angle of BLR is 24–30°. Eracleous et al. [11] estimated the inclination angle of the BLR as $i = 19$–$42°$. It is interesting to note that the polarimetric observations do not contradict these estimates [12, 13, 14].

The virial coefficient $f$ depends strongly on the BLR geometry, velocity field and orientation. Usually the authors used the value $f \approx 1$. Peterson and Wandel [15] used $f = 3/4$. Onken et al. [16] suggested to use the mean value of the virial coefficient $f = 1.4$. McLure and Dunlop [17] have shown that for a disk inclined at an inclination angle $i$ the virial coefficient value is

$$f = \frac{1}{4\sin^2 i}. \tag{2}$$

Collin et al. [5] have considered the situation when the opening angle of the BLR disk should be large, i.e. $\Omega/4\pi \geq 0.1$. It means that the half thickness $H$ of the disk should be large enough and the ratio of $H$ to the radius $R$ should be larger than $H/R = 0.1$. As a result, the virial coefficient has a form:

$$f = \frac{1}{4\left[(H/R)^2 + \sin^2 i\right]}. \tag{3}$$

This way, the relation (2) is a particular case of Eq. (3).

We adopted the disk-like model for the BLR of a number of Seyfert galaxies and, therefore, will use the expression for the virial coefficient $f$ given by Eq. (2). The required value of the inclination angle can be determined from polarimetric observations using the standard Chandrasekhar–Sobolev theory [18, 19] of multiple scattering of the radiation on free electrons and Rayleigh scattering on gas molecules and small dist particles. According to these classical works, the polarization degree of scattered radiation depends strongly on the inclination angle. The scattered radiation has the maximum linear polarization $P_l = 11.7\%$ when the line of sight is perpendicular to the normal to the semi-infinite atmosphere (Milne problem). Chandrasekhar [18] and Sobolev [19] presented the solution of this Milne problem. They have considered the multiple scattering of light in optically thick flattened atmospheres. The Milne problem corresponds to the propagation and scattering of light in optically thick disk-like region, i.e. this solution can be directly applied to BLR problem.

## 2 Polarimetric determining of the virial coefficient

We use the theory of multiple scattering of polarized radiation [18, 19, 20] and the disk-like model for the BLR.

We take into account the process of absorption of radiation. In this case the degree of polarization depends on the parameter $q = \sigma_a/(\sigma_a + \sigma_{SC})$, where $\sigma_a$ is the cross-section of intrinsic absorption and $\sigma_{SC}$ is the scattering cross-section. For the standard accretion disk model $\sigma_{SC} = \sigma_{Th}$, where $\sigma_{Th} = 6.65 \times 10^{-25}$ cm$^2$ is the cross-section for scattering of radiation on electrons, which in the non-relativistic case is the classical Thomson scattering cross-section. For some cases it is convenient to use the analytical formula for the polarization degree $P_l(\mu)$ obtained by Silant'ev et al. [21]. Using Eq. (2), we can obtain the values of the polarization degree and their dependence of the virial coefficient $f$. The results of these calculations are presented in Table 1.

For the model of a disk shaped BLR, the sine of the inclination angle is determined by [22]

$$\sin i = \frac{1}{2}\left(\frac{R_{BLR}}{R_g}\right)^{1/2}\frac{FWHM}{c}, \tag{4}$$

where $i$ is the inclination angle, $R_{BLR}$ is the radius of the BLR, $FWHM$ is the full width half maximum of a given line, which can be measured directly, $c$ is the speed of light, $R_g = GM_{BH}/c^2$ is the gravitational radius. The value of $R_{BLR}V_{BLR}^2/G$ is called the "virial product" (VP). This quantity is based on two observable quantities: BLR radius and emission line width and has units of mass. The VP corresponds to the quantity of the virial coefficient $f = 1$ and with accordance to Eq. (2) corresponds to the inclination angle $i = 30°$. According to [18] and [19], it means that the observed BLR emission polarization is $P_l(\mu) = 0.43\%$.

