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Comparison of XPM and UCAC4 catalogues in the galactic coordinate system

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This paper presents the results of the comparison of the galactic reference frames realized by the catalogues XPM and UCAC4. Based on about 40 million stars common to both catalogues, the systematic differences of the galactic coordinates and proper motions have been derived for 12 magnitudes in 0.5^m width bins with the mean J-values from $10^m.25$ to $15^m.75$. The systematic differences were represented by vector spherical harmonics, with the magnitude equation taken into consideration. The mutual orientation of the frames was found to be at the level of 10 mas. It is concluded that these differences are negligible in comparison with the accuracy of the implementation of the standard galactic coordinate system MAS 1958. We investigated two features of the XPM catalogue. First, unlike the HCRF and UCAC4 catalogues, whose proper motions are tied to the quasars and galaxies, the XPM catalogue implements a reference system based only on galaxies. Second, the XPM catalogue has two systems of proper motions—XPM_x and XPM_p, referred to the two galaxy sub-catalogues of the project 2MASS—PSC and XSC. The study of the differences $\mathrm{XPM}_x-\mathrm{XPM}_p$ showed that they are free of the magnitude equation. The speed of relative rotation of XPM_x over XPM_p was found to be $\omega = 0.453 \pm 0.003$ mas/year, which exceeds the residual rotation of the HCRF (0.25 mas/year). Analysis of systematic differences XPM_x-UCAC4 and XPM_p-UCAC4 showed that both frames XPM_x and XPM_p have an appreciable rotation speed relative to the UCAC4 (hence the ICRF), especially large (up to 2 mas/year) for the brightest stars in our range. This shows that a relatively high speed of rotation of the two frames XPM_x and XPM_p with respect to the UCAC4 is a consequence of the transition from a combined "quasar-galaxy" to a purely "galactic" reference system. It is shown that the systematic difference between the proper motions of stars can be interpreted within the Ogorodnikov–Milne kinematic model of the velocity field.

KEYWORDS
astrometry, reference systems, catalogues

1 | INTRODUCTION

This article is devoted to deriving and analysing the systematic differences of the coordinates and proper motions of the XPM (Fedorov, Akhmetov, Bobylev, & Bajkova, 2010; Fedorov, Myznikov, & Akhmetov, 2009) and UCAC4 (Zacharias et al., 2013) catalogues in the galactic coordinate system. It is a continuation of the work of Vityazev and Tsvetkov (2015b), which describes these catalogues and the results of their comparison in the equatorial coordinate system. As a rule, the comparison of the catalogues is fulfilled in the equatorial coordinate system and, as far as we know, there are no examples of systematic difference analysis in the galactic system. However, the galactic coordinate system is widely used in various problems of astronomy, such as the study of galactic structure, stellar kinematics, and dynamics. The currently used galactic coordinate system was introduced by the International Astronomical Union in 1958 (Blaauw, Gum, Pawsey, & Westerhout, 1960) based on a study of the distribution of neutral hydrogen in the galaxy. In recent

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papers (Liu, Zhu, & Hu, 2011; Liu, Zhu, & Zhang, 2011), the problem is posed of improving the galactic coordinate system by identifying the galactic plane with new observational data on the coordinates of the various objects in the infrared range of the catalogue 2MASS (Skrutskie, Cutri, & Stiening, 2006) and radio frequency (Vollmer et al., 2010). All of these motivate the study of different implementations of the galactic coordinate system. In this paper, we solve this problem by representing the systematic differences between the coordinates and the proper motions by vector spherical harmonics complemented with functions that take into account the magnitude equation. Such expansions are made for two sets of proper motions presented in the XPM catalogue.

The catalogue UCAC4 is constructed in the HCRF (Hipparcos Celestial Reference) system with the proper motions tied to the quasars and galaxies (Kovalevsky et al., 1997). The accuracy of the corrections applied to the proper motions of H37 preliminary catalogue to tie it to the ICRF system is estimated as ±0.25 mas/year. Obviously, it measures the residual rotation of HCRF and UCAC4 with respect to the combined quasar-galactic frame. Although quasars and galaxies can theoretically provide a basis for the construction of inertial reference systems, the difference in the specific observations of these objects may lead to systematic differences in the proper motions of stars (especially regarding the magnitude equation). The use of all currently available observational data suggests that the residual rotation of the HCRF system is determined with an error of 0.1 mas/year (Bobylev, 2015).

Unlike UCAC4, where proper motions are defined in the reference system implemented with the help of quasars and galaxies, the main purpose of the XPM was to obtain the absolute proper motions with respect to the extragalactic reference system, constructed on galaxies only. For this reason, we study the expansion coefficients of systematic differences in order to clarify the effects of the transition from the reference system constructed on galaxies and quasars (UCAC4) to the reference system implemented with the help of galaxies (XPM). Moreover, since the proper motions of stars in the galactic coordinate system are used for the analysis of stellar kinematics, we solve the problem of reducing the kinematic parameters of the Ogorodnikov–Milne velocity field from the system of one catalogue to the system of another catalogue. Special attention is paid to clarify the influence of systematic differences on the estimation of the Oort constants.

2 | REPRESENTATION OF SYSTEMATIC DIFFERENCES WITH VECTOR SPHERICAL HARMONICS AND LEGENDRE POLYNOMIALS

The decomposition of systematic differences on the system of complex vector spherical functions has been proposed (Mignard & Froeschle, 2000; Mignard & Klioner, 2012; Mignard & Morando, 1990). The real form of spherical harmonics (henceforth VSH) was used in our previous papers (Vityazev & Tsvetkov, 2013) without taking into account the magnitude equation. In this paper, the systematic differences between the XPM and UCAC4 catalogues are obtained by a modified method based on real vector spherical harmonics and Legendre polynomials to take into account the magnitude equation—henceforth VSHL. This approach was developed by us earlier (Vityazev & Tsvetkov, 2015a).

