The Local Stellar System:
Kinematics Derived from Proper Motions

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The rotational parameters of the Local stellar system are derived from proper motions of the GC reduced into fundamental systems of catalogues N30, FK4 and FK5. A Weak dependence of the parameters on the choice of the fundamental system is found. Rotation of stars of different spectral types is investigated. The most likely geometrical and kinematical descriptions of the Local system are made.

Key words: Proper motions, Local stellar system.

1. Introduction

It is a tradition to interpret proper motions by a kinematic model based on considering the residual precessional motion of the Earth axis, the solar motion and the Galactic rotation. The basic equations for such a model may be found, for instance, in Fricke (1977).

It was noticed long ago that the use of this model does not give a full description of kinematic effects contained in the observational material. For example, it does not describe the North-South asymmetry of Galactic proper motions and the terms proportional to $\sin 3l$ and $\cos 3l$ (these terms are called Dyson's terms 1929). Further investigations by Brosche and Schwan (1981) give evidence that the remaining proper motions after the subtraction of all the standard effects do contain some systematic components. The existence of the systematic parts that cannot be interpreted within the limits of the standard model is found in the works by Vityazev and Tsvetkov (1989,1990).

All these things force supplementing of the standard model with new terms describing the kinematics beyond the standard model. What is the physical nature of these effects?

One of the possible explanations of Dyson's terms and of the North-South asymmetry is the hypothesis of the Local Stellar System (henceforth the LSS) and its rotation. In 1879, a condensation of bright stars along a large circle inclined to the Milky Way was discovered. This star formation is known as the Gould's Belt by the name of its pioneer. The investigation made by Charlier (1921) showed that the system of B-stars was compressed very much in one direction and stretched out in the two others. The center of this system was found to be located at $l = 277°$, $b = 14°$; the distance from the Sun was evaluated to be 65 pc. The same result was derived by Kunitsky (1935). Searse (1928) demonstrated that the stars brighter than $13^m.5$ have a maximum density that is four times more then in neighbouring zones for a given latitude in direction $l = 293°$.

One of the earliest works devoted to the LSS kinematics was the paper by Mineur (1929) in which the author came to the conclusion that the nearest stellar group rotates about a center with longitude $l = 273°$.

In 1950 Shatsova obtained equations which describe the influence of the LSS rotation on proper motions. In this fundamental work, she developed a method to study the LSS kinematics from proper motions of the GC and found the parameters of the axis orientation and angular velocity. However, because of computational limitations of that time, she used a specific method that allowed to solve only one of the equations for all stars together without any division of them

by spectral types. Besides, the GC has large systematic errors both in positions and in proper motions of stars (Vityazev and Vityazeva, 1985).

At present, when powerful computers and new fundamental systems are available, it is expedient to pursue a more detail research of the LSS kinematics. In this work we suggest a new method for the determination of the LSS parameters. Solutions of the problem in the systems of catalogues GC, N30, FK4 and FK5 and the characteristics of the LSS for stars of different spectral types are made.

2. The Equations of the LSS Rotation
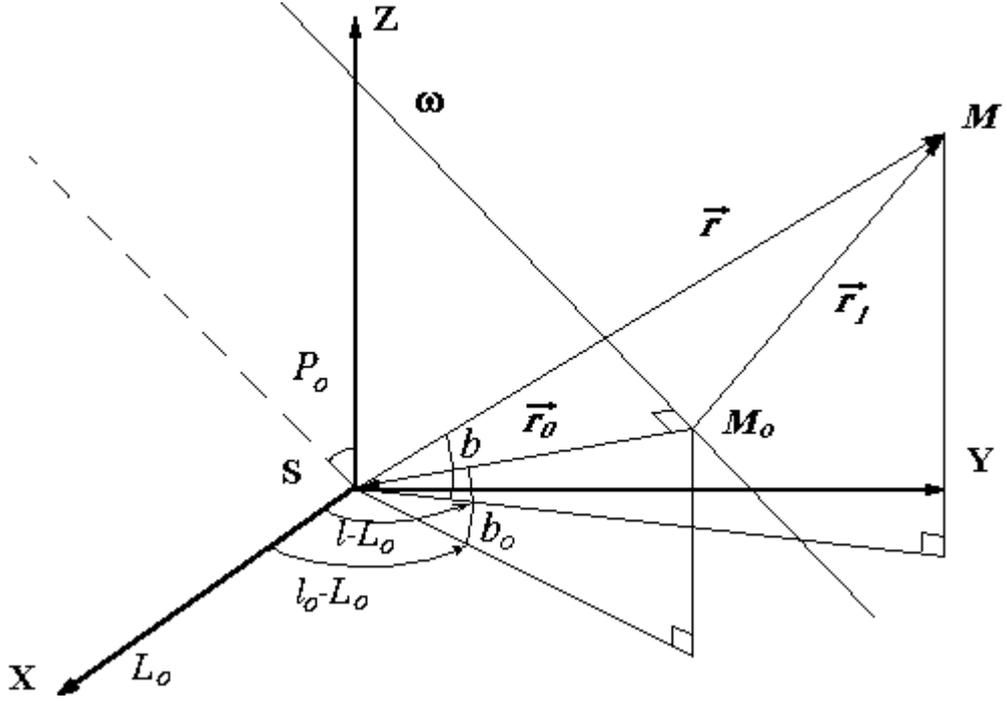
Figure 1. *Rotation of the Local stellar system*

Following Shatsova (1950), we give a short description of the basic equations of the LSS rotation. Consider the coordinate system with the origin at the center of the Sun and the Z-axis directed to the North Galactic pole (see Fig. 1). We arrange the X-axis in such a way that the LSS rotational vector ω is parallel to the XZ-plane and the Y-axis is normal to both others. The longitude of the X-axis coincides with the longitude of the rotational pole L_0; its polar distance we denote as P_0. Draw the perpendicular from point S to ω; the point of their intersection M_0 we call the center of rotation of the LSS. Extend the vectors $\vec{r}$ from point S to point M (an arbitrary star) and $\vec{r_1}$ from M_0 to M. The longitudes of these points are $l - L_0$ and $l_0 - L_0$; the latitudes are b and b_0.

For the velocity of the point M we can write

$$\vec{v} = \vec{\Omega} \times \vec{R} + \vec{\omega} \times \vec{r_1} \quad , \tag{2.1}$$

where $\vec{\Omega}$ is the angular velocity of the Galaxy, $\vec{R}$ is the distance from the Galactic center.