Equation (2) has been used for determining the virial coefficient. The polarimetric data that are necessary for determining the value of the inclination angle and the virial coefficient are presented in the spectropolarimetric atlas of Smith et al. [12]. They obtained the values of polarization degree and position angle for 36 Type 1 Seyfert galaxies during a number of different runs at the Anglo-Australian and William Herschel telescopes. From 36 objects presented in [12] 13 AGNS have the equal polarization degree values for $H_\alpha$ line and continuum. Also these objects have equal values of the position angle with the error limits. For most of the observed objects, there is a difference between

Table 1: Polarization degree $P_l$ [%] dependence on $q$ and $\mu$, $f$ is virial coefficient

| $\mu$ | $f$ | $q = 0.0$ | $q = 0.01$ | $q = 0.05$ | $q = 0.1$ | $q = 0.2$ | $q = 0.3$ |
|---|---|---|---|---|---|---|---|
| 0.000 | 0.2500 | 11.713 | 12.592 | 16.074 | 20.349 | 28.636 | 36.642 |
| 0.025 | 0.2502 | 10.041 | 10.937 | 14.488 | 18.852 | 27.324 | 35.516 |
| 0.050 | 0.2506 | 8.986 | 9.889 | 13.473 | 17.879 | 26.440 | 34.723 |
| 0.075 | 0.2514 | 8.150 | 9.057 | 12.656 | 17.085 | 25.691 | 34.019 |
| 0.100 | 0.2525 | 7.449 | 8.357 | 11.961 | 16.395 | 25.015 | 33.354 |
| 0.125 | 0.2540 | 6.844 | 7.751 | 11.349 | 15.776 | 24.382 | 32.704 |
| 0.150 | 0.2558 | 6.312 | 7.215 | 10.799 | 15.209 | 23.776 | 32.056 |
| 0.175 | 0.2579 | 5.838 | 6.735 | 10.297 | 14.679 | 23.187 | 31.403 |
| 0.200 | 0.2604 | 5.410 | 6.301 | 9.834 | 14.178 | 22.607 | 30.737 |
| 0.225 | 0.2633 | 5.022 | 5.904 | 9.401 | 13.699 | 22.031 | 30.056 |
| 0.250 | 0.2667 | 4.667 | 5.539 | 8.994 | 13.237 | 21.455 | 29.356 |
| 0.275 | 0.2705 | 4.342 | 5.201 | 8.608 | 12.789 | 20.876 | 28.636 |
| 0.300 | 0.2747 | 4.041 | 4.887 | 8.240 | 12.352 | 20.291 | 27.895 |
| 0.325 | 0.2795 | 3.762 | 4.594 | 7.887 | 11.922 | 19.700 | 27.132 |
| 0.350 | 0.2849 | 3.502 | 4.318 | 7.547 | 11.498 | 19.100 | 26.346 |
| 0.375 | 0.2909 | 3.260 | 4.059 | 7.217 | 11.078 | 18.491 | 25.538 |
| 0.400 | 0.2976 | 3.033 | 3.813 | 6.897 | 10.660 | 17.872 | 24.707 |
| 0.425 | 0.3051 | 2.820 | 3.581 | 6.584 | 10.244 | 17.242 | 23.854 |
| 0.450 | 0.3135 | 2.620 | 3.359 | 6.278 | 9.829 | 16.602 | 22.979 |
| 0.475 | 0.3228 | 2.431 | 3.148 | 5.977 | 9.414 | 15.950 | 22.083 |
| 0.500 | 0.3333 | 2.252 | 2.947 | 5.681 | 8.997 | 15.287 | 21.166 |
| 0.525 | 0.3451 | 2.083 | 2.753 | 5.389 | 8.580 | 14.612 | 20.230 |
| 0.550 | 0.3584 | 1.923 | 2.568 | 5.100 | 8.160 | 13.926 | 19.275 |
| 0.575 | 0.3735 | 1.771 | 2.389 | 4.813 | 7.737 | 13.230 | 18.302 |
| 0.600 | 0.3906 | 1.627 | 2.217 | 4.529 | 7.312 | 12.522 | 17.312 |
| 0.625 | 0.4103 | 1.489 | 2.050 | 4.246 | 6.883 | 11.803 | 16.306 |
| 0.650 | 0.4329 | 1.358 | 1.889 | 3.965 | 6.452 | 11.074 | 15.285 |
| 0.675 | 0.4592 | 1.233 | 1.732 | 3.684 | 6.016 | 10.335 | 14.250 |
| 0.700 | 0.4902 | 1.113 | 1.580 | 3.404 | 5.577 | 9.586 | 13.202 |
| 0.725 | 0.5270 | 0.998 | 1.433 | 3.124 | 5.135 | 8.828 | 12.143 |
| 0.750 | 0.5714 | 0.888 | 1.289 | 2.844 | 4.688 | 8.061 | 11.072 |
| 0.775 | 0.6260 | 0.783 | 1.148 | 2.563 | 4.237 | 7.285 | 9.992 |
| 0.800 | 0.6944 | 0.682 | 1.011 | 2.282 | 3.782 | 6.502 | 8.903 |
| 0.825 | 0.7828 | 0.585 | 0.876 | 2.001 | 3.323 | 5.710 | 7.807 |
| 0.850 | 0.9009 | 0.492 | 0.744 | 1.719 | 2.861 | 4.912 | 6.704 |
| 0.875 | 1.0667 | 0.402 | 0.615 | 1.435 | 2.394 | 4.107 | 5.595 |
| 0.900 | 1.3158 | 0.316 | 0.488 | 1.151 | 1.923 | 3.296 | 4.482 |
| 0.925 | 1.7316 | 0.233 | 0.363 | 0.865 | 1.448 | 2.479 | 3.365 |
| 0.950 | 2.5641 | 0.152 | 0.240 | 0.578 | 0.969 | 1.658 | 2.245 |
| 0.975 | 5.0633 | 0.075 | 0.119 | 0.290 | 0.486 | 0.831 | 1.123 |
| 1.000 | – | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