In this method, the vector field of longitude and latitude differences $\Delta l \cos b$ and Δb, and the vector field of the proper motions differences, $\Delta\mu_l \cos b$ and $\Delta\mu_b$, given by

$$\Delta\mathbf{F}(l,b,m) = \begin{cases} \Delta l \cos b\, \mathbf{e}_l + \Delta b\, \mathbf{e}_b, \\ \Delta\mu_l \cos b\, \mathbf{e}_l + \Delta\mu_b\, \mathbf{e}_b, \end{cases} \tag{1}$$

are decomposed on the VSHL basis as follows:

$$\Delta\mathbf{F}(l,b,m) = \sum_{nkpr} t_{nkpr}\mathbf{T}_{nkp}(l,b)\, Q_r(\bar{m}) + \sum_{nkpr} s_{nkpr}\mathbf{S}_{nkp}(l,b)\, Q_r(\bar{m}). \tag{2}$$

In Equation 1, $\mathbf{e}_l$ and $\mathbf{e}_b$ denote the unit vectors in the tangential plane along the longitude and latitude directions. In Equation 2, the toroidal ($\mathbf{T}_{nkp}(l,b)$) and spheroidal ($\mathbf{S}_{nkp}(l,b)$) harmonics describe the dependence of systematic differences on longitude and latitude, whereas the normalized Legendre polynomials ($Q_r(\bar{m})$) take into account the dependence of systematic differences on the brightness of stars. The expression

$$\bar{m} = 2\frac{m - m_{\min}}{m_{\max} - m_{\min}} - 1 \tag{3}$$

transforms the interval $[m_{\min} \leq m \leq m_{\max}]$ into $[-1 \leq \bar{m} \leq +1]$.

A list of 41 316 676 stars common to our catalogues was compiled by the star identification procedure in the J band (2MASS photometric system). We considered the stars in different catalogues identical if the position difference did not exceed 500 mas and the J magnitude difference was less than 0.01. Actually, the magnitudes just coincided, and there were no difficulties with the cross-identification of stars. For all the stars in our sample, the galactic coordinates and proper motions were calculated. The standard procedure to derive positions and proper motions in galactic coordinate system was used (ESA, 1997).

To derive the coefficients t_{nkpr} and s_{nkpr} for the decompositions of the position and proper motion differences, we averaged the values of the fields (1) in each of the 1200 pixels built by the Healpix procedure (Gorski et al., 2005), for stars belonging to the $0.^m5$ wide bins from $J_{\min} = 10^m.25$ to $J_{\max} = 15^m.75$. Then, the averaged values of the fields related to the centres of the areas were represented by Equation 2. A detailed description of this procedure is given elsewhere (Vityazev & Tsvetkov, 2015a, 2015b). Compared to the traditional methods of using vector spherical harmonics, our approach has two novel features: first, we determine the significance of all the harmonics that can be implemented on the selected pixels; and second, we introduce the magnitude equation model and calculate its parameters. All the values of the coefficients t_{nkpr} and s_{nkpr} are derived with a reliability better than 97.9%. Tables A1 and A2 contain the coefficients of $\Delta l \cos b\, e_l + \Delta b\, e_b$ decompositions. Tables A3 and A4 list the coefficients of the $XPM_x - UCAC4$ decompositions, while Tables A5 and A6 show the same for the $XPM_p - UCAC4$ decompositions.

3 | ANALYSIS OF THE XPM–UCAC4 COORDINATE SYSTEMATIC DIFFERENCES

In this section, using Tables A1 and A2, we investigate how the systematic differences of longitudes and latitudes between the XPM and UCAC4 may affect the relative orientation of galactic reference frames implemented by these catalogues.

It is known (Froeschle & Kovalevsky, 1982) that the mutual orientation of coordinate systems can be derived from the analysis of the systematic coordinate differences. These effects must also appear in the coefficients of expansion of systematic differences on a system of orthogonal functions. Within the model of rigid-body rotation, the connection of the expansion coefficients and the angles of rotation of one coordinate system relative to the other was found by Vityazev (1994) for the scalar case. When using the complex vector spherical harmonics (Mignard & Morando, 1990), the mutual orientation of reference systems related to the catalogues under consideration is determined via the toroidal coefficients of the first order. In the notation of the present article (real vector spherical harmonics), the working formulae establishing the ties between the components of the rotation vector (rotation) and toroidal coefficients of the first order are given in Vityazev and Tsvetkov (2009, 2013, 2014).

Returning to our catalogues, denote by $\varepsilon_x, \varepsilon_y, \varepsilon_z$ the angles to rotate the UCAC4 system to bring it into coincidence with the the XPM system. With these designations, the XPM−UCAC4 systematic difference of the galactic coordinates are modelled by the following equations:

$$\Delta l \cos b = \varepsilon_x \sin b \cos l + \varepsilon_y \sin b \sin l - \varepsilon_z \cos b, \tag{4}$$

$$\Delta b = -\varepsilon_x \sin l + \varepsilon_y \cos l. \tag{5}$$

For each value of the magnitude m, one may use the following expressions to calculate the rotation angles via the toroidal coefficients:

$$\varepsilon_x = -t_{1,1,1}(m)/\sqrt{8\pi/3}, \tag{6}$$

$$\varepsilon_y = -t_{1,1,0}(m)/\sqrt{8\pi/3}, \tag{7}$$

$$\varepsilon_z = -t_{1,0,1}(m)/\sqrt{8\pi/3}, \tag{8}$$

where

$$t_{nkp}(m) = \sum_r t_{nkpr}\, Q_r(\bar{m}), \tag{9}$$

while the value $\bar{m}$ is determined by Equation 3.

The expressions (6)–(8) are easily derived by decomposing the vector fields with the right-hand sides of Equations 4 and 5 in the system of vector spherical harmonics.

The angles of mutual orientation of the galactic frames realized by the XPM and UCAC4 catalogues calculated from the formulae (6)–(8) are shown in Figure 1. Here we see that the angles of rotation around the axis OY depend on the brightness of stars and are in the range 3.51 ± 0.26–6.54 ± 0.55 mas. The angle of rotation around the axis OX is -4.58 ± 0.16 mas, and the angle of rotation around the Z-axis is equal to 6.55 ± 0.16 mas.

