# THE LOCAL STAR SYSTEM: KINEMATICS DERIVED FROM RADIAL VELOCITIES

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The rotational parameters of the Local star system are derived from radial velocities of 2537 stars of FK5. The most likely geometrical description of the Local system are made. The results obtained from the radial velocities do not contradict to the ones derived from the proper motions and may be considered as an evidence of the existence of the Local Star system.

KEY WORDS Radial velocities, Local star system.

## 1. INTRODUCTION

In the previous work (Tsvetkov, 1994) the results based on an investigation of the proper motions of stars were obtained with the evidence of the existence of the Local Star system (henceforth LSS). As known, the rotation of the LSS influences not only the proper motions but the radial velocities, too. For complete knowledge of the LSS kinematics a study of the radial velocities is desirable.

## 2. THE EQUATION OF THE LOCAL STAR SYSTEM ROTATION

In 1950 Shatsova derived equations that describe the contribution of the Local Star system to the proper motions and to the radial velocities. In this paper for the convenience we give the short description of the basic equation, which outlines the influence of the LSS rotation on the radial velocities.

Let us fix a coordinate system with its origin placed at the center of the Sun and the *Z*-axis directed to the North Galactic pole (fig. 1). We arrange the *X*-axis in such a way that the LSS rotation vector $\boldsymbol{\omega}$ be parallel to *XZ*-plane and the *Y*-axis be normal to both others. The longitude of the *X*-axis coincides with the longitude of the rotational pole $L_0$; we denote its polar distance as $P_0$. Draw the perpendicular from point *S* to $\boldsymbol{\omega}$; the point of their intersection $M_0$ we call the center of rotation of the LSS. Extend the vector $\vec{\mathbf{r}}$ from point *S* to point *M* (an arbitrary star) and $\vec{\mathbf{r}}_1$ from $M_0$ to *M*. The longitudes of these points are $l - L_0$ and $l_0 - L_0$; the latitudes are $b$ and $b_0$.

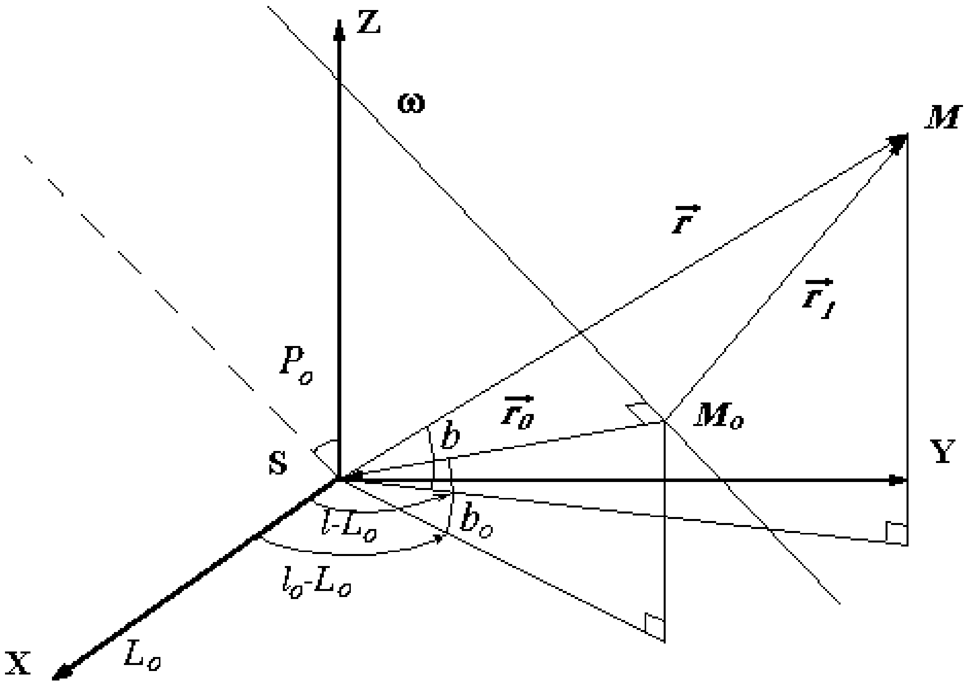


**Figure 1.** *To the theory of the LSS rotation*

For the velocity of point *M* one can write

$$\vec{\mathbf{v}} = \vec{\Omega} \times \vec{\mathbf{R}} + \vec{\omega} \times \vec{\mathbf{r}}_1 , \qquad (1.1)$$

where $\vec{\Omega}$ is the angular velocity of the Galaxy, $\vec{\mathbf{R}}$ is the distance from the Galactic center.