the values of polarization degree and the position angle. This phenomenon can testify to the difference in the inclination between the disk shaped BLR and the accretion disk that can be described by Shakura–Sunyaev model [23]. Continuum polarization degree of AGN from Palomar–Green catalog has been measured on 6 m telescope of Special Astrophysical Observatory [14].

Some results for the virial coefficient $f$ are presented in Table 1 of our paper [24]. In the second column of this table the values for the BLR inclination angles are presented. These data are obtained from polarimetric data derived by Smith et al. [12]. The values of inclination angles have been obtained with use of the standard Chandrasekhar–Sobolev theory of multiple scattering of polarize radiation in the disk-like electron atmosphere and with use of Eq. (4). The virial factor for most of data in Table 1 from [24] corresponds to $f \sim 1$ within error limits, in accordance with [25]. For some objects, including Mrk 841, Mrk 896, Mrk 976, NGC 3516, NGC 3783 and NGC 5548, the virial coefficient is close to the mean value of $f = 1.4$ [16].

For a number of objects, including Fairall 51, Mrk 6, MC 1849.2-78.32, NGC 6814, UGC 3478 and WAS 45, their inclination angle exceeds considerably the value of $i = 30°$. It means that their virial factor $f < 1.0$. This situation corresponds better to the BLR model of random orbits.

## 3 Determining SMBH masses

Equation (1) allows to derive the SMBH mass if the virial coefficient is known. The observed full width of emission lines is presented in [1, 26, 27, 28]. The radius $R_{BLR}$ is estimated usually by the reverberation mapping that is related to the continuum monochromatic luminosity $\lambda L_\lambda$. An increasing body of measurements is now available for $H_\beta$ time lags [29, 30, 4, 31]. For example, from Bentz et al [31] one can obtain the following estimate for $R_{BLR}$:

$$\log\left(R_{BLR}/1\,\mathrm{ltd}\right) = K + \alpha \log \frac{\lambda L(5100\,\text{Å})}{10^{44}\,\mathrm{erg/s}}, \tag{5}$$

where $K = 1.527^{+0.031}_{-0.03}$, $\alpha = 0.533^{+0.035}_{-0.039}$ and $R_{BLR}$ is measured in light days (ltd).