From observational data we can determine only a differential effect, if we assume that the Sun takes part in the rotation of the LSS:

$$\Delta\vec{v} = \vec{v} - \vec{v_0}. \tag{2.2}$$

Using the relations

$$\vec{r}_1 = \vec{r}_0 + \vec{r} \quad \text{and} \quad \vec{R} = \vec{R}_0 + \vec{r} \ , \tag{2.3}$$

from Eqs. (2.1-2.2) we get:

$$\Delta\vec{v} = (\vec{\Omega} - \vec{\Omega}_0) \times \vec{R}_0 + \vec{\Omega} \times \vec{r} + (\vec{\omega} - \vec{\omega}_0) \times \vec{r}_0 + \vec{\omega} \times \vec{r} \ , \tag{2.4}$$

e.g., the Galactic motion and the LSS rotation are separated (in Eqs. (2.2-2.4) the subscript "0" refers to the Sun). Suppose now that the Galactic rotation is known; we project only $(\vec{\omega} - \vec{\omega}_0) \times \vec{r}_0 + \vec{\omega} \times \vec{r}$ on the axes. This gives us the following expressions for the contribution of the LSS rotation to the proper motions and radial velocities:

$$\Delta\mu_l \cos b = (\omega - \omega_0)\frac{r_0}{r}\left[\sin P_0 \sin b_0 \cos(l - L_0) - \cos P_0 \cos b_0 \cos(l - l_0)\right] +$$
$$+ \omega\left[\cos P_0 \cos b - \sin P_0 \sin b \cos(l - L_0)\right], \tag{2.5}$$

$$\Delta\mu_b = (\omega - \omega_0)\frac{r_0}{r}\left[\cos P_0 \cos b_0 \sin b \sin(l - l_0) -\right.$$
$$\left. - \sin P_0 \cos b_0 \cos b \sin(l_0 - L_0) - \sin P_0 \sin b_0 \sin b \cos(l - L_0)\right] + \tag{2.6}$$
$$+ \omega \sin P_0 \sin(l - L_0),$$

$$\Delta v_r = (\omega - \omega_0)r_0\left[\sin P_0 \sin b_0 \cos b_0 \sin(l - L_0) -\right.$$
$$\left. - \cos P_0 \cos b_0 \cos b \sin(l - l_0) - \sin P_0 \cos b_0 \sin(l_0 - L_0)\sin b\right]. \tag{2.7}$$

We will not examine equation (2.7) in this article; the study of it will be a subject of further investigations.

Let the angular velocity ω be a function of only the distance from rotation axis ρ:

$$\omega = \omega(\rho(\vec{r}_1)) \ , \tag{2.8}$$

and be independed of the distance to the symmetry plane. This is supported by theoretical investigations, which tell us that the angular velocity of a stable stellar system is independent of Z.

Since we do not know the function $\omega = \omega(\vec{r}_1)$ we may adopt it as truncated Taylor series in powers of *r*. The existence of the third harmonics in the representations of observational proper motions indicates that the rotational center is very close to us and we cannot ignore the second terms:

$$\omega(\vec{r}) = \omega_0 + \left(\frac{d\omega}{dr}\right)_0 r + \frac{1}{2}\left(\frac{d^2\omega}{dr^2}\right)_0 r^2 \ , \tag{2.9}$$

here $\left(\dfrac{d\omega}{dr}\right)_0$ and $\left(\dfrac{d^2\omega}{dr^2}\right)_0$ are the first and the second derivatives of ω along the $\vec{r}$ direction in the solar vicinity.

After evaluating the derivatives it turns out that Eq. (2.9) can be rewritten as

$$\omega - \omega_0 = \Omega' + \Omega'' \sin b \ ; \tag{2.10}$$

where Ω' and Ω'' are functions symmetrical about the Galactic plane:

$$\Omega' = (\omega_0' r)\left\{-\cos b_0 \cos b \cos(l - l_0) +\right.$$
$$+ \frac{1}{2n_0}\left[1 - \sin^2 b(\sin^2 b_0 + \cos^2 P_0) -\right.$$
$$\left. - \cos^2 b\left(\cos^2 b_0 \cos^2(l - l_0) + \sin^2 P_0 \cos^2(l - L_0)\right)\right]\right\} + \tag{2.11,}$$
$$+ (\omega_0'' r^2)\frac{1}{2}\left\{\cos^2 b_0 \cos^2 b \cos^2(l - l_0) + \sin^2 b_0 \sin^2 b\right\}$$

$$\Omega'' = (\omega_0' r)\left\{-\sin b_0 - \frac{1}{n_0}\cos b[\sin b_0 \cos b_0 \cos(l-l_0) + \sin P_0 \cos P_0 \cos(l-L_0)]\right\} +$$
$$+(\omega_0'' r^2)\{\cos b \sin b_0 \cos b_0 \cos(l-l_0)\} \tag{2.12}$$

here and further $n_0 = \dfrac{r_0}{r}$; ω_0 is angular velocity of the LSS rotation in the Sun vicinity; $\omega_0' = \left(\dfrac{d\omega}{d\rho}\right)_0$, $\omega_0'' = \left(\dfrac{d^2\omega}{d\rho^2}\right)_0$ are the first and the second derivatives in the neighbourhood of the Sun in the normal direction to the rotational axis.

An immediate use of Eqs. (2.5), (2.6) leads to inconvenience, because there are other kinematic phenomena, which interfere with the LSS rotation. It is difficult to separate motions of various kinds due to considerable correlations.

Since the Galactic rotation and the solar motion are symmetric with respect to the Galactic equator we can exclude them by considering the differences of $\Delta\mu_l \cos b$ "North-South":

$$\delta\mu_l \cos b = (\Delta\mu_l \cos b)_\text{N} - (\Delta\mu_l \cos b)_\text{S} \tag{2.13}$$

and the sums of $\Delta\mu_b$ "North+South" :

$$\delta\mu_b = (\Delta\mu_b)_\text{N} - (\Delta\mu_b)_\text{S} \tag{2.14}$$

taken at symmetric points on the sky. These values do not contain the effects discussed earlier. Still the error caused by the incomplete knowledge of the precession constant penetrates into them; and we need to exclude it using other sources. It concerns the solar component $Z_\odot$, which is included in the sums of $\Delta\mu_b$ derived from observations.

Equations (2.5) and (2.6) are followed by

$$\delta\mu_l \cos b = 2\sin b\{-(\omega_0 + \Omega')\sin P_0 \cos(l-L_0) +$$
$$+\Omega''[n_0(\sin P_0 \sin b_0 \cos(l-L_0) - \cos P_0 \cos b_0 \cos(l-l_0)) + \cos P_0 \cos b]\}, \tag{2.15}$$

$$\delta\mu_b + 2Z_\odot \cos b = 2\{-\Omega' n_0 \sin P_0 \cos b_0 \sin(l_0 - L_0)\cos b + (\omega_0 + \Omega')\sin P_0 \sin(l-L_0) +$$
$$+\sin^2 b \Omega'' n_0[-\sin P_0 \sin b_0 \sin(l-L_0) + \cos P_0 \cos b_0 \sin(l-l_0)]\}. \tag{2.16}$$

Equations (2.15) and (2.16) theoretically allow us to determine from proper motions the following eight parameters of the LSS:

L_0, P_0 - the latitude and the polar distance of the rotational pole;
l_0, b_0, r_0 - the coordinates of the rotational center;
$\omega_0, \omega_0', \omega_0''$ - the angular velocity and its derivatives in the Sun neighbourhood.

But because of a strong correlation we refused to obtain L_0 and P_0 from proper motions. Assuming that the position of the LSS is indicated by the Gould Belt, we adopted the coordinates of the LSS rotational pole to be those of the Gould Belt:

$$L_0 = 343°, \quad P_0 = 17° . \tag{2.17}$$

In practice we do not know distances to stars properly that is why following conventional way, we find the following parameters instead of $r_0, \omega_0', \omega_0''$:

$$n_0 = \frac{r_0}{\langle r \rangle}, \quad \omega_0'\langle r \rangle, \quad \omega_0''\langle r^2 \rangle \tag{2.18}$$

where angle brackets denote averaging.