From observational data we can determine only differential effect, if we suppose the Sun takes part in the rotation of the LSS:

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0. \qquad (1.2)$$

Using the relations

$$\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_0 + \vec{\mathbf{r}} \quad \text{and} \quad \vec{\mathbf{R}} = \vec{\mathbf{R}}_0 + \vec{\mathbf{r}} \qquad (1.3)$$

from Eqs. (2.1-2.2) we acquire:

$$\Delta\vec{\mathbf{v}} = (\vec{\Omega} - \vec{\Omega}_0) \times \vec{\mathbf{R}}_0 + \vec{\Omega} \times \vec{\mathbf{r}} + (\vec{\omega} - \vec{\omega}_0) \times \vec{\mathbf{r}}_0 + \vec{\omega} \times \vec{\mathbf{r}} , \qquad (1.4)$$

e.g., the Galactic motion and the LSS rotation are separated (in Eqs. (1.2-1.4) the subscript "0" refers to the Sun). Suppose now that the formulae of the Galactic rotation are known and we project only $(\vec{\omega} - \vec{\omega}_0) \times \vec{\mathbf{r}}_0 + \vec{\omega} \times \vec{\mathbf{r}}$ on the axes. This yields the next expressions for the contribution of the LSS rotation to the radial velocities of stars:

$$\begin{aligned} \Delta v_r = (\omega - \omega_0) r_0 \big[ & \sin P_0 \sin b_0 \cos b_0 \sin(l - L_0) - \\ & - \cos P_0 \cos b_0 \cos b \sin(l - l_0) - \sin P_0 \cos b_0 \sin(l_0 - L_0) \sin b \big] \end{aligned} . \qquad (1.5)$$

Now we suppose that the angular velocity ω depends only on the distance from rotation axis ρ:

$$\omega = \omega(\rho(\vec{\mathbf{r}}_1)) \qquad (1.6)$$

and does not depend upon the distance from the symmetry plane. This is supported by theoretical investigations (Ogorodnikov, 1944), which tell us that the angular velocity of a stationary stellar system is independent of $Z$.

Since we do not know the function $\omega = \omega(\vec{\mathbf{r}})$ we may expand it in the Tailor's series by powers of $r$. Existence of the third harmonics in the representations of observational proper motions points that the rotational center is very close and we can not ignore the second terms:

$$\omega(\vec{\mathbf{r}}) = \omega_0 + \left(\frac{d\omega}{dr}\right)_0 r + \frac{1}{2}\left(\frac{d^2\omega}{dr^2}\right)_0 r^2 \quad ; \tag{1.7}$$

here $\left(\frac{d\omega}{dr}\right)_0$ and $\left(\frac{d^2\omega}{dr^2}\right)_0$ are the first and the second derivatives of $\omega$ along the $\dot{\mathbf{r}}$ direction in the Sun's vicinity.

After evaluating of the derivatives it turns out that Eq. (1.7) can be rewritten as

$$\omega - \omega_0 = \Omega' + \Omega'' \sin b \ ; \tag{1.8}$$

where Ω' and Ω" are functions symmetrical about the Galactic plane:

$$\begin{aligned}\Omega' = (\omega_0' r)\Big\{&-\cos b_0 \cos b \cos(l-l_0) + \\ &+\frac{1}{2n_0}\big[1-\sin^2 b(\sin^2 b_0 + \cos^2 P_0) - \\ &-\cos^2 b\big(\cos^2 b_0 \cos^2(l-l_0) + \sin^2 P_0 \cos^2(l-L_0)\big)\big]\Big\} + \\ &+(\omega_0'' r^2)\frac{1}{2}\left\{\cos^2 b_0 \cos^2 b \cos^2(l-l_0) + \sin^2 b_0 \sin^2 b\right\}\end{aligned} \tag{1.9}$$

$$\begin{aligned}\Omega'' = (\omega_0' r)&\left\{-\sin b_0 - \frac{1}{n_0}\cos b\big[\sin b_0 \cos b_0 \cos(l-l_0) + \sin P_0 \cos P_0 \cos(l-L_0)\big]\right\} + \\ &+(\omega_0'' r^2)\left\{\cos b \sin b_0 \cos b_0 \cos(l-l_0)\right\}\end{aligned} . \tag{1.10}$$

In these equations and further $n_0 = \frac{r_0}{r}$; $\omega_0$ is the angular velocity of the LSS rotation in the Sun's vicinity; $\omega_0' = \left(\frac{d\omega}{d\rho}\right)_0$, $\omega_0'' = \left(\frac{d^2\omega}{d\rho^2}\right)_0$ are the first and the second derivatives in the Sun neighbourhood along the normal direction to the rotation axis.