Equation (5) allows to present the BLR radius in the form

$$R_{BLR} = 10^{16.94} \left(\frac{\lambda L_\lambda(5100\,\text{Å})}{10^{44}\,\mathrm{erg/s}}\right)^{0.533}. \tag{6}$$

Then, using Eqs. (2) and (4), one can get the values of the SMBH masses. The value of $\sin i$ can be derived from the data on the degree of polarization that is strongly dependent on the inclination angle.

The results of our calculations of the SMBH masses are presented in Table 2 [24]. In the last column of this table the published values for SMBH masses are presented. There is a difference for a number of objects. For example, for Fairall 9 our estimate of the SMBH mass looks lower, but our upper

Table 2: The masses of SMBH in AGNs determined via measured polarization of broad $H_\alpha$ emission

| Object | $M_{BH}/M_\odot$ (observations) | $M_{BH}/M_\odot$ (literature) |
|---|---|---|
| Akn 120 | $(3.23^{+0.44}_{-0.32}) \times 10^8$ | $(4.49 \pm 0.93) \times 10^8$ |
| Akn 564 | $(1.2^{+0.94}_{-0.49}) \times 10^6$ | $\sim 1.1 \times 10^6$ |
| Fairall 9 | $(1.34^{+0.76}_{-0.36}) \times 10^8$ | $(2.55 \pm 0.56) \times 10^8$ |
| I Zw 1 | $10^{7.6 \pm 0.17}$ | $10^{7.441^{+0.093}_{-0.119}}$ |
| Mrk 6 | $(1.09^{+0.37}_{-0.25}) \times 10^8$ | $(1.36 \pm 0.12) \times 10^8$ |
| Mrk 279 | $(8.13^{+1.24}_{-1.26}) \times 10^7$ | $(15.2^{+3.25}_{-3.18}) \times 10^7$ |
| Mrk 290 | $(3.94 \pm 0.19) \times 10^7$ | $(2.43 \pm 0.37) \times 10^7$ |
| Mrk 304 | $10^{8.4^{+0.09}_{-0.02}}$ | $10^{8.511^{+0.093}_{-0.113}}$ |
| Mrk 335 | $(1.56^{+0.19}_{-0.15}) \times 10^7$ | $(1.42 \pm 0.37) \times 10^7$ |
| Mrk 509 | $(1.35 \pm 0.12) \times 10^8$ | $(1.39 \pm 0.12) \times 10^8$ |
| Mrk 705 | $10^{7.07^{+0.11}_{-0.09}}$ | $10^{6.79 \pm 0.5}$ |
| Mrk 841 | $10^{8.55 \pm 0.1}$ | $10^{8.523^{+0.079}_{-0.052}}$ |
| Mrk 871 | $10^{7.04^{+0.09}_{-0.06}}$ | $10^{7.08 \pm 0.5}$ |
| Mrk 876 | $10^{8.57^{+0.19}_{-0.52}}$ | $10^{9.139^{+0.096}_{-0.122}}$ |
| Mrk 896 | $10^{7.07 \pm 0.06}$ | $10^{7.01}$ |
| Mrk 926 | $10^{8.8^{+0.19}_{-0.11}}$ | $10^{8.36 \pm 0.02}$ |
| Mrk 985 | $3.18 \times 10^7$ | $5.71 \times 10^7$ |
| NGC 3516 | $10^{8.06 \pm 0.27}$ | $10^{7.88^{+0.04}_{-0.03}}$ |
| NGC 3783 | $10^{7.7 \pm 0.11}$ | $10^{7.47^{+0.07}_{-0.09}}$ |
| NGC 4051 | $(1.64^{+0.67}_{-0.55}) \times 10^6$ | $(1.58^{+0.50}_{-0.65}) \times 10^6$ |
| NGC 4593 | $(8.25^{+3.46}_{-3.06}) \times 10^6$ | $(9.8 \pm 2.1) \times 10^6$ |
| NGC 5548 | $(7.84^{+0.53}_{-0.46}) \times 10^7$ | $(7.827 \pm 0.017) \times 10^7$ |
| NGC 6104 | $10^{7.16^{+0.09}_{-0.08}}$ | $10^{7.39}$ |
| NGC 6814 | $10^{6.94^{+0.077}_{-0.09}}$ | $10^{7.02 \pm 0.5}$ |
| NGC 7213 | $10^{6.83^{+0.84}_{-0.33}}$ | $10^{6.88 \pm 0.5}$ |
| NGC 7469 | $10^{7.54^{+0.17}_{-0.22}}$ | $10^{7.19 \pm 0.13}$ |
| PG 1211+143 | $10^{8.34^{+0.29}_{-0.20}}$ | $10^{7.961^{+0.082}_{-0.101}}$ |

