Ionization from Excited Levels as a Cause of Hydrogen Level Non-Stationary Occupation during Radiative Cooling behind a Shock Wave

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It is shown that for physical conditions typical of late giant star atmospheres, the discrete level occupations are of necessity non-stationary in the case of radiative cooling of hydrogen behind a shock wave. They depend on the whole cooling process as well as on the current values of temperature and electronic density. This fact is due mainly to impact ionization from the excited discrete hydrogen levels.

1 Introduction

Hydrogen contributes significantly to the cooling rate of a shocked gas and defines emission spectrum during flares in stellar atmospheres of the stars whose spectral class is later than A. Both the cooling rate and emission spectra are determined by the relative occupations of discrete levels ν_k and the ionization state x. We assume that N_a and N_p are the atomic and ion hydrogen number densities, respectively, N_k is the number density of hydrogen on the level with the main quantum number k, and N is the total hydrogen number density. So,

$$N = N_a + N_p, \quad x = N_p/N, \quad \nu_k = N_k/N, \quad y_k = N_k/N_1. \tag{1}$$

The discrete level occupations of hydrogen and its ionization state are non-stationary as it follows from our calculations of radiative cooling of shocked gas [1]. This result calls for an explanation. Indeed, transitions between discrete levels are rather quick in stellar atmospheres, and one could expect that both functions of time $\nu_k(t)$ and $x(t)$ are nearly completely determined by the current values of electron temperature $T_e(t)$ and electron density $N_e(t)$, i.e. there is a "quasi-stationary" flow. The Lagrangian coordinate t is the time passed from the beginning of the process, i.e. since the moment when the gas element passed the shock front.

Let us consider a simple model of the two-level atom where the level occupations are controlled by electron impact and spontaneous radiative

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transitions. The occupation y_2 of the excited level is described by the simple formula

$$y_2(t) = y_{2\infty} - (y_{2\infty} - y_{20})\, e^{-\lambda t}, \tag{2}$$

where $y_{20} = y_2(0)$ and $y_{2\infty}$ is the steady state value. The parameter λ is the inverse e-folding time which is expressed through the interaction rates as follows:

$$\lambda = A^*_{21} + Q_{21} + Q_{12}, \tag{3}$$

where Q_{ij} is the excitation ($i<j$) or deexcitation ($i>j$) rate, A^*_{21} is spontaneous emission probability. The value of $1/\lambda$ is rather large compared with the temperature evolution time scale. So, if the set of processes is restricted only by the transitions between the discrete levels, we could obtain "quasi-stationary" values of ν_k. However, electron impact ionization from the excited levels changes the time scale. It tends to establish the stationary ionization and occupation states simultaneously, and as a result the whole process is slowed down. The influence of ionization can be demonstrated by the analytical solution of a simple two-level system.

2 Analytical solution

Let us write the kinetic equations for a two-level atom taking into account transitions between discrete levels and ionization by electron impact. All the coefficients are suggested to be constant, and Q_{12} is used for impact excitation rate, R_{21} for deactivation rate (radiative and impact), and Q_{ic} for impact ionization from levels $i = 1, 2$. Recombination is not included, and hence both ν_1 and ν_2 tend to zero. The equations describing evolution of the level occupations are as follows:

$$\frac{d\nu_1}{dt} = -(Q_{12} + Q_{1c})\nu_1 + R_{21}\nu_2, \tag{4}$$

$$\frac{d\nu_2}{dt} = -(R_{21} + Q_{2c})\nu_2 + Q_{12}\nu_1. \tag{5}$$

The characteristic equation has two roots

$$\lambda_1 = \frac{s+d}{2}, \quad \lambda_2 = \frac{s-d}{2}, \tag{6}$$

where

$$s = Q_{12} + R_{21} + Q_{1c} + Q_{2c}, \tag{7}$$

$$p = Q_1\, R_{21} + Q_2\, (Q_{12} + Q_{1c}), \tag{8}$$

$$d = \sqrt{s^2 - 4p}\,. \tag{9}$$

With chosen initial conditions

$$\nu_{10} = 1, \quad \nu_{20} = 0, \tag{10}$$

the solution is

$$\nu_1 = \frac{Q_{12} + Q_{1c} - \lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1 - Q_{12} - Q_{1c}}{\lambda_1 - \lambda_2} e^{-\lambda_2 t}, \tag{11}$$

$$\nu_2 = \frac{Q_{12}}{\lambda_1 - \lambda_2} \left(e^{-\lambda_2 t} - e^{-\lambda_1 t} \right). \tag{12}$$