So, we have the following six parameters to be determined from analysis of proper motions:

$$l_0, \quad b_0, \quad n_0, \quad \omega_0, \quad \omega_0'\langle r \rangle, \quad \omega_0''\langle r^2 \rangle .$$

3. Data Preparation

To find the LSS parameters we an extensive catalogue of proper motions. Up to date there is only one fundamental catalogue of this type. This is GC of Boss that contains 33343 stars.

It is well known that positions and proper motions of GC are polluted by large systematic errors (Vityazev and Vityazeva, 1985). Random errors can be compensated for by suitable averaging. More exact catalogues of the FK-series contains less stars and hardly fits our aim. A way out is a compromise: we should reduce GC to the FK5 system.

To compare it with FK4, the authors of FK5 (1988) chose an analytical model of systematic differences based on the complete orthogonal system of Hermit-Legendre-Fourier functions. To reduce GC to FK4 (Fricke, 1963), the systematic differences FK4-GC may be taken from Brosche *et al.* (1964). Obviously, one can calculate the differences FK5-GC by means of a simple relation:

$$(FK5\text{-}GC) = (FK4\text{-}GC) + (FK5\text{-}FK4). \tag{3.1}$$

We represented the differences FK5-FK4 in the form as FK4-GC and found the systematic differences FK5-GC we need (Vityazev and Tsvetkov, 1991). To study the dependence of the LSS parameters on the fundamental system we reduced GC to the N30 and FK4 systems using tables of systematic differences N30-GC and FK4-GC (Fricke, 1963).

It is impossible to solve the LSS equations directly for 33342 stars because of three reasons: (a) we are to know the differences of $\mu_l \cos b$ and sums of μ_b for opposite points of the sky and this leads us to troubles to find a suitable couple for each star; (b) we are to reduce the random component before solving equations by averaging the data over certain areas on a sphere; (c) it is not easy to solve a system of more than 30000 nonlinear equations even with modern computers.

For this reasons we divided the celestial sphere into some zones symmetric about the Galactic equator and subdivided them by trapezia of equal size in longitude. So, we have a set of trapezia to average the data from GC.

Now, we present the algorithm that was applied to prepare initial data:
1. Select a star of required spectral type;
2. Reduce proper motions $\mu \cos \delta$ and μ' onto the systems of the catalogues FK5, FK4 and N30 by means of the tables of systematic differences FK5-GC, FK4-GC and N30-GC. Subtract from proper motions the precessional corrections:

$$p_\alpha = \Delta n \sin \alpha \sin \delta + \Delta k \cos \delta, \tag{3.2}$$

$$p_\delta = \Delta n \cos \alpha, \tag{3.3}$$

where $\Delta n = 0".437$ /century and $\Delta k = -0".191$ /century (Fricke, 1977).
3. Calculate the modulus of the proper motion:

$$|\vec{\mu}| = \sqrt{(\mu \cos \delta)^2 + (\mu')^2} \ . \tag{3.4}$$

If $|\vec{\mu}| \geq 20''$ per century, the star is rejected as having large peculiar component. The total number of such stars has been found to be 1% of the whole list.
4. Transform proper motions $\mu \cos \delta$ and μ' into Galactic proper motions using the formulae:

$$\mu_l \cos b = \mu \cos \delta \cos \varphi + \mu' \sin \varphi, \tag{3.5}$$

$$\mu_b = -\mu \cos \delta \sin \varphi + \mu' \cos \varphi, \tag{3.6}$$

where φ is the parallactic angle defined by

$\sin \varphi = \sin i \cos(l - \Omega) / \cos \delta,$

$\cos \varphi = (\cos b \cos i - \sin b \sin i \sin(l - \Omega)) / \cos \delta,$

$i = 62°.2$ is the declination of the Galactic equator,

$\Omega = 33°.0$ is the longitude of the Galactic center.
5. Transform equatorial coordinates of the stars into Galactic ones.

6. Define an trapezium, within which the star is located. The coordinates of the center of an *i*-trapezium in a *j*-zone are

$$l_i = \frac{180°}{n} + \frac{360°}{n}(i-1), \qquad i=1,2,...,n \qquad (3.7)$$

$$b_j = 90° - \frac{90°}{m} - \frac{180°}{m}(j-1), \qquad j=1,2,...,m \qquad (3.8)$$

Here *m* is the number of latitude zones ($\frac{m}{2}$ is the number of the zones in one hemisphere) and *n* is the number of trapezia in every zone. The indices for a star may be found as

$$i = \left[\frac{l}{360°}n\right] + 1 , \qquad (3.9)$$

$$j = \left[\frac{90° - b}{180°}m\right] + 1 , \qquad (3.10)$$

where brackets denote an integer part.

7. Average $\mu_l \cos b$ and μ_b over the trapezia; count stars in every trapezium.

This algorithm yields two tables (of size $m \times n$), which contain averaged values of $\mu_l \cos b$ and μ_b for all the trapezia as well as a table of numbers of stars considered.

For our task we need to know not Galactic proper motions but their differences and sums only. We can easily obtain these tables of size $\frac{m}{2} \times n$:

$$\delta\mu_l \cos b = (\mu_l \cos b)_N - (\mu_l \cos b)_S , \qquad (3.11)$$
$$\delta\mu_b = (\delta\mu_b)_N + (\delta\mu_b)_S . \qquad (3.12)$$

Each trapezium was assigned a weight, which is a sum of stars corresponding North-South couple of areas contains:

$$w = (n)_N + (n)_S. \qquad (3.13)$$

To solve Eq. (2.16) for $\delta\mu_b$ we must know the solar parallactic component $Z_\odot$. We found it from the solution of the Airy-Kovalsky equations:

$$X_\odot \sin l - Y_\odot \cos l = \mu_l \cos b, \qquad (3.14)$$
$$X_\odot \sin b \cos l + Y_\odot \sin b \sin l - Z_\odot \cos b = \mu_b \qquad (3.15)$$

for data of the same tables. We solved these equations for the solar motion without taking into consideration other kinematical effects since the function in Eqs. (2.5,2.6) are orthogonal to those in Eqs. (3.14 and 3.15). We found that for stars of GC in fundamental system FK5

$$Z_\odot = 1''.13 \pm 0''.05 \text{ per century.}$$

4. The Method to Solve the LSS Equations

Equations (2.15) and (2.16) may be represented as
$$y = f(\vec{\mathbf{x}}, \vec{\mathbf{t}}), \qquad (4.1)$$
where y is either $\delta\mu_l \cos b$ or $\delta\mu_b$;

$\vec{\mathbf{x}}$ is the vector of unknown variables l_0, b_0, n_0, ω_0, $\omega_0'\langle r \rangle$ and $\omega_0''\langle r^2 \rangle$;

$\vec{\mathbf{t}}$ is the vector of coordinates of the northern trapezium center;

f is a nonlinear function of $\vec{\mathbf{x}}$ and $\vec{\mathbf{t}}$.

To apply conventional LSM technique for solving Eq. (4.1), the latter is to be linearized.