limit value coincides with the low order limit value of BH mass obtained by Reynolds [32]. For Ark 120 the situation looks the same: there is a coincidence between our estimated value and the low order value of BH mass obtained by Vestergaard and Peterson [1]. The similar situation occurs also for NGC 3516, NGC 3783, NGC 5548, and Mrk 279. The extreme case is Mrk 290. Our low order estimate appears 1.34 times higher than the upper limit of BH mass presented by Feng et al. [28].

This difference between our and other estimates of mass values for some black holes can be associated with the real determination of the virial coefficient $f$ from the Eq. (1). Value of virial coefficient from the papers [17, 25] allows us to determine this parameter directly from Eq. (2) but only in the situation when the inclination angle of the BLR is certain. Polarimetric observations have preference because the value of polarization degree is directly associated with the inclination angle value, especially for standard Chandrasekhar–Sobolev theory of the generation of polarization in the plane-parallel atmosphere. Other methods for determination of inclinations of BLRs and accretion disks are considerably uncertain. Unfortunately, the size of the BLR cannot be directly measured from single epoch spectra, which are used for estimation of BH masses. Most popular estimates of BLR size rely on the $R_{BLR}$ scales with a certain power of continuum luminosity of the AGN.

## 4 Determining recoiling velocities of black holes ejected by gravitational radiation in galaxy mergers

We demonstrate that polarimetric observations allow to derive the recoiling velocity of black hole ejected by gravitational radiation produced in the process of black holes merging. Really merging of spinning black holes can produce recoil velocities ("kicks") of the final merged black holes via anisotropic gravitational radiation up to several thousands km/s [33, 34, 35]. The basic feature of the gravitational wave (GW) recoil effect is the situation when the SMBH spends a significant fraction of time off nucleus at scales beyond that of the molecular obscuring torus. For example, according to [36], isophotal analysis of M87, using data from Advanced Camera for Surveys, reveals a projected displacement of $68 \pm 0.8$ pc ($\sim 0.''1$) between the nuclear point source and the photo-center of the galaxy itself.

It is very important that a recoiling SMBH in an AGN retains the inner part of its accretion disk. Bonning et al. [37] have shown that the accretion disk will remain bound to the recoiling BH inside the radius $R_{out} = 1.3 \times 10^{18} M_8/V_3^2$ cm ($M_8 = M_{BH}/10^8 M_\odot$, $V_3 = V/10^3$ km/s), where the orbital velocity $V$ is equal to the recoil velocity $V_K$. In this case the retained disk mass appears less than the BH mass because of stability requirement [33].

If we suggest that $R_{BLR} \approx R_K$, we obtain the following relation instead of Eq. (4):

$$\sin i = 0.492 \left( \frac{FWHM}{V_K} \right). \qquad (7)$$

In this case, when $FWHM = 1500$ km/s and $V_K = 10^3$ km/s, we obtain $P_l(H_\beta) = 1.24\%$. For $V_K = 3 \times 10^3$ km/s, we obtain $P_l(H_\beta) = 0.1\%$. According to [38], for merging black hole with spin value $a = 1.0$, the kick velocity has magnitude of $V_K = 4925.94$ km/s. For $FWHM = 5 \times 10^3$ km/s the polarization degree is $P_l(H_\beta) = 0.624\%$. For $H_\alpha$ emission line, the values of polarization degree are practically the same because of the practical equality of FWHMs [39].

According to [33], the accretion disk remains bound to the ejected BH within the region where the accreting matter orbital velocity is larger than the ejection speed. For example, the ejected disk of $10^7 M_\odot$ BH has a characteristic size of tens of thousands of Schwarzschild radii and an accretion lifetime of $\sim 10^7$ years. But in this case the ejected BH could traverse a considerable distance and appears as an off-center galaxy region. Loeb [33] claimed that only small fraction of all quasars could be associated with an escaping SMBH.

