

# LUNI-SOLAR PRECESSION DERIVED FROM PROPER MOTIONS OF PPM

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## Abstract

From the proper motions of 222185 PPM stars the correction to the value of the IAU 1976 luni-solar precession was found to be  $-0.38 \pm 0.05$  arcsec per century. The value is in a good agreement with the result obtained from VLBI observations of extragalactic radio sources. (Walter and C.Ma, 1994). In the magnitude ranges 6-8 and 10-13 the component  $\mu \cos(\delta)$  yields the correction to precession which strongly deviates from the above mentioned value. The reduction procedure to free the proper motions of the PPM of quasi-precessional magnitude equation and to make them compatible with the newest value of the luni-solar precession is proposed.

## 1 INTRODUCTION

For more than 70 years of the 20th century the Newcomb's value of the luni-solar precession ( $p = 5025.64$  arcsec/cy  $B1900.0$ ) was in general use. The new value of precession ( $p = 5029.0966$  arcsec/cy  $J2000.0$ ) adopted by IAU in 1976 resulted from intensive kinematical investigations of rather small catalog containing proper motions of 512 FK4 and FK4 Sup stars (Fricke, 1967 a,b, 1977 a,b) Since then in the field concerning precession, the astrometry made at least two tremendous achievements. The first one is a compilations of large catalogs containing proper motions for more than 100 000 stars (HIC, ACRS, PPM) in the standard system FK5. The second attainment is an establishment of the IERS, in frame of which the measurement of precessional constant can be done independently of the classical approach based on proper motions. Naturally, the both methods should be compared. The results obtained by Walter and Ma (1994) from VLBI observations and by Miyamo and Soma (1993) from proper motions of 30000 K-M giants listed in the ACRS may be regarded as the first result of the comparison.

The main goal of the present paper is continuation of this comparison this time employing the proper motions listed in the catalog PPM.

## 2 DERIVATION OF THE PRECESSIONAL CONSTANT FROM THE PROPER MOTIONS OF THE STARS

It is known that an erroneous value of the luni-solar precession yields the following contributions to the proper motions of the stars:

$$\Delta\mu\cos\delta = \Delta m + \Delta n \sin(\alpha) \tan(\delta), \quad (1)$$

$$\Delta\mu' = \Delta n \cos(\alpha), \quad (2)$$

here:

$\Delta n$  - correction to precession in declination;

$\Delta m$  - correction to precession in right ascension;

The values  $\Delta m$  and  $\Delta n$  are linked by the relation:

$$\Delta m = \Delta n \cot(\epsilon) - \Delta \lambda, \quad (3)$$

where

$\Delta \lambda = -0.029$  arcsec/cy - correction to the correction to the planet precession; (Licske et al., 1977)

$\epsilon$  - the tilt of the ecliptic onto equator;

From (1) and (2) one can see that the value  $\Delta n$  can be derived from both  $\mu$  and  $\mu'$  independently. The practice shows that though  $\Delta n_\alpha$  and  $\Delta n_\delta$  turn out to be in good agreement, still Eq.(3) is never satisfied. In order to link the values  $\Delta m$  and  $\Delta n$  one must adopt the hypothesis that Eq.(3) besides  $\Delta \lambda$  contains an additional term (the rate of fictitious motion of the equinox  $\Delta e$ ). For this reason Eq.(3) is replaced by equation

$$\Delta k = \Delta n \cot(\epsilon) - (\Delta e + \Delta \lambda) = \Delta n \cot(\epsilon) - \Delta E, \quad (4)$$

where

$$\Delta E = (\Delta e + \Delta \lambda); \quad (5)$$

It is important to emphasise that we need to replace Eq.(3) by Eq.(4) even in the case when only proper motions in RA are considered.