The direct use of Eqs. (1.5) leads to inconvenience, because of other kinematic phenomena, which interfere with the LSS rotation. It is difficult to separate the motions of various kinds due to considerable correlations.

Since the influences of the Galactic rotation and the Solar motion are both symmetrical with respect to the Galactic equator we can exclude them if we introduce the differences of radial velocities "North-South":

$$\delta v_r = (\Delta v_r)_{\mathrm{N}} - (\Delta v_r)_{\mathrm{S}}, \tag{1.11}$$

taken in symmetrical points of the sky. These values will not contain previous effects even if we do not know the exact law of the Galactic rotation. Still, the Solar motion component $Z_\odot$ penetrates into them and we need to exclude it. Really, from Eq. (1.5) we find

$$
\begin{aligned}
\delta v_r = & -2r_0 \sin b \cos b_0 \\
& \{-\Omega' \sin P_0 \sin(l_0 - L_0) + \\
& +\Omega''[\sin P_0 \sin b_0 \sin(l - L_0) - \cos P_0 \cos b \sin(l - l_0)]\} - \\
& -2Z_\odot \sin b .
\end{aligned}
\tag{1.12}
$$

Theoretically, the Eq. (1.12) allows us to find the next seven LSS parameters from proper motions:

$L_0, P_0$ - the latitude and the polar distance of the rotational pole;

$l_0, b_0, r_0$ - the coordinates of the rotational center;

$\omega'_0, \omega''_0$ - the angular velocity derivatives in the Sun neighbourhood.

However, because of strong correlation between some parameters, we refused to acquire $L_0$ and $P_0$ from the radial velocities. In the previous work the values of $L_0, P_0$ were adopted to be equal the coordinates of the Gould's Belt pole:

$$L_0 = 343° , \quad P_0 = 17° . \tag{1.13}$$

In the present paper we shall make the same suggestion intentionally that the results can be compared with ones derived from proper motions. The common problem in the stellar kinematics is poor knowledge of the stellar parallaxes. Therefore, following conventional way, instead of $r_0, \omega'_0, \omega''_0$ we will determine the next parameters:

$$n_0 = \frac{r_0}{\langle r \rangle}, \quad \omega'_0 \langle r \rangle, \quad \omega''_0 \langle r^2 \rangle , \tag{1.14}$$

where angle parentheses denote averaging of the corresponding values. In these designations the multiplier $r_0$ in the equation (1.12) can be expressed as

$$r_0 = n_0 \langle r \rangle . \tag{1.15}$$

Unfortunately, we are forced to find an average distance to stars listed in a catalogue before solving equation (1.12). This obstacle makes troubles to get the LSS parameters from the radial velocities as compared with the proper motions case. Also, we are not able to find so important parameter as the angular velocity because it does not enter the equation (1.12). In order to improve the comparison of the results and to set the initial approximation for solving the system of equations (1.12) we measure $\omega'_0 \langle r \rangle, \quad \omega''_0 \langle r^2 \rangle$ in arcsec per century. For this we multiply these values by $k$=0.0474 to measure them in km·s$^{-1}$·pc$^{-1}$. After this the equation (1.12) may be represented as

$$
\begin{aligned}
\delta v_r = & -2k n_0 \langle r \rangle \sin b \cos b_0 \\
& \{-\Omega' \sin P_0 \sin(l_0 - L_0) + \\
& +\Omega''[\sin P_0 \sin b_0 \sin(l - L_0) - \cos P_0 \cos b \sin(l - l_0)]\} - \\
& -2Z_\odot \sin b .
\end{aligned}
\tag{1.16}
$$

So, we have the next six parameters to be determined from analysis of the radial velocities:

$$l_0, \quad b_0, \quad n_0, \quad \omega_0, \quad \omega'_0 \langle r \rangle, \quad \omega''_0 \langle r^2 \rangle .$$

