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Determination of kinetic energies of stars using Hipparcos data ⋆

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Abstract. It is well known that the stars of late spectral types from the main sequence of the HR diagram have larger proper motions than the stars of earlier types. This is due to two reasons: firstly, these stars are generally closer because of the observational selection, and secondly, there is a physical reason: the space velocities of K-M stars are twice or more larger than the velocities of A-F stars. In the present paper we try to estimate the kinetic energies of the stars from the Hipparcos catalogue using the catalogue of stellar masses. It is shown that the kinetic energies are approximately the same for all stars.

Key words. stellar kinematic Hipparcos

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Introduction

The completion of the Hipparcos space mission has resulted in kind of a U-turn of the astronomical community towards stellar kinematics. This is not surprising, in view of the fact that the previous peak of the amount of works devoted to stellar kinematics was in 1930s when the General Catalogue by B. Boss became available (Boss, B. 1937). The main feature of Hipparcos in comparison with the rest of astrometric catalogues is the fact that it contains individual trigonometric parallaxes for each individual star. The absolute error of one measurement is about 1 mas, which produces the relative error better than 25% for more than a half (i.e., about 50 000) of the stars in the Hipparcos catalogue (Mignard, F. 1997). This observational material allows one to solve a class of stellar kinematics problems whose solution was scarcely imaginable before.

General directions of research in Hipparcos proper motions were established at the Hipparcos Venice'97 symposium. In particular, the questions of determining the average density (Pham, H.-A. 1997) and escape velocity (Meillon, L. et al. 1997) in the vicinity of the Sun, were discussed. Our research is along these lines. More precisely, we attempt to find the average kinetic energy per star in the vicinity of the Sun.

As is well known, the stars of late spectral types have larger proper motions than the stars of early spectral types. There is a good deal of different hypotheses that aim to explain this phenomenon. One of them asserts that the late type stars have smaller masses, which is why their velocities are larger. The theoretical base for this statement can be found in stellar dynamics which uses the notion of "stellar gas" in order to describe large systems of stars. Using this notion is apparently inevitable, because of computational limits that occur when one describes an ensemble containing a substantial number of stars. However, the stellar gas is not a complete analogue of the ideal gas in thermodynamics, as the energy interchanges in the ideal gas are due to elastic impacts, whereas the stellar gas has no such mechanism. In principle one cannot exclude energy interchanges that are in fact caused by gravity during long evolution of a system. If this is indeed the case, one may expect that a sufficiently old stellar system would evolve towards a dynamic equilibrium that can be characterized by means of the average kinetic energy of a single star.

The Hipparcos catalogue which contains individual trigonometric parallaxes, made it possible for the astronomers to obtain the tangential velocity of each star.

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We can thus estimate the kinetic energy of a star by combining velocities and masses of stars.

The present paper is aimed to use the mass difference of stars with approximately the same kinetic energy, in order to explain the difference between remainder proper motions of stars that belong to different spectral classes (after we removed the influences of standard kinematic effects).

Estimation of stellar spatial velocity

Observational material

The nearby stars (closer than 250 pc) from Hipparcos (Mignard, F. et al. 1997) have been used as our observational material.

Table 1 illustrates the obvious fact that the late spectral type stars have substantial proper motions in comparison with the stars of early and middle types. Only stars from the main sequence are represented in this table, and we calculate the average module of proper motions for each spectral type.

Table 1. *Average module of proper motion for stars of main sequence that are closer than 250 pc.*

Spectral type	Average module of proper motion in "/cy
B	2.33 ± 2.45
A	3.03 ± 3.72
F	5.88 ± 6.74
G	10.8 ± 14.3
K	17.2 ± 25.3
M	44.6 ± 60.7