Besides the precessional contribution the proper motions reflect other kinematical effect, among them the Solar motion and the galactic rotation are the most prominent. In this paper the galactic rotation is considered in the frames of the Oort-Lindblad model. With regard to all these effects the basic kinematical equations for the proper motions can be written in the next form

$$\mu \cos \delta = \sum_{i=1}^6 L_i \phi_i(\alpha, \delta), \quad (6)$$

$$\mu' = \sum_{i=1}^6 L'_i \phi'_i(\alpha, \delta), \quad (7)$$

where

$$L_i = \{X, Y, P, Q, \Delta n, \Delta k\}, \quad (8)$$

$$L'_i = \{X, Y, Z, P, Q, \Delta n\}. \quad (9)$$

$$\phi_1(\alpha, \delta) = \sin(\alpha), \quad \phi_2(\alpha, \delta) = -\cos(\alpha), \quad (10, 11)$$

$$\phi_3(\alpha, \delta) = 0.416 \cos(\delta) \sin(2\alpha) - 0.407 \cos(\delta) \cos(2\alpha) + \quad (12)$$

$$0.280 \sin(\delta) \sin(\alpha) - 0.437 \sin(\delta) \cos(\alpha),$$

$$\phi_4(\alpha, \delta) = 0.198 \sin(\delta) \sin(\alpha) + 0.868 \sin(\delta) \cos(\alpha) + 0.456 \cos(\delta), \quad (13)$$

$$\phi_5(\alpha, \delta) = \sin(\delta) \sin(\alpha), \quad \phi_6(\alpha, \delta) = \cos(\delta). \quad (14, 15)$$

$$\phi'_1(\alpha, \delta) = \sin(\delta) \cos(\alpha), \quad (16)$$

$$\phi'_2(\alpha, \delta) = \sin(\delta) \sin(\alpha), \quad (17)$$

$$\phi'_3(\alpha, \delta) = -\cos(\delta), \quad (18)$$

$$\begin{aligned} \phi'_4(\alpha, \delta) = & 0.204 \sin(2\delta) \sin(2\alpha) + 0.208 \sin(2\delta) \cos(2\alpha) - \\ & 0.437 \cos(2\delta) \sin(\alpha) - 0.280 \cos(2\delta) \cos(\alpha) - 0.542 \sin(2\delta), \end{aligned} \quad (19)$$

$$\phi'_5(\alpha, \delta) = 0.198 \cos(\alpha) - 0.868 \sin(\alpha), \quad (20)$$

$$\phi'_6(\alpha, \delta) = \cos(\alpha). \quad (21)$$

Numerical values of the coefficients in these formulae correspond to the angles of mutual orientation between galactic and equatorial coordinate systems at the epoch J2000.0.

Now we make a short review of the rest of the values that enter Eqs.(8) and (9).

1. Parameters of the Solar motion:

$$X = (V/kr) \cos D \cos A, \quad (22)$$

$$Y = (V/kr) \cos D \sin A, \quad (23)$$

$$Z = (V/kr) \sin D, \quad (24)$$

where

$V$  - the velocity of the Solar motion with respect to centroid of the chosen group of stars in  $km s^{-1}$ ;

$r$  - the distance of a star from the Sun in  $kps$ ;

$A, D$  - equatorial coordinates of the apex of the Solar motion;

$k = 47.4$  - the factor to reduce dimension  $[arcsec/cy]$  into  $[km s^{-1}kps^{-1}]$ ;

2. Parameters of the Galaxy rotation ( the Oort coefficients), expressed in  $[arcsec/cy]$ :

$$P = A/k; \quad Q = B/k \quad (25)$$

$$A = -\frac{1}{2}(R \frac{d\Omega}{dR}), \quad B = A - \Omega, \quad (26)$$

where  $\Omega$  - the angular speed of the Galactic rotation.

### 3 NUMERICAL MATERIAL

At present, the catalog PPM consists of two parts: PPM North (1988) and PPM South (1993), containing correspondingly 181731 and 191179 stars. Total number of stars in catalog is 378910. Positions and proper motions of stars in PPM are given in the system FK5 J2000. The average errors of proper motions are equal  $0.44 ''/cy$  and  $0.30 ''/cy$  for northern and southern hemisphere of the celestial sphere. In addition to positions and proper motions the photographic magnitudes for all the stars and the spectral types for 296547 stars are available.