## 3. DATA PREPARING

We used a subset of 2537 stars of FK5/FK5Sup for which the radial velocities are available. Following our method we need not the radial velocities but their differences "North - South" - $\delta v_r$. It is difficult to find the couple for each star in the other hemisphere. The way out is to use the averaged radial velocities over trapezia in the zones symmetrical with respect to the Galactic equator. The coordinates of the centers of such trapezia are determined by the next expressions:

$$l_i = \frac{180^\circ}{n} + \frac{360^\circ}{n}(i-1) , \qquad i=1,2,...,n \tag{2.1}$$

$$b_i = 90^\circ - \frac{90^\circ}{m} - \frac{180^\circ}{m}(j-1) , \qquad j=1,2,...,m \tag{2.2}$$

where $m$ is the number of the latitude zones ($m/2$ is the number of the zones in one hemisphere), $n$ is the number of trapezia in every zone. The indexes of a trapezium for a star can be found as

$$i = \left[\frac{l}{360^\circ} n\right] + 1, \; j = \left[\frac{90^\circ - b}{180^\circ} m\right] + 1, \tag{2.3}$$

where brackets denote an integer part. This procedure yields a grid $m \times n$ of points (2.1), (2.2) and as table, which contains averaged values of the $\delta v_r$ for all the trapezia, as a table of numbers of employed stars. To each trapezium a weight may be assigned that is a sum of stars corresponding to North-South couples of areas. We get the final numerical material for our investigation after subtracting the South values from the North ones. For more detailed studies, we can construct such tables for star groups marked with some specific features, for instance, the spectral type.

Besides the differences of the radial velocities we must know the component $Z_\odot$ of the Solar motion and the average distance of stars of the catalogues. The $Z_\odot$-component follows from solution of the Airy-Kovalsky equation for radial velocities:

$$-X_\odot \cos b \cos l - Y_\odot \cos b \sin l - Z_\odot \sin b = v_r . \tag{2.4}$$

The 2537 stars of the FK5 provide

$$Z_\odot = 8.08 \pm 0.77 \text{ km/sec} . \tag{2.5}$$

The determination of the average distance of the FK5 stars $\langle r \rangle$ is more difficult problem. Lucky, the 1170 stars of used set have the trigonometric parallaxes. Fig. 2 illustrates the distribution of the stars on the distance. This permits us to estimate the $\langle r \rangle$ to be

$$\langle r \rangle = 36 \text{ pc}. \tag{2.6}$$

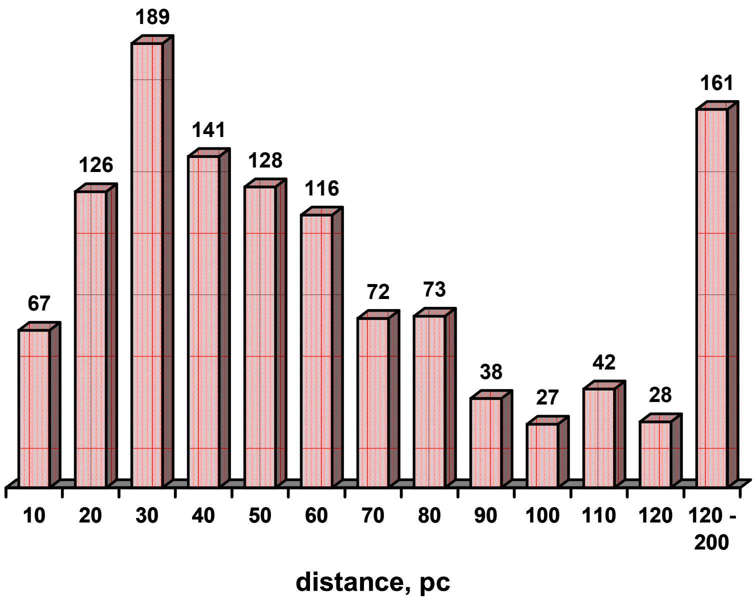


**Figure 2.** *Dependence of the number of stars on distance*

A few stars have large radial velocities and it is reasonable to exclude them from examining. Taking into account the distribution of the stars on radial velocities we rejected these stars, for which $|v_r| > 40\,\mathrm{km/sec}$. The number of such stars was found to be equal 109.