Assume that the first approximation $\vec{\mathbf{x}}_0$ to $\vec{\mathbf{x}}$ is known from preliminary consideration. Then one can calculate

$$y_0 = f(\vec{\mathbf{x}}_0, \vec{\mathbf{t}}) \qquad (4.2)$$

for every point $\vec{\mathbf{t}}$ and write, using Tailor series with accuracy up to first power term

$$\Delta y = y - y_0 \approx \sum_j \Delta x_j \frac{\partial f}{\partial x_j}\bigg|_{\vec{\mathbf{x}}_0} \tag{4.3}$$

The system generated by Eq. (4.3) is linear with respect to unknown corrections Δx_j to components of $\vec{\mathbf{x}}$. Now, one can solve this system,

$$\hat{\mathbf{A}}\,\Delta\vec{\mathbf{x}} = \vec{\mathbf{Y}}, \tag{4.4}$$

by the conventional least square technique. Elements of the matrix $\mathbf{A}$ are

$$\mathbf{A}_{ij} = \frac{\partial f(\vec{\mathbf{x}}, \vec{\mathbf{t}}_i)}{\partial x_j}\bigg|_{\vec{\mathbf{x}}_0} \qquad \begin{array}{l} i = 1,2,\ldots m \\[4pt] j = 1,2,\ldots 6 \end{array} \tag{4.5}$$

where $\vec{\mathbf{t}}_i = (l_i, b_i)$ are coordinates of the center of the northern i-trapezium. Partial derivatives can be calculated by symmetric formula:

$$\frac{\partial f}{\partial x_j} = \frac{f(x_1, x_2, \ldots, x_j + \Delta_j, \ldots, x_{n,}\vec{\mathbf{t}}_i) - f(x_1, x_2, \ldots, x_j - \Delta_j, \ldots, x_{n,}\vec{\mathbf{t}}_i)}{2\Delta_j} \tag{4.6}$$

The elements of column $\mathbf{Y}$ are written as

$$\mathbf{Y}_i = y_i - f(\vec{\mathbf{x}}_0, \vec{\mathbf{t}}_i); \tag{4.7}$$

where y_i are average values of $\delta\mu_l \cos b$ or $\delta\mu_b$ in the i-trapezium.

The weights calculated from Eq. (3.13) were assigned to all of the equations.

After corrections Δx_j have been obtaned from a separate or combined solution of Eqs. (2.15) and (2.16), they can be added to the components $\vec{\mathbf{x}}_0$ to get the next approximation $\vec{\mathbf{x}}_1$:

$$\vec{\mathbf{x}}_1 = \vec{\mathbf{x}}_0 + \Delta\vec{\mathbf{x}} \tag{4.8}$$

We repeat this process until the condition

$$\left|\Delta x_j\right| < \varepsilon_j \tag{4.9}$$

is fulfilled. Here ε_j is the required accuracy of computation of the j-component of $\vec{\mathbf{x}}$. After the first numerical experiment had been made, we adopted ε_j as an average square errors of Δx_j. As for Δ_j, they were taken as

$$\Delta_j = \frac{1}{2}\varepsilon_j \tag{4.10}$$

This was done to make the length of the interval $\left[x_j - \Delta_j, x_j + \Delta_j\right]$ be more than the mean square error of Δx_j.

5. Test of the Method

The method described above was tested on artificial data. We took the following values of the LSS parameters (close to real ones):

$$l_0 = 286^\circ, \quad b_0 = -5^\circ, \quad n_0 = 1.5,$$
$$\omega_0 = 1'', \quad \omega_0' r = -1'', \quad \omega_0'' r^2 = 2''.5,$$

and computed model catalogues of proper motions using Eqs. (2.15) and (2.16) with random components with m.s.r.e. varying from 0".0 to 0".5. (The accuracy of data from GC is characterized by same values.)

The results of testing are given in Tables 1 and 2. We can see from the tables that even in the absence of noise the method by Shatsova distorts the initial values of the parameters. The values of the parameters are completely destroyed when the noise level is increased. For instance, some parameters changed their signs (ω_0 in the zones 79° and 34°); solutions obtained for different zones do not correspond to each other; parameters had anomalously large values ($\omega_0' r$ and $\omega_0'' r^2$ in the zone 79° and $\omega_0' r$ in the zone 11°). We have underlined such values in

Table 1. However, one can notice that geometric characteristics l_0, b_0 and n_0 are not affected so strongly as kinematic ones ω_0, $\omega'_0 r$ and $\omega''_0 r^2$. More detailed analysis of Shatsova's formulae shows that Eqs. (30, $30^{\text{I-V}}$) lead to instability of the whole algorithm. The linearization method, on the contrary, reconstructs the input values exactly if $\sigma = 0''$, and is more stable to random component. Shatsova's method can neither solve the second equation for $\delta\mu_b$ nor make a combined solution. Our method shows good agreement between both separated and combined solutions of Eqs. (2.15) and (2.16).

After testing we can conclude that the linearization method is a tool to find all the parameters, whereas Shatsova's method allows to estimate only geometric parameters l_0, b_0 and n_0. One should not use this method to get the kinematic parameters.

6. Results of Solution

We carried out separate and combined solutions of Eqs. (2.15) and (2.16) for proper motions derived from GC after reducing them to the fundamental systems FK5, FK4, N30 in the way described above. Shatsova's values of the parameters were used as first-order approximations. The parameters were determined for three spectral groups of stars: O-B9, A0-G5 and G6-M9, and also for all stars together. For each group except O-B9 the celestial sphere was divided into 8 zones in longitude and 18 trapezia in latitude. This provided 72 equations. Since the O-B9 group is poor of stars, the number of trapezia was taken 6×12. In spite of a larger size of a trapezium, three of them turned out to contain less than four stars and were rejected. So, for this group of stars the number of equations was 33.

The values of the LSS parameters and their mean square errors are given in Table 3. All the solutions are weighted except that for early stars. For them the data were assigned equal weights because of strong concentration of these stars to the Galactic plane where their number is hundreds times more than in the polar areas.

Solutions for stars of individual spectral types B, A, F and for all stars together are given in Table 4.

7. The Influence of Fundamental Systems on the LSS Parameters

From the astrometric point of view, it is interesting to see how the system of fundamental catalogue effects the LSS parameters. To investigate this we used combined solutions for stars of all spectral types (Table 5). One can see immediately that the values of the parameters in all the fundamental systems are similar to those in the FK5 system with some exceptions: only values of ω_0 derived from equation for $\delta\mu_b$ in the N30 system and GC are close to zero.

To illustrate the dependence of the parameters on catalogue we plotted the functions (2.15) and (2.16) for some values of latitude (Figs. 2 and 3). From their analysis we can conclude that the asymmetry caused by the LSS rotation for longitude $\delta\mu_l \cos b$ in the FK4 system is nearly identical to that in FK5. The asymmetry in GC is also very close to that in FK5. The behaviour of the curve corresponding to N30 has some peculiarities. From the study of curves for $\delta\mu_b$ we can derive the same conclusion on identity of FK5 and FK4. The curves for the catalogues GC and N30 form the own system but their general character does not contradict to the FK5 system.

In order to clarify finally the degree of influence of systematic errors in catalogues on LSS parameters we calculated the correlation coefficients of $\delta\mu_l \cos b$ and $\delta\mu_b$ for separate and combined solutions in different fundamental systems (Tables 6 and 7). One can see that correlation of $\delta\mu_l \cos b$ between GC and FK5 is 0.99; the same values applies to FK4 and FK5. The smallest value of correlation coefficient is 0.62 between GC and N30 for $\delta\mu_b$.

Thus we can say that systematic errors in fundamental catalogues only weakly affect the derivation of the LSS parameters.

8. Dependence of the Solutions on Spectral Type

In this section we try to find which stars from GC belong to the LSS. The information contained in this catalogue permits to study the dependence of the LSS parameters only on spectral type of the stars. All further results are given in the FK5 system because discussion in sect. 7 has shown the proximity of all fundamental systems.