The magnitude distributions for the PPM stars is shown in Fig.1. From this diagram it is visible, that down to  $9^m$  the catalog PPM, probably, completely reflects the stellar population, and for fainter stars there is only partial information.

In Fig. 2 we exhibit the spectral distributions of the PPM stars. The majority of stars belongs to a type K, however, PPM lists sufficient number of stars in spectral types A,F,G. On the contrary, the types O,B,M,N are rather poor.

From Fig. 3, on which distribution of PPM stars on module of proper motion is shown, we conclude, that the majority of the PPM stars are slow and that only the insignificant part of them (10% ) has large proper motions, exceeding  $6 ''/cy$  . In kinematical investigations such stars are rejected, since one fast star can spoil the result which follows from hundreds slow stars.

The distribution of stars on an average error of determination of proper motions is shown on Fig. 4. We see, that for 80% of stars the average error of the proper motions in the catalog PPM does not exceed  $0.6 \text{ arcsec/cy}$ .

In present work the first version of the catalog, consisting of 326518 stars is used. The material, contained in the catalog PPM, was subjected to selections with purpose to choose the stars with the most exact proper motions and to reject the stars with large proper motions. We selected the stars to be processed, for which the following conditions are valid:

$$|\mu| < 6 \text{ arcsec/cy}, \quad |\sigma\mu| < 0.6 \text{ arcsec/cy}, \quad m > 6.$$

In all, 222185 stars have been found to satisfy this filter, from them only for 203181 stars the spectral classes are available. The solutions were carried out for stars in the following magnitude intervals

$$6^m - 8^m, 8^m - 9^m, 9^m - 10^m, 10^m - 11^m, 11^m - 13^m$$

and for the spectral classes

$$O - B, A, F, G, K, M - N.$$

In each of these cases the proper motions were averaged over spherical trapeze obtained by an uniform division of equator and of a circle of declinations on 24 and 18 parts respectively. Numerical runs have shown, that the results remain practically the same for divisions on 12 and 9 as well as 48 and 36 parts. In the least squares solutions, the weights proportional to the number of stars in each zone were assigned to each equations.

## 4 PRECESSIONAL CORRECTIONS

From physical reasons it follows, that precessional quantities  $\Delta n$  and  $\Delta m$  (but not  $\Delta k$ ) should not depend on the brightness of stars. The catalog PPM gives the following results:

Table 2. Precessional corrections in seconds of arc per 100 years. The indexes  $\alpha$  and  $\delta$  designate the solutions from  $\mu$  and  $\mu'$  accordingly

m	$\Delta n_\alpha$	$\Delta n_\delta$	$\Delta k$	$\Delta e$	$N$
6 - 8	$-0.28 \pm 0.05$	$-0.16 \pm 0.03$	$0.01 \pm 0.04$	$-0.63 \pm 0.12$	20300
8 - 9	$-0.18 \pm 0.04$	$-0.15 \pm 0.02$	$0.01 \pm 0.03$	$-0.40 \pm 0.10$	62832
9 - 10	$-0.17 \pm 0.03$	$-0.18 \pm 0.02$	$-0.09 \pm 0.02$	$-0.27 \pm 0.07$	99811
10 - 11	$-0.04 \pm 0.04$	$-0.16 \pm 0.03$	$-0.23 \pm 0.03$	$0.17 \pm 0.10$	32880
11 - 13	$0.20 \pm 0.09$	$-0.18 \pm 0.08$	$-0.34 \pm 0.06$	$0.83 \pm 0.22$	6362
<i>All</i>	$-0.10 \pm 0.03$	$-0.15 \pm 0.02$	$-0.05 \pm 0.02$	$-0.15 \pm 0.07$	222185

Here one can see that the values  $\Delta n_\delta$  practically do not change in the whole range of magnitudes, and the values  $\Delta n_\alpha$  are essentially the same in the range from  $8^m$  up to  $10^m$ . Nevertheless, in the range  $6^m - 8^m$  and  $10^m - 13^m$  the correction  $\Delta n_\alpha$  differs from  $\Delta n_\delta$ . This circumstance, if attributed to the magnitude equation, can be explained by pure astrometric reasons. We can suppose that since it is known that the systematic differences FK5-FK4 for  $\mu'$  are free from the magnitude equation, while the same differences for  $\mu \cos \delta$  are m-dependent (Fricke et al., 1988).