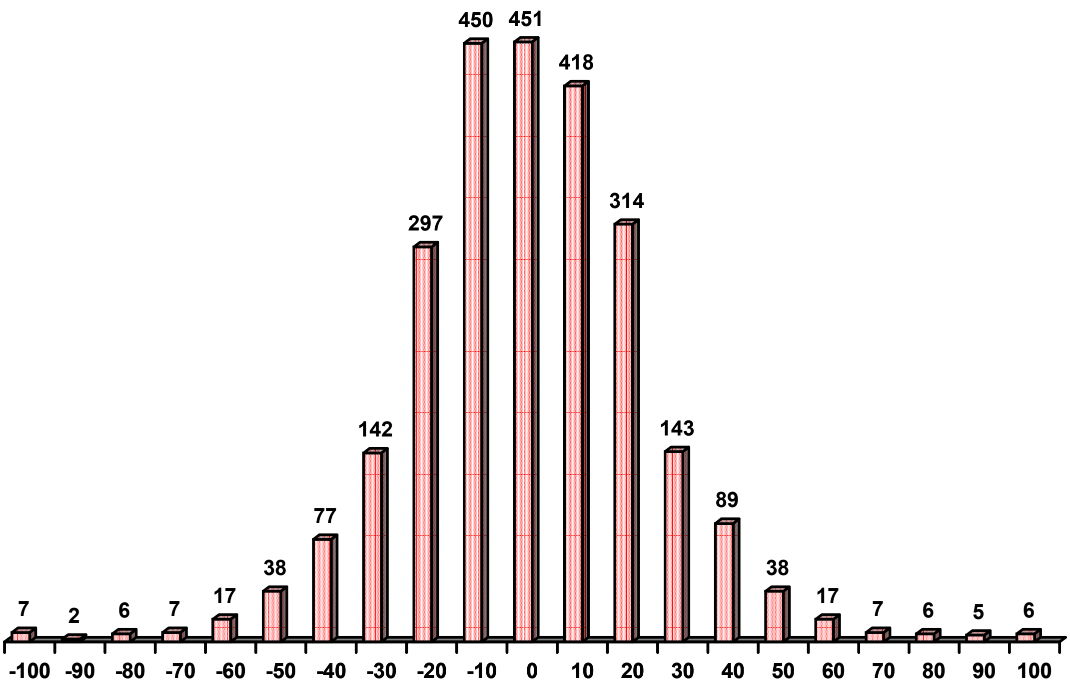


**Figure 3.** *Dependence of the number of stars on radial velocity*

The most of the stars in the FK5 having data about radial velocity, belong to the spectral type **K** though there is a large group of stars of middle and early spectral types. Our previous paper shows that **O-F** stars satisfy the equations of the LSS rotation better than others. In present work we will try to do find the dependence of the rotational parameters on spectral types of stars, too.

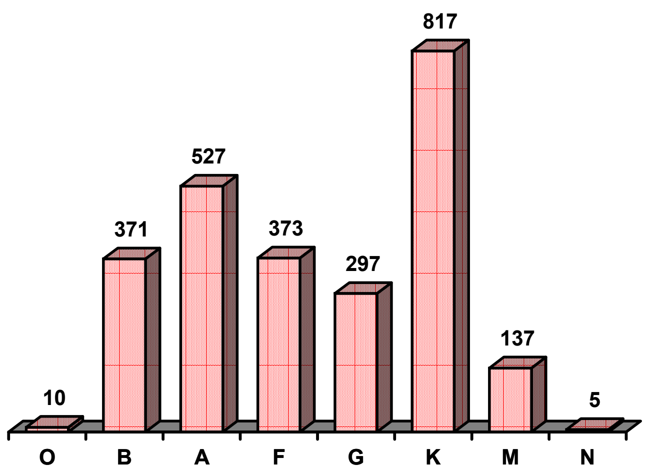


**Figure 4.** *Distribution of the stars by spectral types*

The differences of the radial velocities "North-South" show the significant asymmetry, in which one can notice a systematic part due to the 1-st and 2-nd harmonics by *l*. This circumstance confirms the LSS rotation. Fig. 5 shows the $\delta v_r$ after subtracting the Sun motion component $Z_{\odot}$ from $(v_r)_N - (v_r)_S$