Equation (7) allows us to find the candidates for escaping BHs. According to [40] and [5], the virial coefficient can be presented in the form

$$f = \frac{1}{4\left[\left(\frac{V_K}{V_{orb}}\right)^2 + \sin^2 i\right]}. \quad (8)$$

In this case Eq. (4) is transforming in the expression

$$\sqrt{\left(\frac{V_K}{V_{orb}}\right)^2 + \sin^2 i} = \frac{FWHM}{2c}\left(\frac{R_{BLR}}{R_g}\right)^{1/2}. \quad (9)$$

The value of $\sin i$ can be obtained from polarimetric observations, naturally if we use the theory of multiple scattering of polarized radiation developed by Chandrasekhar and Sobolev. If we consider the situation $R_K \approx R_{BLR}$ and $V_K < V_{orb}$, we obtain instead of Eq. (7) the following relation:

$$\sqrt{\left(\frac{V_K}{V_{orb}}\right)^2 + \sin^2 i} = 0.492\left(\frac{FWHM}{V_K}\right). \quad (10)$$

For example, for $V_K = 0.5\,V_{orb}$, $\sin i = 0.847$, that corresponds to $P_l(\mu) = 2.05\%$. We estimated the values of kick velocity from a number of AGN using the polarimetric data from [12] and estimates of $R_{BLR}$ obtained in [41]. For estimates of $M_{BH}$, we used the relation (6) from [1].

As a result we obtained the following estimates of the recoiling velocities: $V_K = 0.22\,V_{orb}$ for Ark 120 and Mrk 279, $V_K = 0.42\,V_{orb}$ for NGC 3516, $V_K = 0.288\,V_{orb}$ for NGC 3783, $V_K = 0.31\,V_{orb}$ for NGC 4593. It is interesting that for NGC 4051 the situation occurs when $V_K \approx V_{orb}$.

Of course, our results have a preliminary character, because another situation can be considered when the coefficient $A = H/R$, where $H$ is the geometrical thickness of the BLR [5]. It means that the BLR is the geometrically thick accretion disk compared with the geometrically thin accretion disk that provides the continuum polarization.

# 5 Conclusions

We demonstrated that V.V. Sobolev and S. Chandrasekhar theory of the multiple scattering of polarized radiation is an efficient instrument for an analysis and interpretation of the polarimetric data of AGNs. The virial theorem accepted to a disk-like configuration of BLR in AGN allows us to determine the real mass of AGN if the inclination angle of the accretion flow is known. Namely, the polarimetric observations allow us to derive the inclination angle itself using the theory for the generation of polarized radiation developed by Sobolev and Chandrasekhar. As a result we have demonstrated the possibility of polarimetric determining of the virial coefficient and, as the final result, determining the mass of SMBH in AGN. In principle, polarimetric observations allow us to derive the recoil velocity of a black hole ejected by gravitational radiation as a result of a galaxy merger.