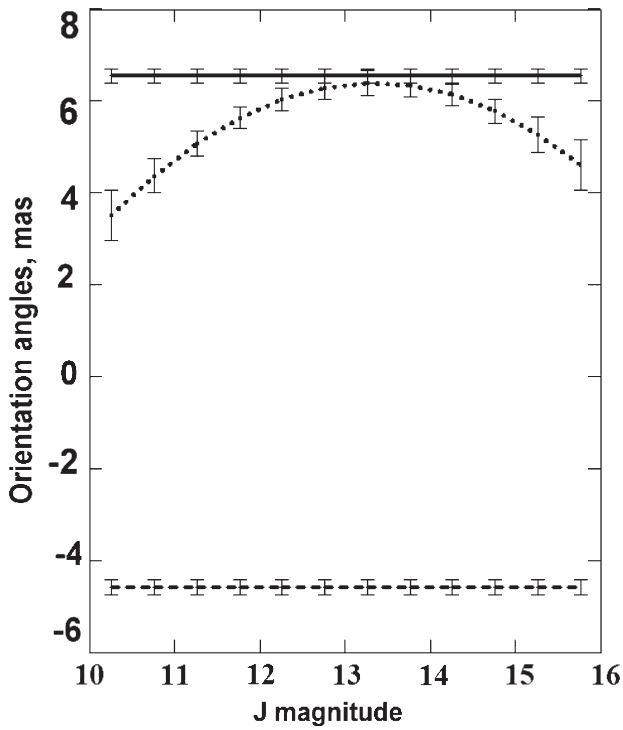

FIGURE 1 Mutual orientation angles to bring the system UCAC4 in coincidence with the XPM system: ε_x – dashes, ε_y – dots, ε_z – solid line.

There is reason to believe (Liu, Zhu, & Hu, 2011; Liu, Zhu, & Zhang, 2011) that the accuracy of fixing the axes of the standard galactic coordinate system MAS 1958 (Blaauw et al., 1960) was done within a few minutes of arc. Therefore, we can say that the orientations of the XPM and UCAC4 galactic axes are virtually identical, since the differences found (although they have a high statistical reliability) are significantly less than the precision of the standard galactic reference frame. More details on this issue are given in the article by Vityazev and Tsvetkov (2016).

4 | ANALYSIS OF THE XPM–UCAC4 SYSTEMATIC DIFFERENCES OF THE PROPER MOTIONS

The XPM catalogue contains two entries for the proper motions (henceforth referred to as XPM_x and XPM_p) obtained with the data from two catalogues of extended sources (XSC and PSC) of the 2MASS project (Fedorov, Akhmetov, et al., 2010; Fedorov, Myznikov, et al., 2009).

This section is devoted to the study of the systematic differences between the two variants of the XPM proper motions (XPM_x and XPM_p) as well as how each variant differs from the UCAC4 proper motions. In addition, we will also show how the results of the decomposition of the proper motions on the VSHL can be used to reduce the values of the kinematic parameters from the system of one catalogue to the system of another catalogue.

4.1 | Systematic differences of the proper motions $XPM_x - XPM_p$

For a detailed study of the two variants of the proper motions XPM_x and XPM_p in the galactic coordinate system, we averaged the differences between them in each of the 1200 HealPix areas for stars belonging to the same intervals of magnitudes in the J band used by us in the processing of the differences XPM−UCAC4. Then, the average values of the differences of the proper motions, referred to the centres of the pixels, were represented by Equation 2 as was described by Vityazev and Tsvetkov (2015b). The toroidal and spheroidal coefficients of these expansions are shown in Table 1. The analysis of this table allows us to state the following facts:

1. The systematic differences $XPM_x - XPM_p$ do not depend on the magnitude equation. This is evidenced by the zero value of the index r of all toroidal and spheroidal coefficients.

2. In contrast to the equatorial coordinate system (Vityazev & Tsvetkov, 2015b), the galactic zonal harmonics are not of primary importance among all other harmonics. Apparently, this is an evidence that the differences of coordinates of galaxies in the sub-catalogues XSC and PSC are not due to the avoidance of external galaxies along the plane of the Milky Way depending mainly on the galactic latitude, and particularly strong at low galactic latitudes, but due to the specifics of the observations in the 2MASS project in the equatorial coordinate system.

3. The first-order toroidal coefficients of the $XPM_x - XPM_p$ systematic differences of the proper motions yield the components of the vector field that is generated by the rotation of the reference system XPM_p about XPM_x system with the angular velocities $\omega_x, \omega_y, \omega_z$:

$$\Delta\mu_l \cos b = \omega_x \sin b \, \cos l + \omega_y \sin b \sin l - \omega_z \cos b, \quad (10)$$

$$\Delta\mu_b = -\omega_x \sin l + \omega_y \cos l, \quad (11)$$

where

$$\omega_x = -t_{1,1,1,0}(m)/\sqrt{8\pi/3}, \quad (12)$$

$$\omega_y = -t_{1,1,0,0}(m)/\sqrt{8\pi/3}, \quad (13)$$

$$\omega_z = -t_{1,0,1,0}(m)/\sqrt{8\pi/3} \quad (14)$$

are the analogues of Equations 6–8.

4. It is obvious that the components of the angular speed $\omega_x, \omega_y,$ *and* ω_z allow us to determine the coordinates of the pole of the relative rotation:

$$L_{\text{rot}} = \text{arctg}\left(\frac{\omega_y}{\omega_x}\right); \quad (15)$$

$$B_{\text{rot}} = \text{arctg}\left(\frac{\omega_z}{\sqrt{\omega_x^2 + \omega_y^2}}\right). \quad (16)$$

In addition, the full mutual angular speed of rotation about the pole can be obtained from the formula

$$\Omega_{\text{rot}} = \sqrt{\omega_x{}^2 + \omega_y{}^2 + \omega_z{}^2}. \quad (17)$$

The vector map of the field with components (10) and (11) is shown in Figure 2. The coordinates of the rotation pole turned out to be $L_{\text{rot}} = 124.8° \pm 0.5°$, $B_{\text{rot}} = 30.4° \pm 0.4°$. They are very close to the galactic coordinates of the north pole of the equatorial coordinate system, $L = 122.9°$, $B = 27.1°$. The rotation is clockwise with angular velocity $\omega = 0.453 \pm 0.003$ mas/year, which exceeds the residual rotation of the HCRF (0.25 mas/year). This fact is consistent with fig. 7 in Vityazev and Tsvetkov (2015b), which shows the vector field generated by the rotation of the system XPM_p with respect to XPM_x in the equatorial coordinate system.