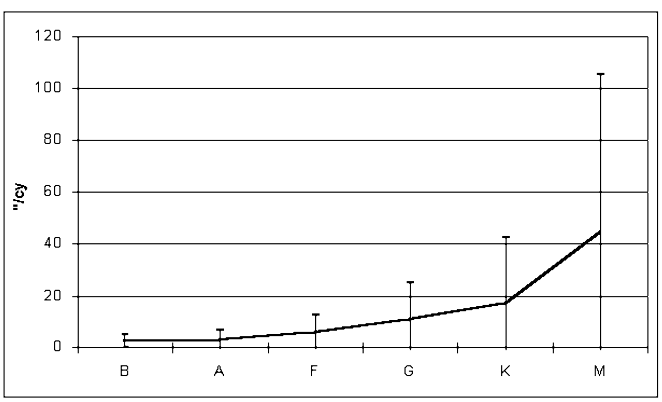

Fig. 1. *Dependence of the module of proper motion on spectral type for stars of main sequence that are closer than 250 pc.*

Analysis of Table 1 shows systematic growth of both the proper motion module and dispersion. The proper motion module of the M-stars is 20 times larger than the one of the B-stars, whereas the corresponding scattering ratio is 24. Figure 1 illustrates this tendency. Note that only this type of behaviour could have been observed in the pre-Hipparcos epoch.

If the distances to stars were not be known at all (or known with a bad precision rate), one would speculate that the M-stars are closer due to observational selection, as for more distant stars their proper motions of are not so visible.

Knowing accurate distances to stars allows us to examine this hypothesis. The tangential velocities can be calculated as

$$V_\tau = kr\mu, \tag{1}$$

where

$k = 0.0474$ – the factor to convert "$/cy$ into km/s;
r – the distance to a star in pc;
$\mu = \sqrt{(\mu_\alpha \cos_\delta)^2 + \mu_\delta^2}$ – the full proper motion in "$/cy$.

Table 2. *Average tangential velocity for the main sequence stars that are closer than 250 pc.*

Spectral type	Average tangential velocity in km/s
B	16.9 ± 13.8
A	19.1 ± 15.6
F	28.3 ± 23.3
G	40.3 ± 34.0
K	41.7 ± 37.2
M	43.5 ± 38.2

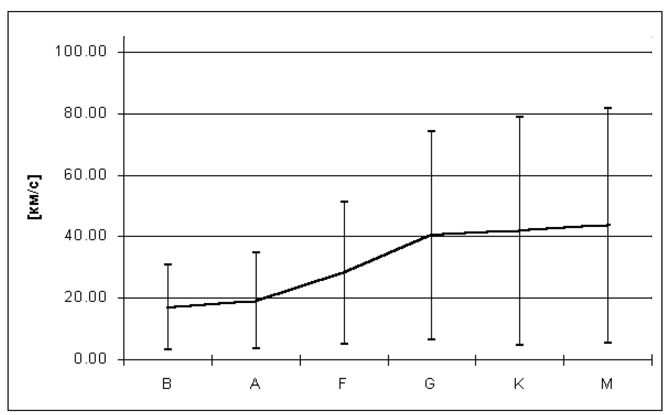

Fig. 2. *Dependence of tangential velocity on spectral type.*

The graphic representation of Table 2 is Fig. 2.

The analysis of this data shows that in comparison with the increment of the corresponding proper motions, the tangential velocity increases not so drastically when passing to late type stars. However, the difference in tangential velocities amounts to 2.5, and the difference in root-mean-square errors runs up to 3 times. So, we can establish the fact that the proximity of K- and M-stars is only a partial explanation of their large proper motions.

The transition from proper motions to tangential velocities demonstrates the fact that the stars of late spectral types have not only larger proper motions but also larger velocities in comparison with the stars of earlier spectral types.

Residual spatial velocities of stars

Unfortunately, the Hipparcos catalogue does not contain radial velocities. Moreover, there are Hipparcos stars whose radial velocities cannot be found in other sources either. This results in the impossibility to determine the spatial velocity for a particular star in Hipparcos. Nevertheless, it is possible to obtain the average spatial velocity for a group of stars using only Hipparcos data. The algorithm for determining the average residual spatial velocity for a group of stars is described below.