The excitation coefficient Q_{12} and the ionization coefficient from the ground level Q_{1c} are often small compared to the deactivation rate

$$Q_{12} + Q_{1c} \ll R_{21}. \tag{13}$$

Then Eqs. (6) for λ_1 and λ_2 are simplified

$$\lambda_1 \approx s, \quad \lambda_2 \approx m/s, \tag{14}$$

where

$$m = Q_1 \left(Q_2 + R_{21} \right) + Q_{12} Q_2. \tag{15}$$

Note that the simplified value of λ_1 is near to λ from Eq. (2). They coincide if we substitute A^*_{21}+Q_{21} instead of R_{21} and omit Q_{1c} and Q_{2c} in the expression for s. The inequality $m \ll s$ follows from Eq. (13), and hence

$$\lambda_2 \ll \lambda_1. \tag{16}$$

So, ionization changes significantly the evolution of excited level occupation in comparison with Eq. (2). Now the whole process develops in two stages. Occupation changes quickly at the first stage on the "short" time scale which is defined by deactivation. The final value of occupation establishes on the "long" time scale $1/\lambda_2$ in which ionization from the excited level plays the important role.

Equation (12) contains all these features. The occupation ν_2 quickly grows on the time scale $1/\lambda_1$ due to diminishing the negative term $e^{-\lambda_1 t}$ in Eq. (12), reaches a maximum at the moment t_m such as

$$t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}, \tag{17}$$

and drops exponentially after the maximum on the "long" scale $1/\lambda_2$. All these features are drawn on Fig. 1.

3 Radiative cooling behind a shock front

Here we consider numerical results for a multilevel system to which the analytical approach is not applicable. Let the unperturbed equilibrium gas pass through the shock front at the velocity 50 km s^{-1}, with the temperature T_0 and number density N_0 being typical of Mira Ceti atmospheres: $T_0 = 3000$ K and $N_0 = 10^{12}$ cm^{-3}.

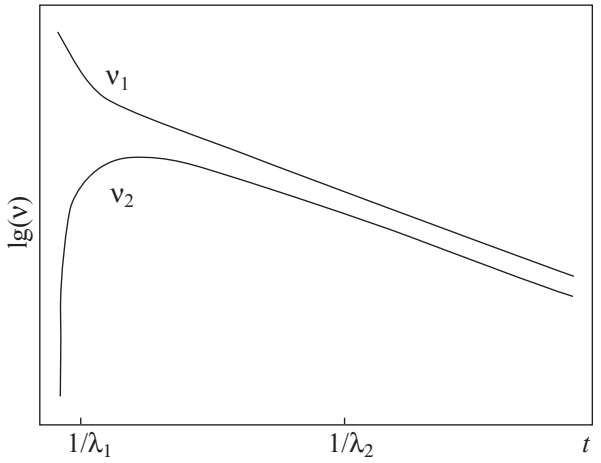

Figure 1: Analytical solution for a two-level system.

We have written the differential equations for the problem in [1]. Here we rewrite the equation describing the occupations of excited hydrogen levels as follows:

$$\begin{aligned}\frac{d\nu_k}{dt} = &-\left[q_{kc}N_e + \sum_{k>i}\left(A^*_{ki} + q_{ki}N_e\right) + \sum_{k<i} q_{ki}N_e\right]\nu_k \\ &+ \left(r_k + \gamma_k N_e\right)N_e x + \sum_{i>k}\left(A^*_{ik} + q_{ik}N_e\right)\nu_i + N_e\sum_{k<i} q_{ik}\nu_i,\end{aligned} \tag{18}$$

where q_{ki} are the coefficients of electron impact transitions between discrete levels k and i for excitation ($k<i$) and deactivation ($k>i$), r_k and γ_k are for radiative and triple recombination on the level k, respectively, x is the proton relative concentration, A^*_{ij} designs the spontaneous radiative transition probability taking into account the line scattering

$$A^*_{ij} = A_{ij}/\zeta_{ij}, \tag{19}$$

with ζ_{ij} being the scattering number before a quantum leaves the cooling gas. For ν_k and x, we have

$$\sum_{k=1}^{K}\nu_k + x = 1. \tag{20}$$

The upper limit K is the main quantum number of the highest level which is realized under the given conditions. We obtain $K = 25$ using Inglis–Teller equation.

Some results of calculations of non-stationary cooling of the shocked gas are drawn on Fig. 2 as functions of time $T_e(t)$ and $\nu_5(t)$. Note that T_e can rise when the gas cools behind the shock. Time t on the bottom axis is measured in seconds. A fragment of gas evolution during about 0.007 s is selected at about 3 seconds