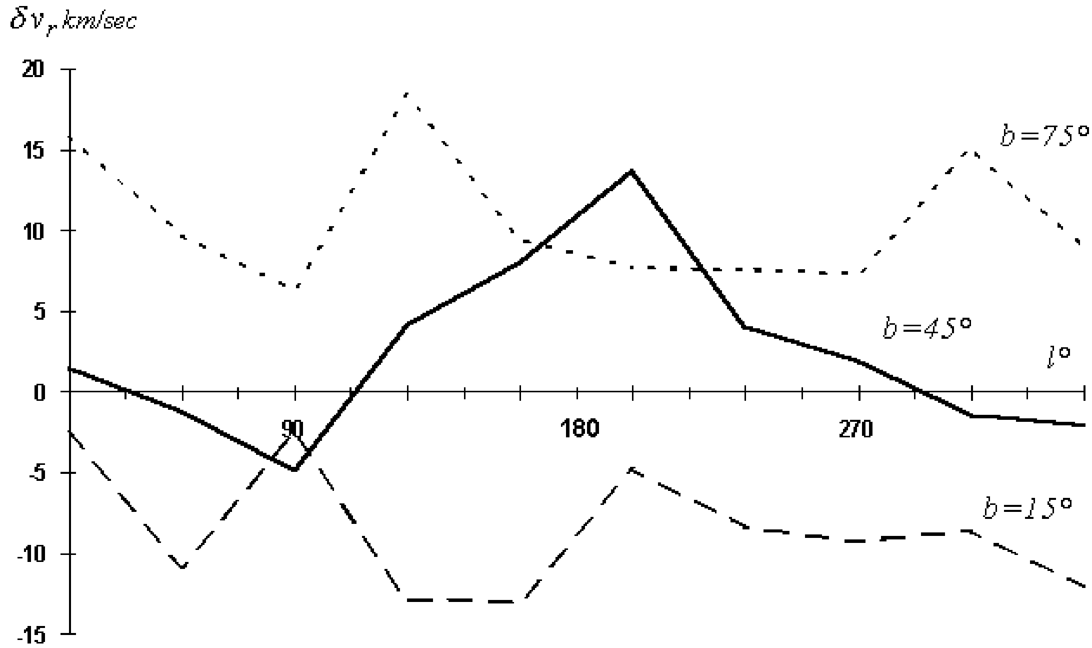


**Figure 5.** *The asymmetry of the radial velocities for all stars*

## 4. THE METHOD TO SOLVE THE LSS EQUATIONS

We use the linearzation method to solve the equation (1.16), which can be represented here as

$$y = f(\vec{\mathbf{x}}, \vec{\mathbf{t}}), \tag{3.1}$$

where $y$ is $\delta v_r$;

$\vec{\mathbf{x}}$ is a vector of unknown variables $l_0$, $b_0$, $n_0$, $\omega_0'\langle r\rangle$, $\omega_0''\langle r^2\rangle$;

$\vec{\mathbf{t}}$ is a vector of coordinates of a North trapezium center;

$f$ is non-linear function of $\vec{\mathbf{x}}$ and $\vec{\mathbf{t}}$.

To apply conventional LSM technique for solving Eq. (3.1) the latter is to be linearized.

Let we know a primary approximation $\vec{\mathbf{x}}_0$ of $\vec{\mathbf{x}}$ from preliminary consideration. Then one can calculate

$$y_0 = f(\vec{\mathbf{x}}_0, \vec{\mathbf{t}}) \tag{3.2}$$

for every point $\vec{\mathbf{t}}$ and write, using the Tailor's series representation with accuracy to the first power term

$$\Delta y = y - y_0 \approx \sum_j \Delta x_j \left.\frac{\partial f}{\partial x_j}\right|_{\vec{\mathbf{x}}_0} \tag{3.3}$$

The system generated by Eq. (3.3) is linear concerning unknown corrections $\Delta x_j$ to components of $\vec{\mathbf{x}}$. Now, one can solve this system

$$\hat{\mathbf{A}}\,\Delta\vec{\mathbf{x}} = \vec{\mathbf{Y}} \tag{3.4}$$

by the conventional least squares technique. The elements of the matrix $\mathbf{A}$ are

$$\mathbf{A}_{ij} = \left.\frac{\partial f(\vec{\mathbf{x}}, \vec{\mathbf{t}}_i)}{\partial x_j}\right|_{\vec{\mathbf{x}}_0} \qquad \begin{matrix} i = 1,2,\ldots m \\ j = 1,2,\ldots 5 \end{matrix} \tag{3.5}$$

where $\vec{\mathbf{t}}_i = (l_i, b_i)$ are the coordinates of the center of North $i$-th trapezium. The partial derivatives may be evaluated by symmetrical formula:

$$\frac{\partial f}{\partial x_j} = \frac{f(x_1, x_2, \ldots, x_j + \Delta_j, \ldots, x_{n,} \vec{\mathbf{t}}_i) - f(x_1, x_2, \ldots, x_j - \Delta_j, \ldots, x_{n,} \vec{\mathbf{t}}_i)}{2\Delta_j} \tag{3.6}$$

The elements of the column $\mathbf{Y}$ are written as

$$\mathbf{Y}_i = y_i - f(\vec{\mathbf{x}}_0, \vec{\mathbf{t}}_i), \tag{3.7}$$

where $y_i$ are the average values of $\delta v_r$ in $i$-th trapezium. The weights may be assigned to all the equations.