In Table 3 we see two blank cells. These reflect attempts to solve equations $\delta\mu_l \cos b$ for stars of G6-M9 types and $\delta\mu_b$ for stars of O-B9 types. In both cases we failed to obtain solutions but the reasons are different. In the former case we had no convergence of successive approximations, e.g., the stars of later spectral types do not participate in the rotation. In the second case the large correlations prevented obtaining a result. These correlations arise because of low stellar density and significant asymmetry in the distribution of early spectral type stars. For example, the number of O-B9 stars spectral type near the Galactic plane is about 400 while their number in polar areas is only 5÷10. This fact explains the larger values of the mean square error in comparison with other spectral types but does not imply that stars of early spectral types do not belong to the LSS.

The best convergence of iterations was observed for A0-G5 stars, which can be explained by their uniform distribution and by belonging of the Sun to this spectral group. The values of the LSS parameters derived for these stars we consider to be the most reliable.

Comparison between solutions for $\delta\mu_l \cos b$ and for $\delta\mu_b$ shows a small systematic difference between them (parameters b_0 and ω_0). Probably, this may be caused by unknown kinematic effects. In spite of this all the solutions show good agreement between them. We may regard this as a proof of the LSS rotation.

When we begin to study the solutions themselves, especially those derived for separate spectral types (Table 4) from B to F, partially G, we see that proper motions of stars in GC of middle and early spectral types do not contradict the fact of existence of the LSS. According to statistical investigations the LSS mainly consists of young O-B stars. Investigating kinematics, we conclude that these stars participate in the rotation but their scarcity and significant asymmetry in their distribution on the celestial sphere prevents obtaining reliable estimates of the parameters. The parameters for stars of later spectral types differ very much up to changing sign of $\omega_0' r$ and of $\omega_0'' r^2$ and almost zero value of n_0. However, from analysis of some parameters, one can note the stability of geometric characteristics l_0 and b_0 - the coordinates of the direction to the center of rotation, whereas the kinematic parameters behave in another way: ω_0 decreases, and $\omega_0' r$ and $\omega_0'' r^2$ are defined badly. These facts indicate slower rotation for stars of later spectral types or deluding the stars belonging to the LSS by stars that have others kinematic characteristics.

All this can be illustrated by plots of $\delta\mu_l \cos b$ (2.16) and $\delta\mu_b$ (2.17) for some values of latitude using the parameters derived form combined solutions for stars of different spectral groups. From these one can see that the behaviour of the curves for O-B9 and A0-G5 stars are similar in spite of large mean square errors of the LSS parameters for early spectral type stars. The character of the curve corresponding to G5-M9 drastically differs from the previous ones. The all-star curve passes closely to those for early and middle spectral type stars because more then a half of stars used in GC - 16545 - have such spectra (4142 stars are of early and 10501 are of later types).

It is necessary to recall that the values of the parameters obtained are based on the hypothesis of the Sun participating in the LSS rotation. The Sun's spectrum essentially differs from an early one. In spite of this circumstance the values obtained for early spectral type stars

are in good agreement with the values calculated for middle types. This means that these stars also take part in the rotation.

It is important to point out that the coordinates of the center of the rotation obtained here correspond to a point at which stellar density is four times higher than in nearby regions. This fact confirms the physical consistency of our solution. The sign of the angular velocity shows that the direction of the LSS rotation is opposite to the Galaxy's one.

9. Conclusions

Results of our investigations can be summarized as follows:
1. Proper motions of early and middle spectral type stars in GC do not contradict the assumption that the LSS rotates.
2. Systematic errors of proper motions in fundamental catalogues have only a weak influence on LSS parameters.
3. We believe that the most probable values of LSS parameters follow from a combined solution for stars of A0-G5 spectral types in the FK5 fundamental system:

coordinates of rotation center: $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad l_0 = 278° \pm 2°,$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad b_0 = -7° \pm 3°;$

the relative distance to the rotation center: $\quad\quad n_0 = 0.94 \pm 0.14;$

with the adopted value of the average distance
of all stars in GC to be 200 pc this yields: $\quad\quad r_0 = 190 \pm 28$ pc;

the angular velocity and its derivatives: $\quad\quad\quad \omega_0 = 1.05 \pm 0.35$ "/century,

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \omega_0' r = -2.20 \pm 0.44$ "/century,

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \omega_0'' r^2 = 3.75 \pm 1.22$ "/century;

the coordinates of the rotation pole: $\quad\quad\quad L_0 = 343°,$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad P_0 = 17°.$

It is obvious that the question of the existence of the LSS deseves further investigation. We plan following:
1. To obtain LSS parameters from radial velocities since the rotation affects them too.
2. To investigate the remainders of proper motions after subtracting from them the LSS rotation and to solve the standard model equations.

In order to improve our knowledge on the structure of stellar velocity field we should have a more extensive new catalogue of proper motions and parallaxes. In the nearest future such a catalogue is expected to be come available from the HIPPARCOS mission.

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Table 1   Tests of Shatsova's Method

Units: l_0, b_0 - *degrees;* n_0 - *dimensionless;*
ω_0, $\omega_0'r$, $\omega_0''r^2$ - *arcsec / cy .*

Zone	Parameters	$\sigma = 0".0$	$\sigma = 0".1$	$\sigma = 0".3$	$\sigma = 0".5$
79°	l_0	274	275	273	273
	b_0	-6.3	-6.4	-6.0	-5.9
	n_0	0.94	0.35	0.41	0.42
	ω_0	1.1	<u>-0.5</u>	<u>-4.5</u>	<u>-8.3</u>
	$\omega_0'r$	-1.4	<u>10.7</u>	<u>39.7</u>	<u>67.</u>
	$\omega_0''r^2$	1.6	11.7	<u>44.</u>	<u>-75.</u>
56°	l_0	272		289	273
	b_0	-5.7		-10.0	-7.9
	n_0	1.18		1.0	1.1
	ω_0	1.2		0.8	0.5
	$\omega_0'r$	-1.9		<u>1.7</u>	<u>-4.6</u>
	$\omega_0''r^2$	1.5		<u>-0.5</u>	<u>-2.6</u>
34°	l_0	270	267		295
	b_0	-5.3	-4.2		-11.0
	n_0	1.4	1.14		2.6
	ω_0	1.2	0.9		<u>-0.8</u>
	$\omega_0'r$	-2.6	-1.4		<u>1.9</u>
	$\omega_0''r^2$	1.6	1.5		<u>-0.4</u>
11°	l_0	270	273	275	275
	b_0	-5.2	-6.1	-6.5	-6.7
	n_0	1.7	1.9	2.0	2.0
	ω_0	1.3	2.5	5.7	9.0
	$\omega_0'r$	-3.4	-7.9	-17.3	-26.6
	$\omega_0''r^2$	1.6	2.7	5.0	7.4

Table 2   Tests of the Linearization Method

Units: l_0, b_0 - degrees; n_0 - dimensionless;
ω_0, $\omega'_0 r$, $\omega''_0 r^2$ - arcsec / cy .