We prefer to take as the final result the precessional correction which follows from the  $\mu'$

$$\Delta n = -0.15 \pm 0.02 \text{ arcsec/cy},$$

from which we obtain the correction to the constant of luni-solar precession

$$\Delta p = -0.38 \pm 0.05 \text{ arcsec/cy.}$$

This value is consistent with results obtained from kinematical investigation of 30 000 K-M giants, the proper motions of which were taken from the catalog ACRS (Miyamoto et al., 1993).

$$\Delta p = -0.27 \pm 0.03 \text{ arcsec/cy.}$$

Moreover, our value is in excellent agreement the results obtained from the VLBI observations of extragalactic radio sources (Walter and Ma, 1994):

$$\Delta p = -0.36 \pm 0.11 \text{ arcsec/cy from annual catalogs}$$

$$\Delta p = -0.38 \pm 0.12 \text{ arcsec/cy from semi - annual catalogs}$$

The values of the fictitious equinox motion  $\Delta e$ , shown in Table 2, are m-dependent. It is likely, that this is caused by the magnitude equation. The confirmation of an magnitude equation in the proper motions  $\mu \cos(\delta)$  of the stars in the catalog PPM follows from table 2-a, in which the same parameters are shown, this time obtained from the stars of various spectral types. Really, except for the spectral types O-B and M-N (10 per cent) all other stars give for  $\Delta n_\delta$  practically the same value, which we see in Table 2. It is not so with  $\Delta n_\alpha$  and  $\Delta e$ . Inside each spectral class there are the stars of all magnitudes, therefore the values  $\Delta n_\alpha$  and  $\Delta e$  being insignificant are scattered around the mean values obtained from the whole material taken in processing.

Table 2-a. Precessional corrections in seconds of arc per 100 years.

The indexes  $\alpha$  and  $\delta$  designate the solutions from  $\mu$  and  $\mu'$

Sp	$\Delta n_\alpha$	$\Delta n_\delta$	$\Delta k$	$\Delta e$	$N$
<i>O - B</i>	$0.05 \pm 0.09$	$-0.05 \pm 0.03$	$0.18 \pm 0.07$	$-0.03 \pm 0.22$	14505
<i>A</i>	$-0.09 \pm 0.04$	$-0.16 \pm 0.02$	$0.03 \pm 0.04$	$-0.21 \pm 0.10$	45686
<i>F</i>	$-0.10 \pm 0.04$	$-0.13 \pm 0.02$	$-0.10 \pm 0.03$	$-0.10 \pm 0.10$	46130
<i>G</i>	$-0.03 \pm 0.04$	$-0.12 \pm 0.02$	$-0.05 \pm 0.03$	$0.01 \pm 0.10$	30855
<i>K</i>	$-0.06 \pm 0.03$	$-0.16 \pm 0.02$	$-0.02 \pm 0.02$	$-0.09 \pm 0.07$	60909
<i>M - N</i>	$0.07 \pm 0.06$	$-0.08 \pm 0.03$	$0.08 \pm 0.05$	$0.11 \pm 0.15$	5096
<i>All</i>	$-0.07 \pm 0.03$	$-0.14 \pm 0.02$	$-0.04 \pm 0.02$	$-0.09 \pm 0.07$	203181

For further treating of the material it is expedient to exclude from the proper motions the precessional effects. It can be made with help of the following formulas (CPPM-Corrected PPM):

$$\mu_{CPPM} = \mu_{PPM} - \Delta k - \Delta n \sin(\alpha) \tan(\delta);$$

$$\mu'_{CPPM} = \mu'_{PPM} - \Delta n \cos(\alpha).$$

The numerical values of  $\Delta n_\alpha$ ,  $\Delta n_\delta$  and  $\Delta k$  should be taken from Table 2 for each interval of magnitudes. If this is done, we would have the proper motions consistent with the improved precession and free of quasi-precessional errors due to magnitude equation.