1. The Hipparcos stars are split into five spherical layers with step of 50 pc. For each layer the parameters of the standard kinematic model are calculated by means of the least squares.

$$\mu_l \cos b = \frac{1}{kr}\left(V_x \sin l - V_y \cos l\right) + \\ + \frac{A}{k}\cos 2l \cos b + \frac{B}{k}\cos b, \quad (2)$$

$$\mu_b = \frac{1}{kr}\left(V_x \sin b \cos l + V_y \sin b \sin l - V_z \cos b\right) + \\ + \frac{A}{k}\sin 2l \sin b. \quad (3)$$

Here

V_x, V_y, V_z – the components of the solar motion,
A, B – the Oort constants,
k – conversion factor.

The values of the parameters for stars of different distances are shown in Table 3.

2. The influence of standard kinematic effects are removed from individual proper motions for each spherical layer. This produces the residual proper motions $\mu_l \cos b^\star$ and $\mu_b^\star$.

$$\mu_l^\star \cos b = \mu_l \cos b - \frac{1}{kr}\left(V_x \sin l - V_y \cos l\right) - \\ - \frac{A}{k}\cos 2l \cos b - \frac{B}{k}\cos b, \quad (4)$$

Table 3. *Parameters of the standard kinematic model.*

r [pc]	V_x [km/s]	V_y [km/s]	V_z [km/s]
$0-50$	11.6 ± 0.5	19.7 ± 0.5	3.7 ± 0.5
$50-100$	12.1 ± 0.3	22.7 ± 0.3	7.5 ± 0.3
$100-150$	9.8 ± 0.3	18.0 ± 0.3	6.7 ± 0.3
$150-200$	9.7 ± 0.3	16.7 ± 0.3	6.9 ± 0.3
$200-250$	9.6 ± 0.3	16.6 ± 0.3	6.3 ± 0.3

r [pc]	A [km/s · Kpc^{-1}]	B [km/s · Kpc^{-1}]
$0-50$	33.4 ± 28.5	12.4 ± 22.3
$50-100$	18.1 ± 5.4	-14.0 ± 4.4
$100-150$	8.2 ± 2.7	-8.7 ± 2.2
$150-200$	9.7 ± 2.0	-12.6 ± 1.5
$200-250$	13.6 ± 1.7	-15.3 ± 1.3

$$\mu_b^\star = \mu_b - \frac{1}{kr}\left(V_x \sin b \cos l + V_y \sin b \sin l - V_z \cos b\right) + \\ - \frac{A}{k}\sin 2l \sin b. \quad (5)$$

3. The tangential velocities for each star were determined from the residual proper motions using equation (1).
4. The average tangential velocity is obtained for each spectral type.
5. The final step consists in calculating the average spatial velocity for each spectral type. It is known from Probability Theory Agekyan T.A. 1974 that the mean value for the length of the planar projection of a unit interval randomly placed in $\mathbf{R}^3$, is $\pi/4$. The tangential velocity is the projection of the full velocity onto the plane perpendicular to the line of sight. Consequently, we obtain the following estimate of the spatial velocity for a group of stars:

$$\langle V \rangle = \frac{4}{\pi}\langle V_t \rangle. \quad (6)$$

The values of the average spatial velocity for the stars from the main sequence are represented in Table 4.

The data of Table 4 are illustrated by Fig 3. One can see that removing the influence of the standard kinematic effects virtually does not alter the curve representing velocity increment (cf. Fig. 2). This means that the main constituent in proper motions has a peculiar or random nature. A drastic change in residual spatial velocity can be observed when crossing from the F to G spectrum. The modification of kinematic characteristics within this spectral range was observed long time ago by Parenago (Parenago P.P. 1954).

Table 4. *The average spatial velocity for the stars from the main sequence that are closer than 250 pc.*

Spectral type	$\langle V \rangle$ [km/s]
B	21.11 ± 17.51
A	24.27 ± 19.79
F	35.92 ± 29.61
G	51.20 ± 43.26
K	52.98 ± 47.25
M	55.35 ± 48.51