Equation	Parameters	$\sigma = 0".0$	$\sigma = 0".1$	$\sigma = 0".3$	$\sigma = 0".5$
$\delta\mu_l \cos b$	l_0	286. ± 0.2	278. ± 4.5	260. ± 17.6	
	b_0	-5.0 ± 0.1	-7.2 ± 2.4	-9.9 ± 9.4	
	n_0	1.49 ± 0.01	1.16 ± 0.17	0.81 ± 0.41	
	ω_0	1.00 ± 0.01	1.07 ± 0.10	1.16 ± 0.35	
	$\omega'_0 r$	-0.99 ± 0.01	-0.82 ± 0.12	-0.60 ± 0.35	
	$\omega''_0 r^2$	2.47 ± 0.03	1.87 ± 0.57	1.42 ± 1.80	
$\delta\mu_b$	l_0	286. ± 0.1	287. ± 1.6	290. ± 5.5	291. ± 10.9
	b_0	-5.0 ± 0.0	-5.0 ± 0.7	-5.1 ± 9.4	-5.3 ± 4.4
	n_0	1.50 ± 0.00	1.59 ± 0.08	1.78 ± 0.32	2.01 ± 0.86
	ω_0	1.00 ± 0.00	0.98 ± 0.06	0.93 ± 0.18	0.86 ± 0.30
	$\omega'_0 r$	-1.00 ± 0.00	-0.92 ± 0.08	-0.76 ± 0.23	-0.62 ± 0.38
	$\omega''_0 r^2$	2.50 ± 0.01	2.29 ± 0.20	1.95 ± 0.58	1.65 ± 0.97
Combine	l_0	286. ± 0.1	286. ± 1.5	286. ± 5.0	287. ± 9.1
	b_0	-5.0 ± 0.0	-4.8 ± 0.05	-4.3 ± 1.5	-3.7 ± 2.6
	n_0	1.50 ± 0.00	1.56 ± 0.06	1.69 ± 0.22	1.96 ± 0.55
	ω_0	1.00 ± 0.00	0.98 ± 0.05	0.94 ± 0.15	0.92 ± 0.25
	$\omega'_0 r$	-1.00 ± 0.00	-0.94 ± 0.05	-0.80 ± 0.18	-0.66 ± 0.31
	$\omega''_0 r^2$	2.50 ± 0.01	2.36 ± 0.16	2.05 ± 0.48	1.65 ± 0.82

Table 3   LSS parameters derived in the FK5 fundamental system

Units: l_0, b_0 - degrees; n_0 - dimensionless;
ω_0, $\omega'_0 r$, $\omega''_0 r^2$ - arcsec / cy .

Equation	Parameters	All Sp	O-B9	A0-G5	G5-M9-N9
$\delta\mu_l \cos b$	l_0	285. ± 10.	298. ± 27.	287. ± 12.	
	b_0	-8. ± 6.	-25. ± 55.	-10. ± 8.	
	n_0	0.89 ± 0.35	1.06 ± 0.39	0.73 ± 0.23	
	ω_0	1.23 ± 0.54	2.34 ± 2.70	1.74 ± 0.65	
	$\omega'_0 r$	-1.35 ± 0.56	-3.04 ± 4.43	-2.20 ± 0.77	
	$\omega''_0 r^2$	4.33 ± 3.20	4.45 ± 6.30	5.62 ± 3.40	
$\delta\mu_b$	l_0	269. ± 4.		258. ± 7.	275. ± 7.
	b_0	7. ± 3.		7. ± 5.	8. ± 6.
	n_0	1.12 ± 0.15		0.78 ± 0.21	2.50 ± 1.17
	ω_0	0.35 ± 0.33		0.54 ± 0.50	0.51 ± 0.37
	$\omega'_0 r$	-1.36 ± 0.35		-2.44 ± 0.53	-0.47 ± 0.46
	$\omega''_0 r^2$	3.75 ± 0.89		4.07 ± 1.42	2.08 ± 1.43
Combine	l_0	280. ± 5.	291. ± 13.	278. ± 6.	270. ± 2.
	b_0	-4. ± 2.	-6. ± 4.	-7. ± 3.	-4. ± 1.
	n_0	1.29 ± 0.16	1.83 ± 0.82	0.94 ± 0.14	0.10 ± 0.06
	ω_0	0.69 ± 0.25	1.09 ± 0.99	1.05 ± 0.35	0.34 ± 0.41
	$\omega'_0 r$	-1.26 ± 0.30	-1.75 ± 1.29	-2.20 ± 0.44	1.26 ± 0.23
	$\omega''_0 r^2$	3.46 ± 0.80	5.93 ± 3.80	3.75 ± 1.22	-0.29 ± 1.22

Table 4   LSS parameters derived from combined solution for individual
spectral classes

Units: l_0, b_0 - degrees; n_0 - dimensionless;
ω_0, $\omega'_0 r$, $\omega''_0 r^2$ - arcsec / cy .

Parameters	B	A	F	O-B-A-F
l_0	291. ± 13.	283. ± 10.	285. ± 10.	282. ± 6.
b_0	-6. ± 4.	-2. ± 4.	-9. ± 5.	-2. ± 3.
n_0	1.80 ± 0.76	1.40 ± 0.34	0.62 ± 0.19	1.16 ± 0.16
ω_0	1.08 ± 0.98	0.65 ± 0.42	1.27 ± 0.48	0.68 ± 0.32
$\omega'_0 r$	-1.77 ± 1.39	-0.98 ± 0.49	-2.83 ± 0.70	-1.46 ± 0.38
$\omega''_0 r^2$	5.88 ± 3.80	2.79 ± 1.20	2.78 ± 2.40	3.80 ± 1.00

Table 5   LSS parameters in different fundamental systems

Units: l_0, b_0 - degrees; n_0 - dimensionless;
ω_0, $\omega_0'r$, $\omega_0''r^2$ - arcsec / cy .

Equation	Parameters	FK4	N30	GC
$\delta\mu_l \cos b$	l_0	290. ± 11.	290. ± 11.	286. ± 10.
	b_0	-7. ± 7.	-12. ± 10.	-6. ± 6.
	n_0	1.11 ± 0.39	1.17 ± 0.36	0.99 ± 0.39
	ω_0	1.06 ± 0.51	0.93 ± 0.47	1.45 ± 0.57
	$\omega_0'r$	-1.47 ± 0.60	-1.48 ± 0.75	-1.54 ± 0.60
	$\omega_0''r^2$	5.00 ± 3.00	3.71 ± 2.80	5.12 ± 3.37
$\delta\mu_b$	l_0	269. ± 5.	264. ± 6.	272. ± 12.
	b_0	6. ± 4.	5. ± 5.	19. ± 17.
	n_0	1.14 ± 0.19	1.03 ± 0.23	0.37 ± 0.23
	ω_0	0.36 ± 0.37	-0.05 ± 0.37	-0.09 ± 0.36
	$\omega_0'r$	-1.30 ± 0.39	-0.99 ± 0.38	-0.65 ± 0.25
	$\omega_0''r^2$	3.38 ± 0.98	3.52 ± 1.01	2.38 ± 1.93
Combine	l_0	282. ± 6.	279. ± 6.	284. ± 8.
	b_0	-5. ± 2.	-9. ± 3.	-7. ± 3.
	n_0	1.33 ± 0.17	1.20 ± 0.21	0.95 ± 0.20
	ω_0	0.66 ± 0.27	0.20 ± 0.27	0.66 ± 0.26
	$\omega_0'r$	-1.26 ± 0.30	-1.75 ± 1.29	-1.03 ± 0.31
	$\omega_0''r^2$	3.46 ± 0.80	5.93 ± 3.80	2.35 ± 0.84