5. The vector map $XPM_x - XPM_p$ generated by the vector spherical harmonics higher than the first order is almost fully determined by the large value of the second zonal harmonic in the expansion of systematic differences in the equatorial system: $t_{201} = 2.88 \pm 0.01$ mas/year (Vityazev & Tsvetkov, 2015b). This map corresponds to the toroidal function

$$T^{\alpha}_{2,0,1}(\alpha, \delta) = \sqrt{\frac{15}{32\pi}} \sin 2\delta, \quad (18)$$

TABLE 1 Values of toroidal t_{nkpr} and spheroidal s_{nkpr} coefficients in representation of the differences $XPM_x - XPM_p$ by VSHL

Coeff.	Value (mas/year)	Coeff.	Value (mas/year)	Coeff.	Value (mas/year)
$t_{1,0,1,0}$	-0.94 ± 0.01	$t_{2,2,0,0}$	-1.76 ± 0.01	$t_{6,1,1,0}$	-0.23 ± 0.01
$t_{1,1,0,0}$	-1.31 ± 0.01	$t_{2,2,1,0}$	-0.75 ± 0.01	$t_{6,2,0,0}$	0.27 ± 0.01
$t_{1,1,1,0}$	0.91 ± 0.01	$t_{3,0,1,0}$	0.33 ± 0.01	$t_{6,4,0,0}$	0.24 ± 0.01
$t_{2,0,1,0}$	-0.49 ± 0.01	$t_{3,2,0,0}$	0.43 ± 0.01	$t_{6,6,1,0}$	0.29 ± 0.01
$t_{2,1,0,0}$	1.63 ± 0.01	$t_{4,1,1,0}$	0.20 ± 0.01	$t_{8,1,0,0}$	0.23 ± 0.01
$t_{2,1,1,0}$	-1.32 ± 0.01	$t_{4,4,1,0}$	0.32 ± 0.01	$t_{55,37,1,0}$	-0.20 ± 0.01
$s_{1,1,0,0}$	0.45 ± 0.01	$s_{2,1,1,0}$	-0.43 ± 0.01	$s_{3,3,1,0}$	0.22 ± 0.01
$s_{1,1,1,0}$	-0.35 ± 0.01	$s_{2,2,0,0}$	-0.47 ± 0.01	$s_{5,0,1,0}$	-0.24 ± 0.01
$s_{2,1,0,0}$	0.27 ± 0.01	$s_{3,2,1,0}$	-0.23 ± 0.01	$s_{7,0,1,0}$	0.24 ± 0.01

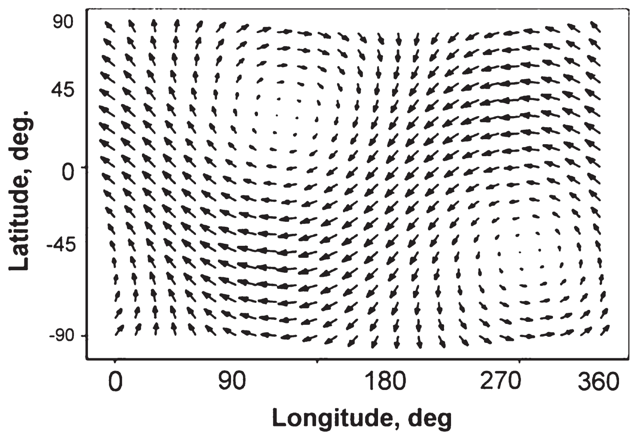

FIGURE 2 Vector map generated by the rotation of the XPM_p about the XPM_x system.

so in galactic coordinates we can see the vortex motion around the north and south poles and zero velocities along the celestial equator. This effect is strong enough because it defines the general structure of the vector field calculated with all significant harmonics of systematic differences $XPM_x - XPM_p$ (Figure 3).

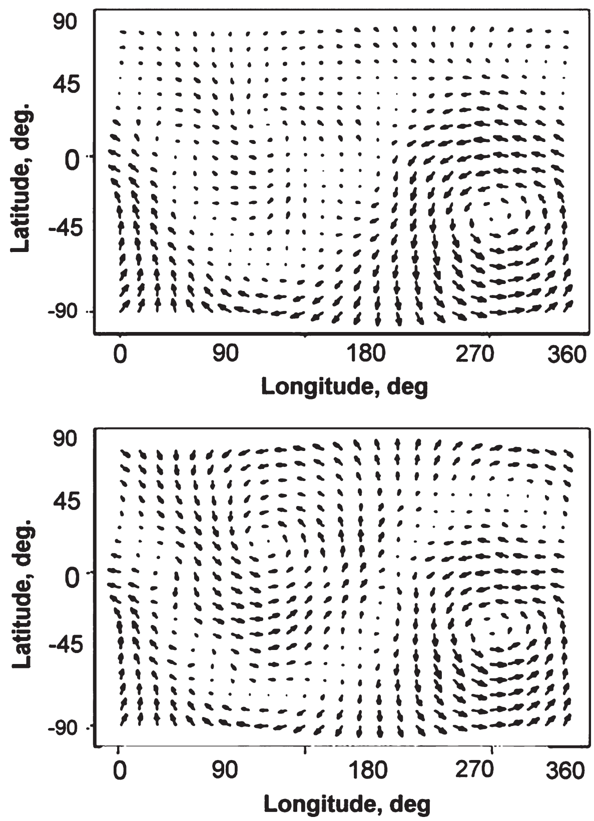

FIGURE 3 Vector maps of the $XPM_x - XPM_p$ systematic differences calculated with different sets of harmonics. (Top) all significant harmonics included; (Bottom) upper ($n \geq 2$) harmonics alone are used.