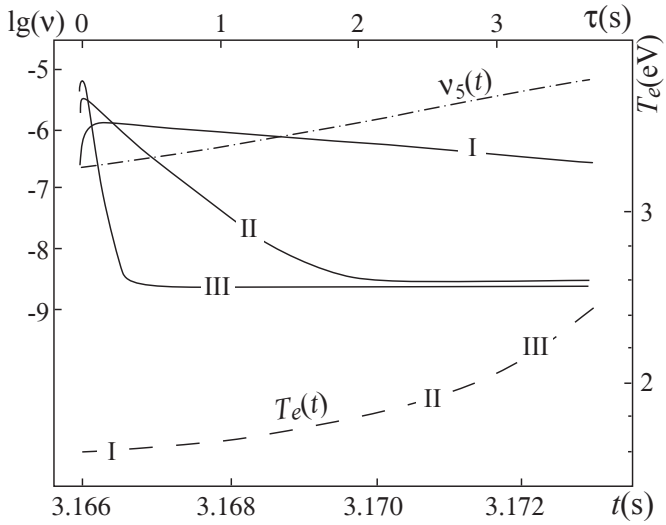

Figure 2: Electron temperature T_e (dashed) and non-stationary occupation ν_5 (dash-dot) evolution of the shocked gas behind a shock: the lower horizontal axis is the time (in sec) elapsed from the shock passed, the right vertical axis is T_e in eV, the left one is occupation in logarithmic scale, the upper horizontal axis is τ (in sec), solid lines I, II and III describe evolution $\nu_5^{(\mathrm{qs})}(\tau)$ as a solution of Eq. (18) for the temperature values I, II, and III.

after passing the shock. The lower dashed line describes the function $T_e(t)$, while $\nu_5(t)$ is drawn by the upper dash-dot line in the logarithmic scale. The right vertical axis is the electron temperature measured in electron-volts, and the left one is decimal logarithm of the occupation ν_5.

Three solid curves (I, II, and III) are the solutions of the system (17) for a given value of $T_e(t)$. These curves show "quasi-stationary" functions $\nu_5^{(\mathrm{qs})}(T_e(t), \tau)$ with the initial conditions $\nu_k(\tau = 0) = \nu_k(t)$, where $T_e(t)$ is a parameter and the independent variable τ is the time elapsed from the moment t. The values of τ are given on the upper horizontal axis. Three values of the parameter $T_e(t)$, chosen and noted by I, II, and III on the lower dashed curve, are as follows: $T_e^{(\mathrm{I})} = 1.6\,\mathrm{eV}$, $T_e^{(\mathrm{II})} = 1.9\,\mathrm{eV}$, $T_e^{(\mathrm{III})} = 2.2\,\mathrm{eV}$.

The functions $\nu_5^{(\mathrm{qs})}(T_e(t), \tau)$ make sure that $\nu_5(t)$ corresponds to a properly non-stationary process. All solid curves are similar to the analytical solution: a fast rise from the initial value followed by slow transition to a maximum and after that to a constant value $\nu_\infty(T_e(t))$.

Comparing the three solid curves with the lower dashed line, we see that the time for any $\nu_5^{(\mathrm{qs})}(T_e(t), \tau)$ to reach its steady state value $\nu_\infty(T_e(t))$ is longer than the time needed for T_e to grow up. This is the first evidence for non-stationary cooling process. And comparing the solid lines with the dash-dot one, we obtain the second evidence. The numerical difference between $\nu_5(t)$ and the corresponding steady state values are very large and can reach orders of magnitude (one for the curve I, and 3–4 orders for II and III).

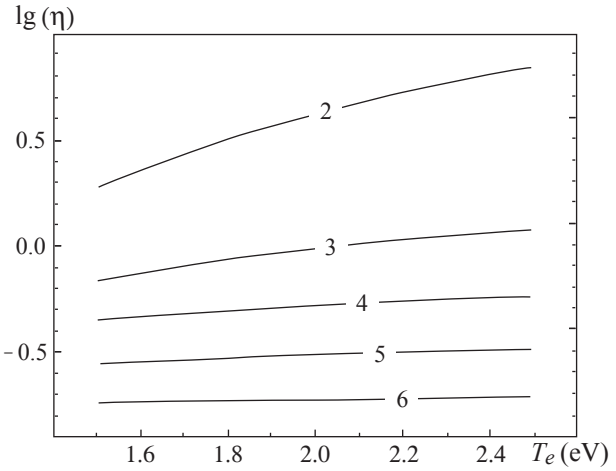

Figure 3: Ratio $q_{kc}/q_{k,k-1}$ vs T_e.

4 Ionization and deexcitation rates

We see that ionization from the excited states is an important factor which determines non-stationary character of shocked gas cooling. Its efficiency depends on the ratio

$$\eta_k = \frac{q_{kc}}{q_{k,k-1}}. \tag{21}$$

The greater η_k, the more effective the influence of ionization is. This relation is drawn on Fig. 3 for the first five excited levels of hydrogen atom as a function of the electron temperature T_e. The temperature range $1.5\,\text{eV} < T_e < 2.5\,\text{eV}$ is typical of a flow behind the front in a cold star atmosphere. The electron impact coefficients q_{kc} and $q_{k,k-1}$ were calculated from the Johnson formulas [2]. As follows from Fig. 3, ionization prevails in the case of the second level ($n = 2$), and the value of η_k remains sufficient up to $n = 6$.

So, electron impact ionization from the excited hydrogen levels ($n \leqslant 6$) is the cause of non-stationary cooling of the shocked gas.

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