Tables 6 Correlation coefficients between rotation of the LSS in different catalogues derived from combined solutions

$\delta\mu_l \cos b$	FK5	FK4	N30	GC
FK5	1.00	0.99	0.71	0.97
FK4	0.99	1.00	0.77	0.98
N30	0.71	0.77	1.00	0.81
GC	0.97	0.98	0.81	1.00

$\delta\mu_b$	FK5	FK4	N30	GC
FK5	1.00	0.99	0.79	0.87
FK4	0.99	1.00	0.81	0.87
N30	0.79	0.81	1.00	0.65
GC	0.87	0.87	0.65	1.00

Tables 7 Correlation coefficients between rotation of the LSS in different catalogues derived from separate solutions

$\delta\mu_l \cos b$	FK5	FK4	N30	GC
FK5	1.00	0.99	0.82	0.99
FK4	0.99	1.00	0.88	0.98
N30	0.82	0.88	1.00	0.78
GC	0.99	0.98	0.79	1.00

$\delta\mu_b$	FK5	FK4	N30	GC
FK5	1.00	1.00	0.87	0.75
FK4	1.00	1.00	0.85	0.74
N30	0.87	0.85	1.00	0.62
GC	0.75	0.74	0.62	1.00
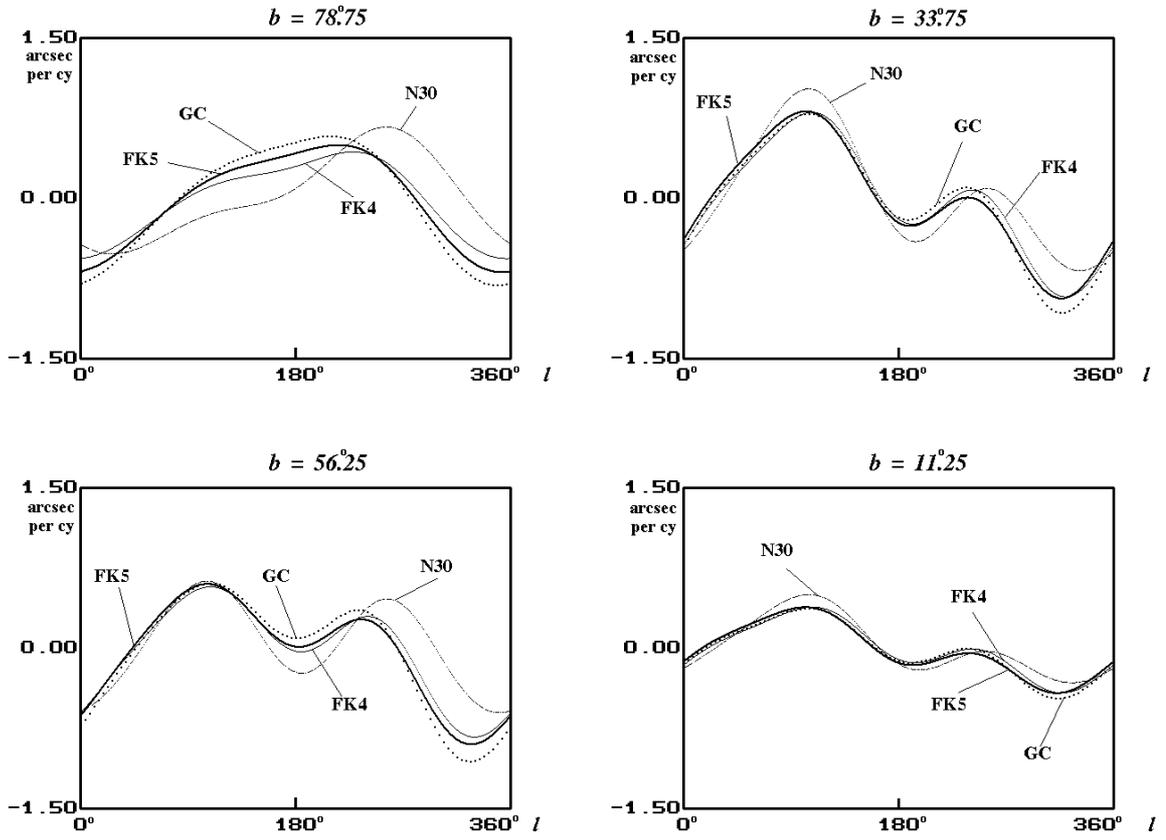
Figure 2. *Dependence of $\delta\mu_l \cos b$ on catalogues*
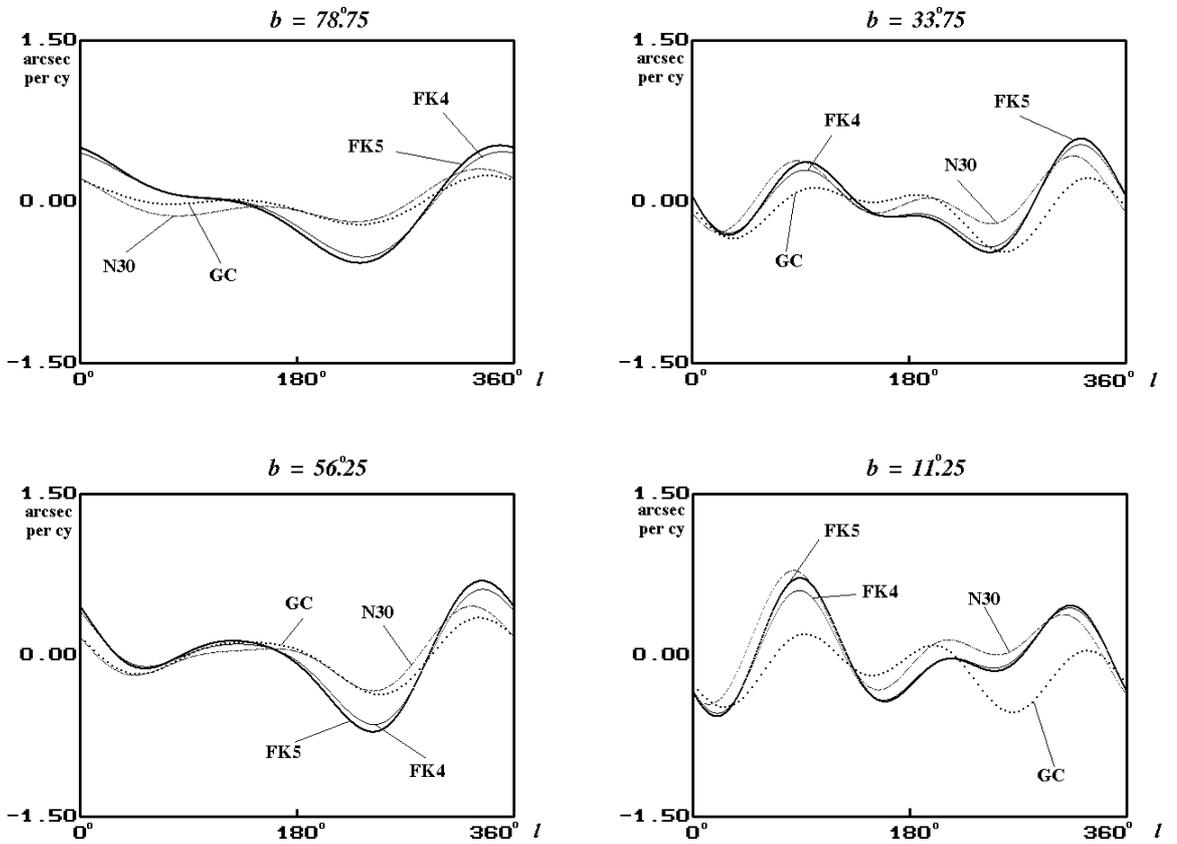
Figure 3. *Dependence of $\delta\mu_b$ on catalogues*
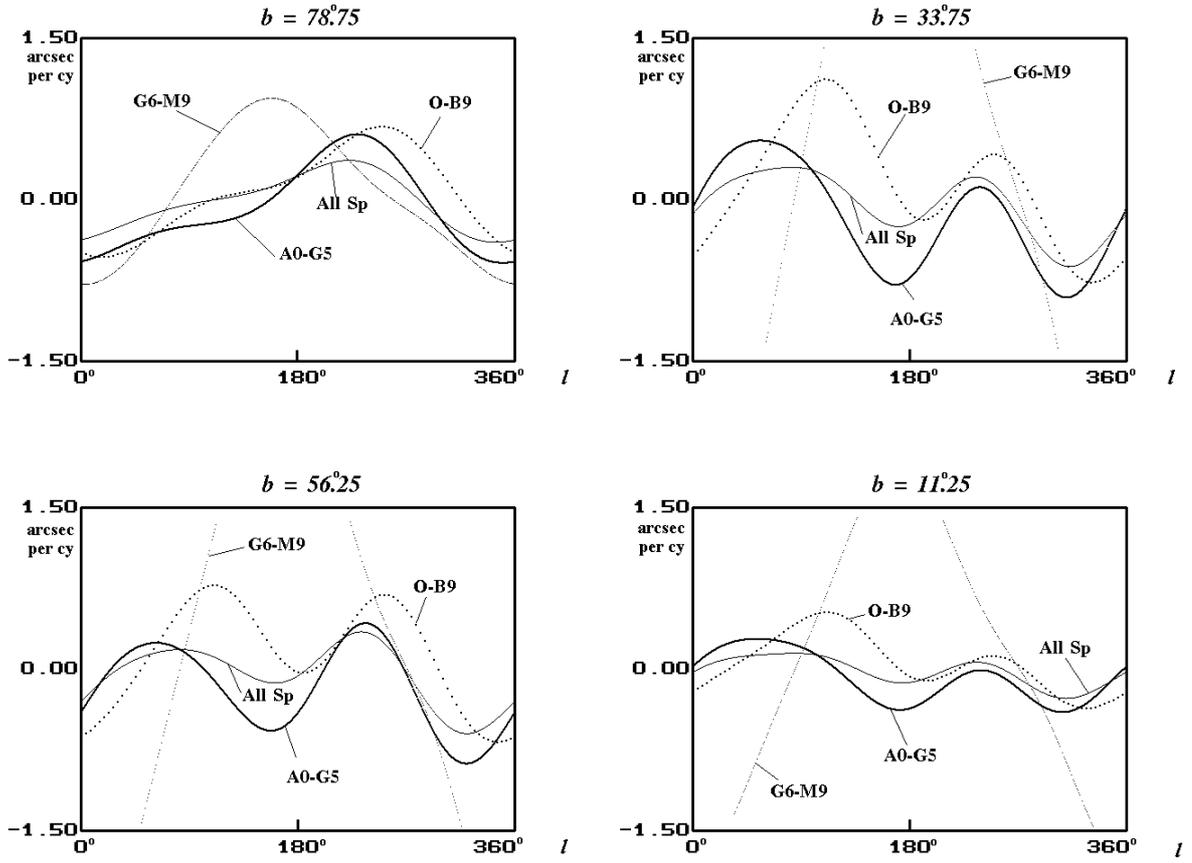
Figure 4. *Dependence of* $\delta\mu_l \cos b$ *on spectral types*
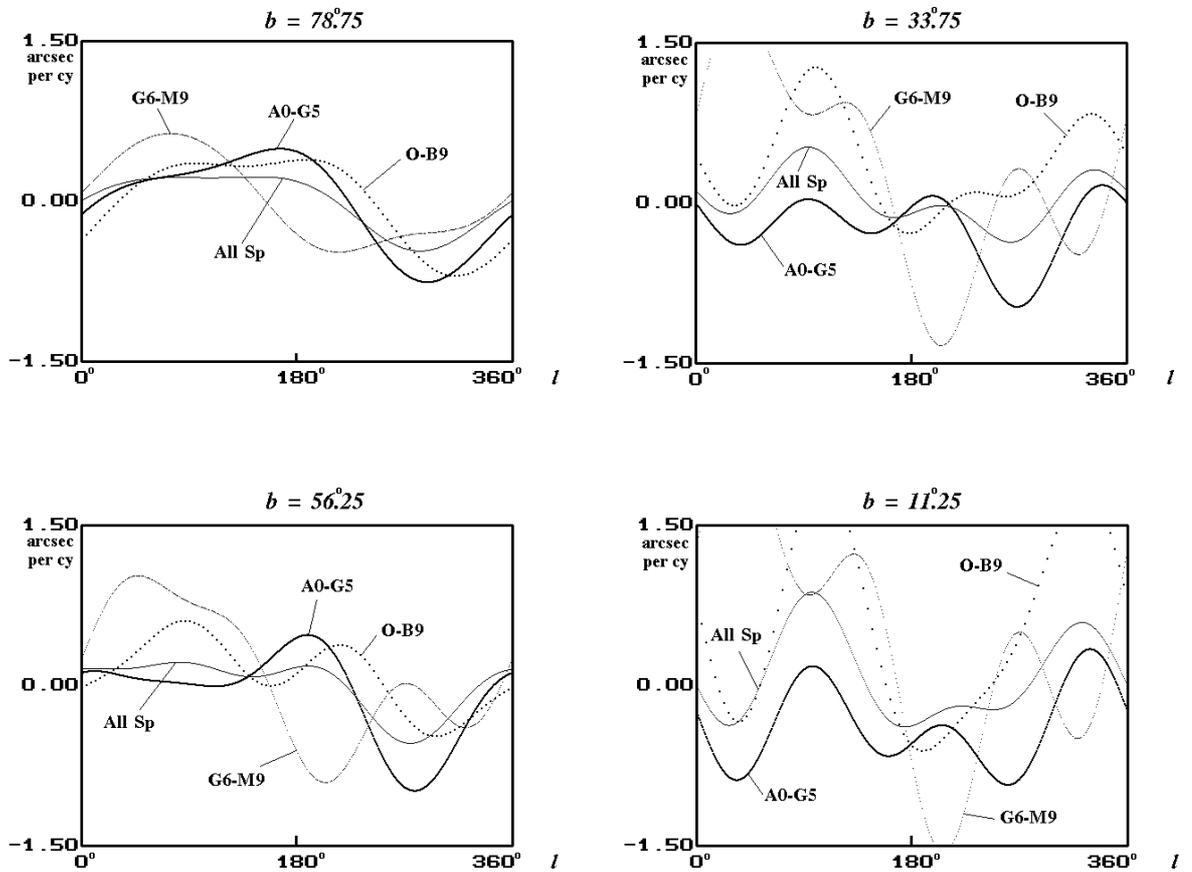
Figure 5. *Dependence of* $\delta\mu_b$ *on spectral types*