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## References

1. *M. Vestergaard, B.M. Peterson*, Astrophys. J., **641**, 689, 2006.
2. *S. Fine, S.M. Croom, P.F. Hopkins et al.*, Mon. Not. Roy. Astron. Soc., **390**, 1413, 2008.
3. *M. Labita, A. Treves, R. Falomo, M. Uslenghi*, Mon. Not. Roy. Astron. Soc., **373**, 551, 2006.
4. *R. Decarli, M. Labita, A. Treves, R. Falomo*, Mon. Not. Roy. Astron. Soc., **387**, 1237, 2008.
5. *S. Collin, T. Kawaguchi, B.M. Peterson, M. Vestergaard*, Astron. Astrophys., **456**, 75, 2006.
6. *S. Tremaine, S.W. Davis*, Mon. Not. Roy. Astron. Soc., **441**, 1408, 2014.
7. *W. Kollatschny, M. Zetzl, M. Dietrich*, Astron. Astrophys., **454**, 459, 2006.
8. *W. Kollatschny, M. Zetzl*, Astron. Astrophys., **558**, A26, 2013.
9. *A. Pancoast, B.J. Brewer, T. Treu, LAMP2008 Collaboration, LAMP2011 Collaboration*, Amer. Astron. Soc. Meet. Abstr., **221**, 309.08, 2013.
10. *M. Eracleous, J.P. Halpern*, Astrophys. J. Suppl., **90**, 1, 1994.
11. *M. Eracleous, J.P. Halpern, M. Livio*, Astrophys. J., **459**, 89, 1996.
12. *J.E. Smith, S. Young, A. Robinson et al.*, Mon. Not. Roy. Astron. Soc., **335**, 773, 2002.
13. *J.E. Smith, A. Robinson, D.M. Alexander et al.*, Mon. Not. Roy. Astron. Soc., **350**, 140, 2004.
14. *V.L. Afanasiev, N.V. Borisov, Y.N. Gnedin et al.*, Astron. Lett., **37**, 302, 2011.
15. *B.M. Peterson, A. Wandel*, Astrophys. J. Lett., **521**, L95, 1999.
16. *C.A. Onken, L. Ferrarese, D. Merritt et al.*, Astrophys. J., **615**, 645, 2004.
17. *R.J. McLure, J.S. Dunlop*, Mon. Not. Roy. Astron. Soc., **327**, 199, 2001.
18. *S. Chandrasekhar*, Radiative Transfer. Oxford: Clarendon Press, 1950.

19. *V.V. Sobolev*, A Treatise on Radiative Transfer. Princeton: Van Nostrand, 1963.

20. *N.A. Silant'ev*, Astron. Astrophys., **383**, 326, 2002.

21. *N.A. Silant'ev, M.Y. Piotrovich, Y.N. Gnedin, T.M. Natsvlishvili*, Astron. Rep., **54**, 974, 2010.

22. *B. Agís-González, G. Miniutti, E. Kara et al.*, Mon. Not. Roy. Astron. Soc., **443**, 2862, 2014.

23. *N.I. Shakura, R.A. Sunyaev*, Astron. Astrophys., **24**, 337, 1973.

24. *M.Y. Piotrovich, Y.N. Gnedin, N.A. Silant'ev et al.*, Mon. Not. Roy. Astron. Soc., **454**, 1157, 2015.

25. *R.J. McLure, M.J. Jarvis*, Mon. Not. Roy. Astron. Soc., **337**, 109, 2002.

26. *L.C. Ho, J. Darling, J.E. Greene*, Astrophys. J. Suppl., **177**, 103, 2008.

27. *J.-G. Wang, X.-B. Dong, T.-G. Wang et al.*, Astrophys. J., **707**, 1334, 2009.

28. *H. Feng, Y. Shen, H. Li*, Astrophys. J., **794**, 77, 2014.

29. *S. Kaspi, P.S. Smith, H. Netzer et al.*, Astrophys. J., **533**, 631, 2000.

30. *M.C. Bentz, B.M. Peterson, H. Netzer et al.*, Astrophys. J., **697**, 160, 2009.

31. *M.C. Bentz, K.D. Denney, C.J. Grier et al.*, Astrophys. J., **767**, 149, 2013.

32. *C.S. Reynolds*, Space Sci. Rev., **183**, 277, 2014.

33. *A. Loeb*, Phys. Rev. Lett., **99**, 041103, 2007.

34. *G.A. Shields, E.W. Bonning*, Astrophys. J., **682**, 758, 2008.

35. *S. Komossa, D. Merritt*, Astrophys. J. Lett., **689**, L89, 2008.

36. *D. Batcheldor, A. Robinson, D.J. Axon et al.*, Astrophys. J. Lett., **717**, L6, 2010.

37. *E.W. Bonning, G.A. Shields, S. Salviander*, Astrophys. J. Lett., **666**, L13, 2007.

38. *C.O. Lousto, Y. Zlochower*, Phys. Rev. Lett., **107**, 231102, 2011.

39. *J.E. Greene, L.C. Ho*, Astrophys. J., **630**, 122, 2005.

40. *X.-B. Wu, J.L. Han*, Astrophys. J. Lett., **561**, L59, 2001.

41. *J.E. Greene, C.E. Hood, A.J. Barth et al.*, Astrophys. J., **723**, 409, 2010.