Table 2-b. Precessional corrections in seconds of an arc per 100 years.

The indexes  $\alpha$  and  $\delta$  designate the solutions from  $\mu$  and  $\mu'$

Sp	$\Delta n_\alpha$	$\Delta n_\delta$	$\Delta k$	$\Delta e$	$N$
<i>O – B</i>	$0.26 \pm 0.09$	$0.12 \pm 0.03$	$0.21 \pm 0.07$	$0.42 \pm 0.22$	14505
<i>A</i>	$0.08 \pm 0.04$	$0.01 \pm 0.02$	$0.07 \pm 0.04$	$0.14 \pm 0.10$	45686
<i>F</i>	$0.03 \pm 0.04$	$0.04 \pm 0.02$	$-0.02 \pm 0.03$	$0.12 \pm 0.10$	46130
<i>G</i>	$0.08 \pm 0.04$	$0.05 \pm 0.02$	$0.04 \pm 0.03$	$0.17 \pm 0.10$	30855
<i>K</i>	$0.06 \pm 0.03$	$0.00 \pm 0.02$	$0.06 \pm 0.02$	$0.11 \pm 0.07$	60909
<i>M – N</i>	$0.16 \pm 0.06$	$0.08 \pm 0.03$	$0.18 \pm 0.05$	$0.22 \pm 0.15$	5096
<i>All</i>	$0.06 \pm 0.03$	$0.03 \pm 0.02$	$0.04 \pm 0.02$	$0.13 \pm 0.07$	203181

In Table 2-b we show results obtained from corrected proper motions  $\mu_{CPPM}$  and  $\mu'_{CPPM}$ . We see that for the stars of spectral types A-F-G-K (90 %) corrections  $\Delta n_\alpha$ ,  $\Delta n_\delta$  and  $\Delta k$  turned out to be insignificant (equal to zero within the r.m.s.e of their determination). This means that our "absolutization" of the proper motions with respect to precession and magnitude equation is worthy of consideration.

From the other hand, the stars of O-B and M-N spectral types still show residual rotation  $\Delta n_\alpha = 0.26 \pm 0.09$ ;  $\Delta n_\delta = 0.12 \pm 0.03$  for O-B stars and  $\Delta n_\alpha = 0.16 \pm 0.06$ ;  $\Delta n_\delta = 0.08 \pm 0.03$  for M-N stars. This indicates that the kinematics of these stars is essentially different from the kinematics of the majority of PPM stars, and this is the reason why these stars should not be used for determination of the precessional constant. In this connection it is worthy to emphasize that the IAU 1976 constant of luni-solar precession was derived by Fricke (1977) from the proper motions of 512 FK4/FK4 Sup stars of which more than 60 per cent were the stars of O-B and M-N spectral types. It is likely that it is the reason why the correction to the Newcomb precession was overestimated by about than 30 per cent.

## 5 CONCLUSION

The main result of this study is: the IAU 1976 value of the luni-solar precession requires the correction

$$\Delta p = -0.38 \pm 0.05 \text{ arcsec/cy}$$

obtained from the proper motion of the PPM stars. This correction is in an excellent agreement with the value found from the VLBI observation  $-0.359 \pm 0.114$  from annual and  $-0.384 \pm 0.116 \text{ arcsec/cy}$  from semi-annual catalogs (Walter and C.Ma, 1994). Our result being quite independent of the proper motions technique gives us the opportunity to find all those  $m$ -dependent regional errors in the  $\mu$ -components that are inconsistent with the precessional effects. To our opinion exclusion them from the components  $\mu \cos \delta$  together with reduction of component  $\mu \cos \delta$  and  $\mu'$  to new value of the precessional constant would significantly improve the proper motion of the catalog PPM.

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