4.2 | Systematic differences of the proper motions XPM_x−UCAC4 and XPM_p−UCAC4

Now, using Equation 17 let us calculate the angular velocity of the XPM system's rotation around the UCAC4 system. Obviously, Equations 15 and 16 enable us to determine the positions of the pole about which the coordinate systems under consideration are involved in mutual spin. The speed of rotation around the poles are shown in Figures 4–7 for stars of different brightness values. From these figures, it follows that the rate of relative rotation of the XPM_x about UCAC4 frame varies from 0.32 ± 0.05 to 1.78 ± 0.05 mas/year, and a similar rate for XPM_p about UCAC4 changes from 0.54 ± 0.06 to 2.22 ± 0.05 mas/year.

As was shown in Vityazev and Tsvetkov (2015a), the spin of the residual UCAC4 rotation practically reproduces the measure of the HIPPARCOS residual rotation (0.25 mas/year). Thus, it can be argued that both galactic frames XPM_x and XPM_p have significant residual speeds of rotation relative to UCAC4 (hence the ICRF), especially for the brightest stars of our range. Since, as was shown above, the relative rotation XPM_p around XPM_x estimated as $\omega = 0.453 \pm 0.003$ mas/year, a relatively high rate of both frames XPM_x and XPM_p with respect to the UCAC4, is caused not by different calibration of the proper motions on the data taken from XSC and the PSC but by the general transition from "quasar-galaxy" to the "galactic" reference system.

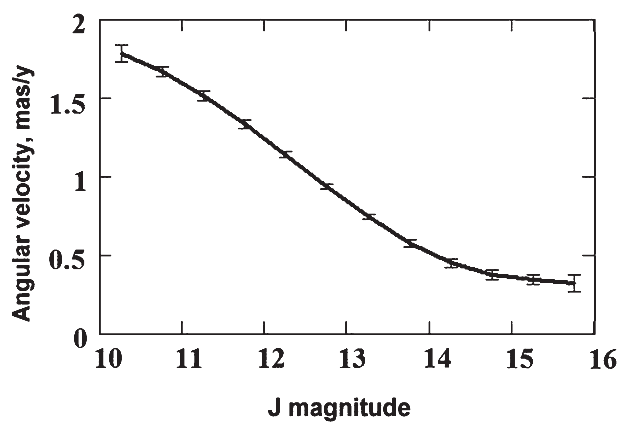

FIGURE 4 Angular velocity of the UCAC4 rotation with respect to XPM_x about the poles with J-dependent coordinates shown in Figure 5.

4.3 | Kinematics of the XPM−UCAC4 systematic differences of proper motions

Very often, the stellar kinematics is studied in the frames of the Ogorodnikov–Milne model (du Mont, 1977; Ogorodnikov, 1965), which represents the velocity field by the following expression:

$$\mathbf{V} = \mathbf{V}_0 + \Omega \times \mathbf{r} + \mathbf{M}^+ \mathbf{r}, \quad (19)$$

where $\mathbf{V}$ is the stellar velocity, $\mathbf{V}_0$ is the effect of the translational solar motion, Ω is the angular velocity of rigid-body rotation of the stellar system, and $\mathbf{M}^+$ is the symmetric velocity field deformation tensor.

The Ogorodnikov–Milne model contains 12 parameters:

U, V, W are the components of the velocity vector of translational Solar motion V_0 relative to the stars;

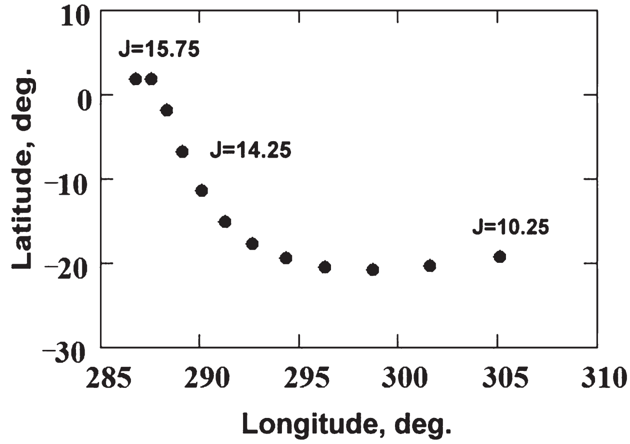

FIGURE 5 J-dependent coordinates of the poles about which the UCAC4 system rotates with respect to XPM_x.

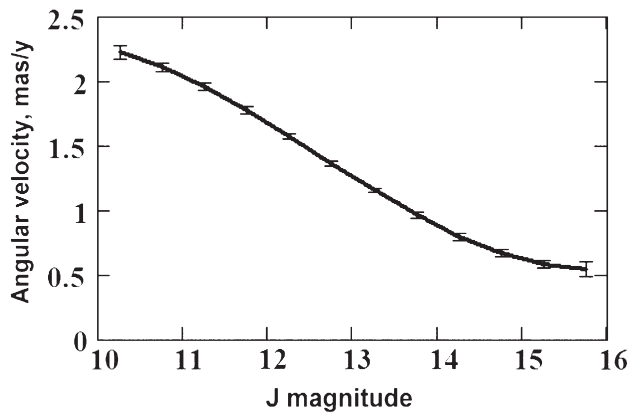

FIGURE 6 Angular velocity of UCAC4 rotation with respect to XPM_p about the poles with J-dependent coordinates shown in Figure 7.