Adding the corrections $\Delta x_j$ received from solution of Eq. (1.16), to the components $\vec{\mathbf{x}}_0$ produces the next approximation $\vec{\mathbf{x}}_1$:

$$\vec{\mathbf{x}}_1 = \vec{\mathbf{x}}_0 + \Delta\vec{\mathbf{x}} \tag{3.8}$$

We repeat this process until the condition

$$\left|\Delta x_j\right| < \varepsilon_j \tag{3.9}$$

is fulfilled. Here $\varepsilon_j$ is the required accuracy of computation of $j$-th component of $\vec{\mathbf{x}}$. After the first numerical experiment had been made, we adopted $\varepsilon_j$ as an average root square error of $\Delta x_j$. As for $\Delta_j$, they were appointed to be

$$\Delta_j = \frac{1}{2}\varepsilon_j \tag{3.10}$$

This was done to make the length of the interval $\left[x_j - \Delta_j, x_j + \Delta_j\right]$ be more than the mean square error of $\Delta x_j$.

## 5. TESTING OF THE METHOD

Before the method was applied to the radial velocities of the FK5, it had been tested on artificial data. We had taken the values of the LSS parameters (close to those, which were obtained from the proper motions):

$$l_0 = 286°,\ b_0 = -5°,\ n_0 = 1.5,\ \omega_0' r = -1'',\ \omega_0'' r^2 = -2''.5\ ,$$

and computed model catalogues of the radial velocities using Eq. (1.12).

If a random component is small and initial approximations are not far from reality, the method works irreproachably and allows to get all the parameters after 2-3 iterations. It is known that the noise level in the determination of a radial velocity is less then 1 km/sec. For testing the method on stability we created some model catalogues, that were constructed as a combination of the equation (1.16) and the random parts with different dispersions (from 0 to 1 km/sec). We found the values of all the parameters if the level of noise was 0.1 ÷ 0.2 km/sec. However, when the dispersion reached 0.5 km/sec, we were able to obtain only the geometric parameters $l_0$, $b_0$ and $n_0$, because strong correlations disturbed the results based on scarce observational data.

**Table 1.** *The results of testing of the method*

| σ km/s | $l_0$ | $b_0$ | $n_0$ | $\omega_0' r$ | $\omega_0'' r^2$ |
|---|---|---|---|---|---|
| 0 | 286 ± 0.4 | -5.0 ± 0.1 | 1.50 ± 0.01 | -1.00 ± 0.01 | 2.49 ± 0.01 |
| 0.1 | 277 ± 6.2 | -8.6 ± 2.5 | 1.56 ± 0.24 | -0.84 ± 0.15 | 2.12 ± 0.55 |
| 0.2 | 272 ± 9.7 | -11.4 ± 4.4 | 1.65 ± 0.56 | -0.74 ± 0.27 | 1.87 ± 0.91 |
| 0.5 | 268 ± 16 | -14.7 ± 7.6 | 1.21 ± 0.36 | --- | --- |
| 1 | 255 ± 26 | -25 ± 16 | 1.08 ± 0.55 | --- | --- |

## 6. THE RESULTS OF THE FK5 PROCESSING

The determination of the LSS rotation parameters was done by the two step procedure. The first step was the preliminary preparing of data by averaging the FK5 stars over trapezia and selecting them by spectral type. The second one is just the solution of the system of the LSS rotation equations by the successive approximation method. The initial approximation for the solution had been taken from our previous investigation of the proper motions.

At first, we found the parameters of the Local Star system rotation from all 2537 stars over the grid 6×10. We could determine three geometrical parameters: the coordinates of the direction to the center of the rotation and relative distance to it. The derivatives of the angular velocity had been fixed on the values derived from proper motions. When we changed the size of the grids, we got the same values but the numbers of stars in one cell became so small that accidental deviations caused by the stars with large $\delta v_r$ , which are not included in the LSS, increased the mean square errors of the parameters. In spite of that, the convergence of approximations was good and the values themselves were close to ones that the proper motions gave.

We tried to reject fast stars. This provided the same values of the parameters but the mean square errors were less than for all stars together.

An attempt to investigate the influence of spectral type on the LSS parameters failed because of too small numbers of stars of selected spectral types per one cell.