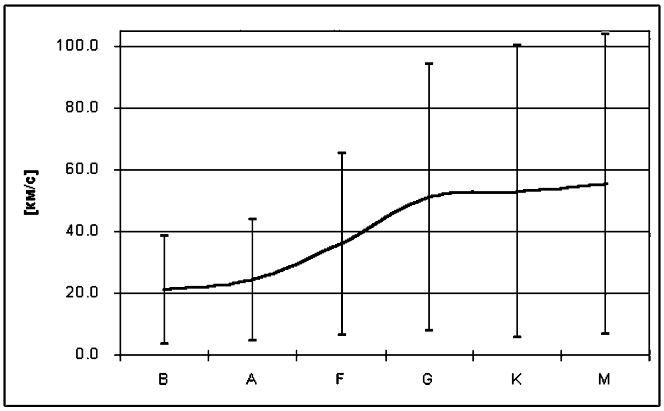

Fig. 3. *Dependence of residual spatial velocity of main sequence stars closer than 250 pc on their spectral type.*

Kinetic energies of stars

Stellar masses and their dispersion

There exist different hypotheses that aim to explain the systematic difference in residual velocities of stars from different parts of the HR-diagram. In the present paper we follow the hypothesis of stellar gas. It was proposed in 1950 by Parenago (Parenago P.P. 1954) and Ogorodnikov (Ogorodnikov K.F. 1958). One may expect the normal distribution of kinetic energies but not velocities, unlike the situation with molecules of gas. The difference between stellar gas and the gas in thermodynamics lies in the way the energies interchange: the elastic collisions are the reason of energy interchange in the "usual" gas, whereas it is apparently only gravity that causes interactions in stellar gas (Zonn V., Rudnitsky K. 1959).

To determine the kinetic energies of stars that belong to different groups, one needs to know not only their average spatial velocities $\langle V \rangle$ but also their average masses $\langle M \rangle$. Provided this is known, one will be able to estimate the average kinetic energy $\langle K \rangle$ by means of the following equation:

$$\langle K \rangle = \frac{M \langle V \rangle^2}{2}. \tag{7}$$

Consider now the question of stellar masses. Originally, the diagram "mass-luminosity" (Kulikovsky K.F. 1985) has been used to obtain the stellar masses. Unfortunately, such an approach does not allow one to estimate the dispersion of stellar masses within one spectral type.

However, we finally succeeded in finding an appropriate stellar mass catalogue, namely, the one by Belikov (Belikov A.N. 1995) in the CDS system. This catalogue contains masses for stars in more than 300 binaries. All these masses have been computed by direct dynamical methods. The catalogue data yield the dispersion of masses, since it contains several ways of mass determination for each spectral type and even for subtypes. The result in question is presented in Table 5:

Table 5. *Average masses of stars in solar unit.*

Spectral type	Masses of stars from the main sequence	Masses of red giants
B	7.84 ± 4.69	
A	2.09 ± 0.65	
F	1.38 ± 0.51	
G	1.01 ± 0.31	2.43 ± 0.53
K	0.85 ± 0.43	1.17 ± 0.85
M	0.43 ± 2.80	

The data from Tables 4 and 5 are used to determine the kinetic energies.

Results

The main results are represented in Tables 6 and 7. The following notation is used (all the values have MSE):

$B-V$ – average value of B-V color index;
$\langle V \rangle$ – spatial velocity in our proper units (one unit = 21 km/s – mean solar motion relative to local standard of rest);
M – average masses in solar units;
K – kinetic energy, kinetic energy of Sun is $\frac{1}{2}$ with accepted units of mass and velocity (it is $4.4 \cdot 10^{38}$watt-second);
N – number of stars.

The specific units are introduced for the velocity and kinetic energy of stars.

The stars of spectral type O were not presented because of their small number. Also, there are no M-giants because there is no mass data for them.

Tables 6, 7 show that the average values of the kinetic energies for the stars in the main sequence are very close each other. The kinetic energies of the giants are comparable with the ones for the main sequence. This conclusion is confirmed by Figures 4 and 5.