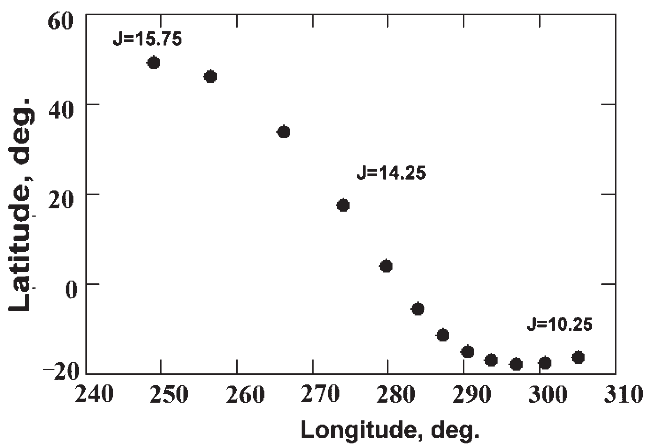

FIGURE 7 J-dependent coordinates of the poles about which the UCAC4 system rotates with respect to XPM_p.

Ω_x, Ω_y, Ω_z are the components of the vector of rigid-body rotation Ω;

M^+_{11}, M^+_{22}, M^+_{33} are the parameters of the tensor $\mathbf{M}^+$ that describes the velocity field contraction–expansion along the principal galactic axes;

M^+_{12}, M^+_{13}, M^+_{23} are the parameters of the tensor $\mathbf{M}^+$ that describes the velocity field deformation in the principal plane and in the two planes perpendicular to it.

Projecting (19) onto the unit vectors of the galactic coordinate system yields (with r standing for the distance to the star and $\mathcal{K} = 4.738$ for converting dimensions mas/year into km/s/kpc):

$$\begin{aligned} \mathcal{K}\mu_l \cos b = {} & U/r\ \sin l - V/r\ \cos l \\ & - \Omega_x \sin b \cos l - \Omega_y \sin b \sin l + \Omega_z \cos b \\ & - M^+_{13} \sin b \sin l + M^+_{23} \sin b \cos l \\ & + M^+_{12} \cos b \cos 2l - \frac{1}{2} M_{11} \cos b \sin 2l \\ & + \frac{1}{2} M^+_{22} \cos b \sin 2l, \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{K}\mu_b = {} & U/r\ \cos l \sin b + V/r\ \sin l \sin b - W/r\ \cos b \\ & + \Omega_x \sin l - \Omega_y \cos l \\ & - \frac{1}{2} M^+_{12} \sin 2b \sin 2l + M^+_{13} \cos 2b \cos l \\ & + M^+_{23} \cos 2b \sin l - \frac{1}{2} M^+_{11} \sin 2b \cos^2 l \\ & - \frac{1}{2} M^+_{22} \sin 2b \sin^2 l + \frac{1}{2} M^+_{33} \sin 2b. \end{aligned} \tag{21}$$

Since there is a linear relationship between the coefficients M^+_{11}, M^+_{22}, and M^+_{33}, the substitutions $M^*_{11} = M^+_{11} - M^+_{22}$ and $M^*_{33} = M^+_{33} - M^+_{22}$ (du Mont, 1977) are often introduced when proper stellar motions are analysed.

In our previous article (Vityazev & Tsvetkov, 2009), the relations connecting the expansion VSH coefficients with the parameters of the Ogorodnikov–Milne model were given. It is obvious that since the Ogorodnikov–Milne equations are linear, the systematic differences of the proper motions may be represented by the same equations with parameters $\Delta U/r$, $\Delta V/r$, ... , ΔM^+_{33} instead of U/r, V/r, ... ,M^+_{33}. In this way, the connections of the systematic difference expansion coefficients with the differences of the kinematic parameters can be found, as shown in Table 2. It should be kept in mind that the components of the solar motion enter into Equations 20 and 21 with the factor $1/r$. When no distances are available, it is common practice to solve our equations assuming $r = 1$. In this case, instead of components U, V, W, we determine the averaged values $\Delta\langle U/r\rangle$, $\Delta\langle V/r\rangle$, and $\Delta\langle W/r\rangle$. Such an approach causes small biases if the stars are taken from narrow interval of distances (or magnitudes). Now, we comment on the term "glide", which for the first time was introduced by Mignard and Klioner (2012). They explain: "From the astronomical point of view, this is a field associated to a motion of the observer toward an apex, with all the stars showing a kinematical stream in the opposite direction." In this connection, it must be said that the standard Ogorodnikov–Milne kinematic model of proper motions always contains the solar motion terms which are nothing but "glide" as was called by Mignard and Klioner (with parameters $G1 = -\langle U/r\rangle, G2 =$

TABLE 2 Connections between the differences of the Ogorodnikov–Milne parameters and the coefficients of the VSH expansion of systematic differences in the proper motions between the two catalogues

VSH coefficients	Ogorodnikov–Milne parameters
t_{101}	$\sqrt{8\pi/3}\ \Delta\omega_3$
t_{110}	$\sqrt{8\pi/3}\,\Delta\omega_2$
t_{111}	$\sqrt{8\pi/3}\,\Delta\omega_1$
s_{101}	$-\sqrt{8\pi/3}\,\Delta\langle W/r\rangle$
s_{110}	$-\sqrt{8\pi/3}\,\Delta\langle V/r\rangle$
s_{111}	$-\sqrt{8\pi/3}\,\Delta\langle U/r\rangle$
s_{201}	$-\sqrt{2\pi/15}\,(\Delta M^+_{11}+\Delta M^+_{22}-2\Delta M^+_{33})$
s_{210}	$\sqrt{8\pi/5}\,\Delta M^+_{23}$
s_{211}	$\sqrt{8\pi/5}\,\Delta M^+_{13}$
s_{220}	$\sqrt{8\pi/5}\,\Delta M^+_{12}$
s_{221}	$\sqrt{2\pi/5}\,(\Delta M^+_{11}-\Delta M^+_{22})$

$-\langle V/r\rangle, G3=-\langle W/r\rangle$). Indeed, the physical meaning of the first-order spheroidal harmonics in terms of the glide components is clarified in Table 2. Within 1 σ precision, the glide parameters for XPM$_x$−UCAC4 and XPM$_p$−UCAC4 turned out to be coincident. Their dependence on the magnitude is shown in Figure 8.