**Table 2** The values of the LSS parameters derived from $v_r$, $\omega_0' r$ = - 1"/100 year, $\omega_0'' r^2$= 2.5"/100 year - fixed.

| M×N | $\|v_r\|$ | Sp | $l_0$ in ° | $b_0$ in ° | $n_0$ |
|---|---|---|---|---|---|
| 6×10 | All | All | 227 ± 34 | 29 ± 45 | 2.6 ± 1.8 |
| 6×12 | All | All | 221 ± 31 | 17 ± 22 | 4.3 ± 3.8 |
| 8×18 | All | All | 221 ± 34 | 17 ± 25 | 4.4 ± 4.3 |
| 6×10 | < 40 | All | 241 ± 22 | 37 ± 26 | 4.0 ± 1.6 |
| 8×18 | < 40 | All | 223 ± 27 | 16 ± 18 | 5.5 ± 3.8 |

Fig. 6 shows the asymmetry of the radial velocities for three values of galactic attitude. One can see that the curves fit the observational data in Fig 5. rather well, although, systematic part of high order is present that our model does not describe.

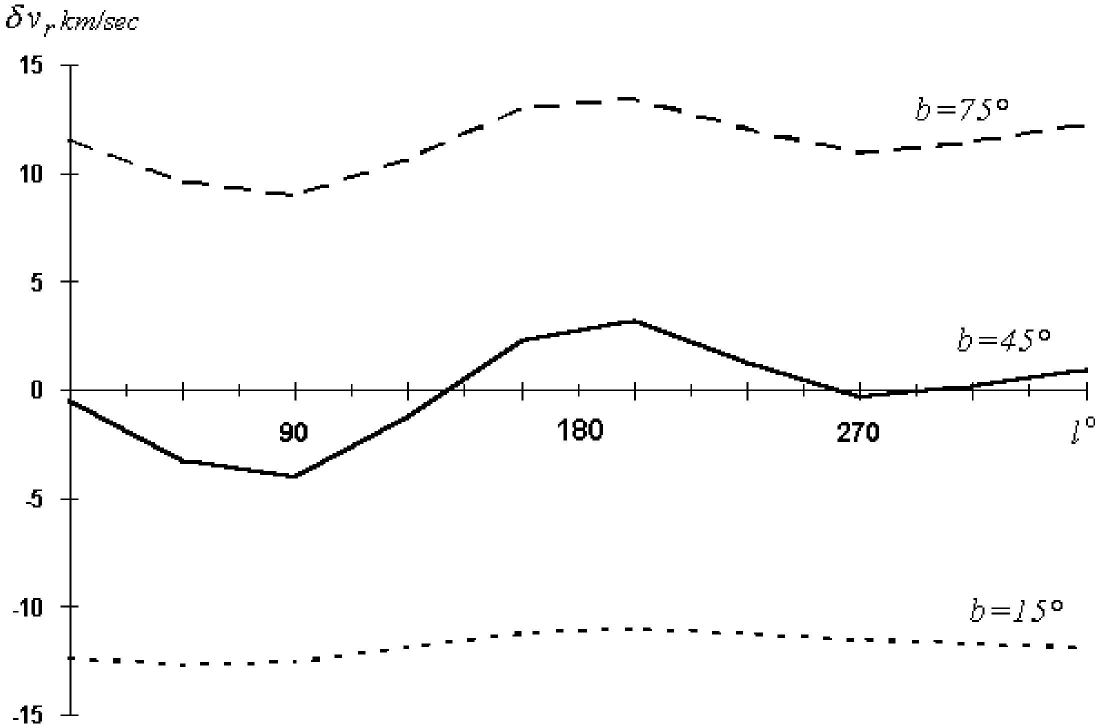


**Figure 6.** *The comparison of the asymmetry of the radial velocities with observational data*

## 7. CONCLUSIONS

The small number of used stars and the distance problem makes the determination of the LSS rotation parameter from the radial velocities less sure than in case of the proper motions. Therefore, we prefer the results of our first work. Nevertheless, the results obtained from the radial velocities do not contradict to the LSS rotation and we may consider them as an evidence of the existence of the Local Star system. What is needed to improve our knowledge of the structure and the kinematics of the LSS? Certainly, it is a mass catalogue of the proper motions and of the radial velocities. The more stars in a catalogue will have the parallaxes and astrophysical data the better it will be suitable. It is hoped that the HIPPARCOS would be a catalogue of this type. It is interesting to compare astrometrical results with astophysical observation of gas cloud, etc. The complex investigation can rectify the understanding of the nearest space in the Sun neighbourhood.

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