Table 6. *Kinetic energies for main sequence stars.*

Sp	$\langle V\rangle$	$B-V$	M	K	N
B	0.00 ± 0.15	1.0 ± 0.8	7.8 ± 4.7	4.0 ± 7.0	2234
A	0.19 ± 0.13	1.2 ± 0.9	2.1 ± 0.6	1.4 ± 2.3	10538
F	0.47 ± 0.10	1.7 ± 1.4	1.4 ± 0.5	2.0 ± 3.4	20727
G	0.69 ± 0.15	2.4 ± 2.1	1.0 ± 0.3	3.0 ± 5.1	14531
K	1.00 ± 0.23	2.5 ± 2.2	0.9 ± 0.4	2.7 ± 5.0	6158
M	1.42 ± 0.21	2.6 ± 2.3	0.4 ± 2.8	$1.5 \pm 10.$	1037

Table 7. *Kinetic energies for red giants.*

Sp	$\langle V\rangle$	$B-V$	M	K	N
G	1.03 ± 0.07	1.9 ± 1.6	2.4 ± 0.5	4.4 ± 7.3	1526
K	1.17 ± 0.16	2.0 ± 1.4	1.2 ± 0.9	2.4 ± 6.2	7085

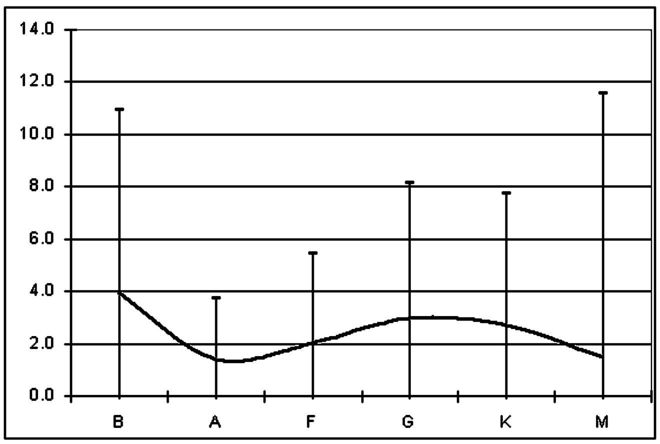

Fig. 4. *Dependence of kinetic energies of main sequence stars on their spectral type.*

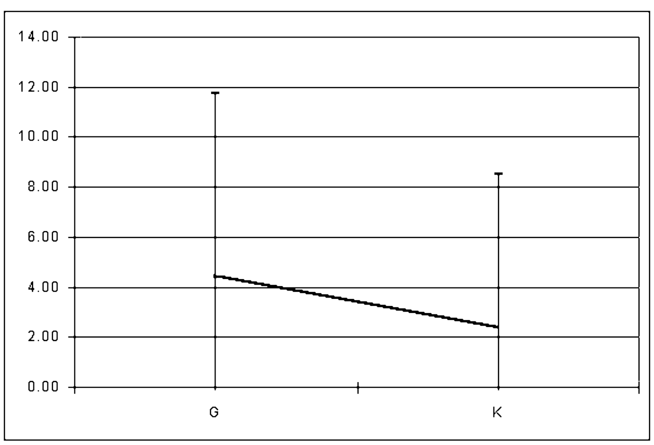

Fig. 5. *Dependence of kinetic energies of red giants on spectral type.*

Conclusions

The principal results of our work can be summarized as follows.

The most important conclusion is that the difference in the kinetic energies of stars from various parts in HR-diagram is not that significant in comparison with the dispersion of their spatial velocities and, especially, with the dispersion of their proper motions. One may say that the stars in the solar neighbourhood possess about equal kinetic energy. If one does not consider the stars of marginal types, the difference in the kinetic energies of A, F, G, K-stars of the main sequence is at most one and half times. The behaviour of the plot in Fig. 4 (in comparison with Figures 2, 3) has changed. There is no systematic increment of its values while moving along the main sequence. The difference in the kinetic energies that remains, can be explained by inaccuracy in mass determination or other reasons and needs to studied in detail.

In our opinion, there are two directions, in which the present paper can be extended:

– The individual radial velocities can be used to obtain the spatial velocity for each star. Unfortunately, only a small part all stars in the Hipparcos catalogue have their radial velocities measured. Nevertheless, at least 10 000 stars that do have measured radial velocities is a sufficient material to attempt this kind of study.
– It would be very interesting to study the profile of kinetic energy dispersion. This may reveal whether this distribution is stationary or not.

We plan to continue this line of research in subsequent papers.

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