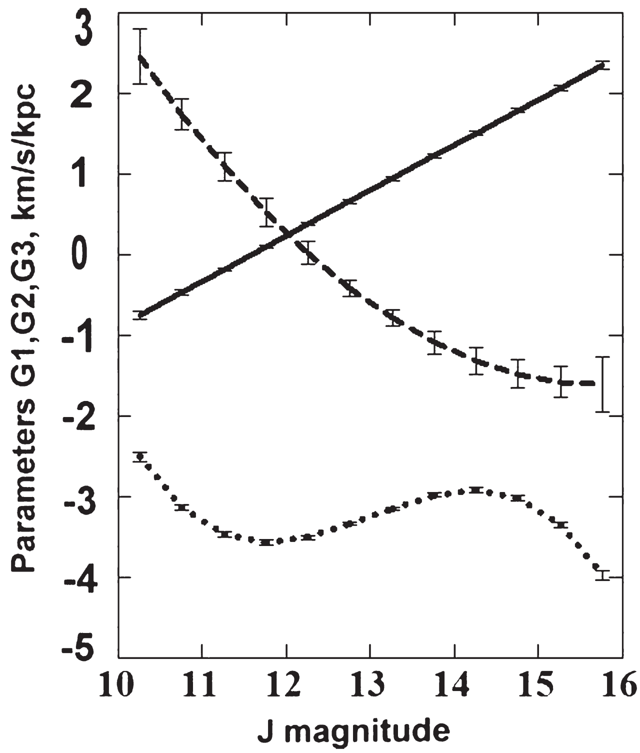

FIGURE 8 Glide parameters G_1 (dashed line), G_2 (points), G_3 (solid line) versus magnitudes for XPM$_p$−UCAC4 and XPM$_x$−UCAC4 differences.

Now, we consider the influence of the XPM−UCAC4 systematic differences of the proper motions on the numerical values of the Oort constants $A = M^+_{12}$ and $B = \omega_3$ derived from each catalogue. Based on the data in Table 2, it can be shown that the differences between the values of the constants Oort ΔA and ΔB obtained in the systems of two different catalogues are connected with the expansion coefficients in the following way:

$$\Delta A(m) = \mathcal{K}\, s_{2,2,0}(m)/\sqrt{8\pi/5}, \tag{22}$$

$$\Delta B(m) = \mathcal{K}\, t_{1,0,1}(m)/\sqrt{8\pi/3}, \tag{23}$$

where

$$s_{2,2,0}(m) = \sum_r s_{2,2,0,r}\, Q_r(\bar{m}), \tag{24}$$

$$t_{1,0,1}(m) = \sum_r t_{1,0,1,r}\, Q_r(\bar{m}). \tag{25}$$

Using these formulae, we calculated the m-dependent differences $\Delta A(m)$ and $\Delta B(m)$ from the VSHL coefficients which correspond to the systematic differences of the proper motions XPM$_x$−UCAC4 and XPM$_p$−UCAC4. The results are shown in Figures 9 and 11. In turn, the m-dependent differences and the sums of these values are shown in Figures 10 and 12. They can be considered as the reductions of A and B and of $A-B$, $A+B$ in the translation them from UCAC4 system to the XPM$_x$ and XPM$_p$ systems. At the same time, as we know, the difference $A-B$ is used to determine the period of the galaxy rotation in the solar neighbourhood, and the sum $A+B$ determines the slope of the rotation curve of the galaxy.

Analysis of these results gives us the following information:

1. In case of XPM$_x$−UCAC4, the difference of the Oort coefficients ΔB ranges from -1.49 ± 0.34 to 3.07 ± 0.34, km/s/kpc, passing through zero value at $J = 13.^m5$. In contrast to the sharp change of the difference ΔB in the transition to fainter stars, the difference of the Oort coefficient ΔA remains practically constant at -2.1 ± 0.2 km/s/kpc in the range of $J = 11^m$ to $J = 15^m$.

2. In case of XPM$_p$−UCAC4, the differences of the Oort coefficients ΔB range from -0.10 ± 0.34 to 4.49 ± 0.34 km/s/kpc, passing through a zero value at $J = 15.^m\,0$. The values ΔB lie in the range of -2.24 ± 0.34 to -0.65 ± 0.24 km/s/kpc.

3. In general, it can be assumed that, with respect to the parameter B, the XPM$_x$ and XPM$_p$ systems are in good agreement with UCAC4 system for faint stars in the range of $J = 13^m - 16^m$ and much worse for the brightest stars in the range of $J = 10^m - 13^m$. The differences of these systems in relation to the Oort parameter A are minimal for the faintest stars at $J = 15^m - 16^m$.

4. It must be said that the dependence of Oort constants A and B and consequently $A-B$ and $A+B$ on magnitude has no physical reason. This is nothing but the consequences of the astrometric measurement techniques when the precision of position determinations depends on brightness of stars. The magnitude equation is inevitable error in photographic catalogues.

Note that the discrepancies between XPM$_x$ and XPM$_p$ affect the results of the kinematic analysis of the proper motions fulfilled in the systems of these catalogues. Really, from Equations 22 and 23 we get $\Delta A = -0.71 \pm 0.02$ km/s/kpc; $\Delta B = -1.11 \pm 0.02$ km/s/kpc. When the

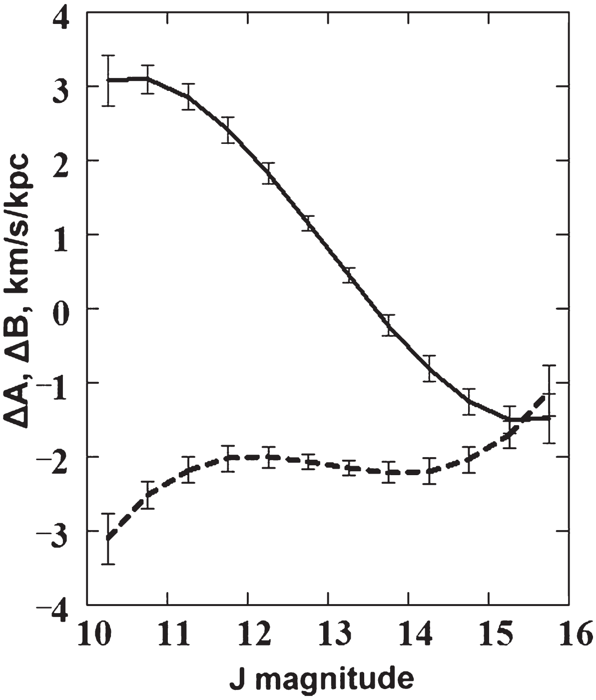

FIGURE 9 *ΔA* (dashed line) and *ΔB* (solid line) versus the magnitudes for XPM_x−UCAC4 differences.

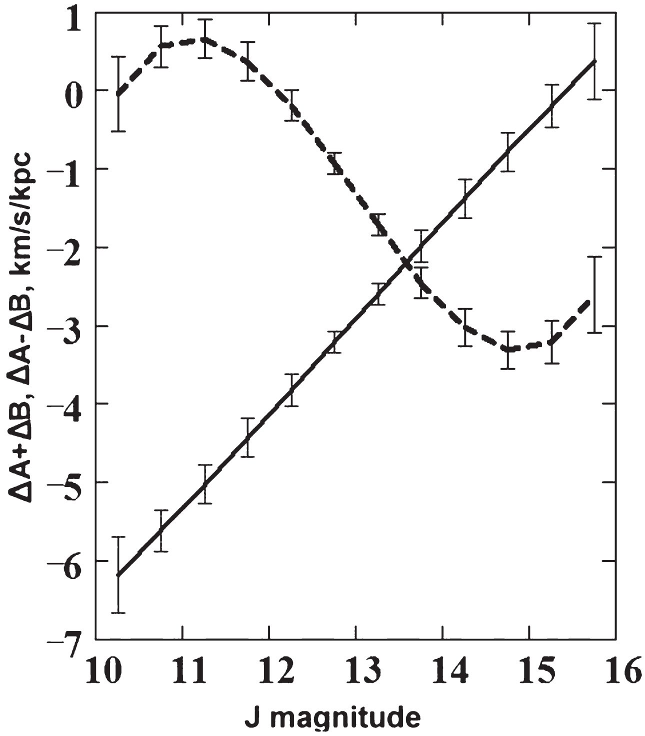

FIGURE 10 ($ΔA + ΔB$) (dashed line) and ($ΔA − ΔB$) (solid line) versus the magnitudes for XPM_x−UCAC4 differences.

UCAC4 is used, the root mean square error in determining the parameters of the Oort constants are at the level of 0.1–0.2 km/s/kpc, which is significantly less than the differences found here. This suggests that the systematic differences in proper motions XPM_x−XPM_p can lead to significant changes in estimates of the Oort constants.

5 | SYSTEMATIC DIFFERENCES IN THE *B* AND *R* PHOTOMETRIC BANDS

Since XPM and UCAC4 are optical catalogues, it is natural to determine the magnitude equation in standard photometric systems *B* (440 nm), *V* (550 nm), or *R* (640 nm). To do this, it is imperative that information about the magnitude for each star in the same photometric band should be taken from the same source. However, the UCAC4 catalogue provides the values of *B*, *V*, and *R* borrowed from the AAVSO Photometric All-Sky Survey (Henden, Levine, Terrell, Smith, & Welch, 2012), and the XPM catalogue gives the values of the *B* and *R* taken from the USNO-A2.0 (Monet, 1998). Thus, for our purposes, the bands *B* and *R* are common only, but, unfortunately, are not homogeneous. The UCAC4 authors write: "UCAC4 is not a photometric catalog", and the PPMXL authors warn: "Magnitudes from USNO-B should be used with care". For this reason, it is dangerous to use such photometry for the identification of stars as well as for deriving the magnitude equation.

In contrast, our catalogues provide the estimates of the brightness of stars borrowed from the 2MASS catalogue. In particular, in both catalogues the magnitudes in the *J* band

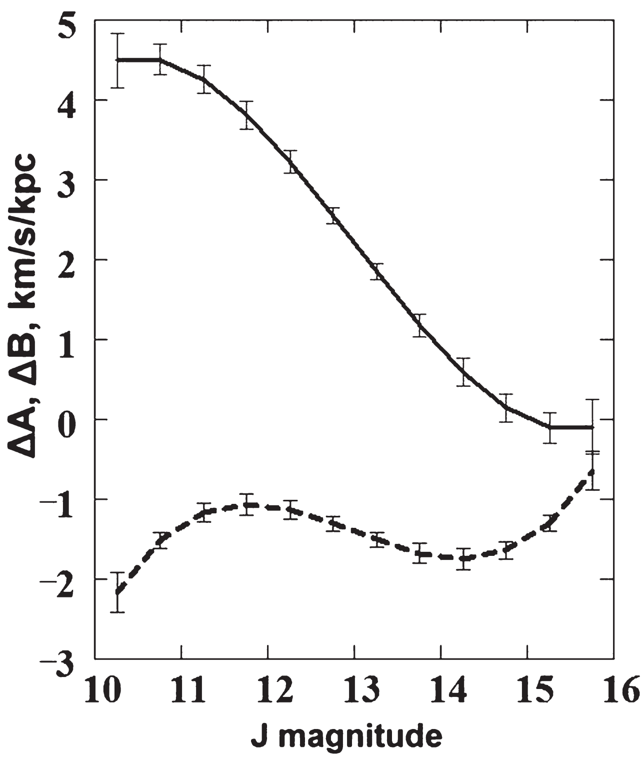

FIGURE 11 *ΔA* (dashed line) and *ΔB* (solid line) versus the magnitudes for XPM_p−UCAC4 differences.

(1280 nm) are available. This justifies the determination of the magnitude equation we have derived in this band. In order to apply the results in the *B* and *R* bands, one must have the conversion formulae between the various magnitude bands. We obtained them from the data in each catalogue.

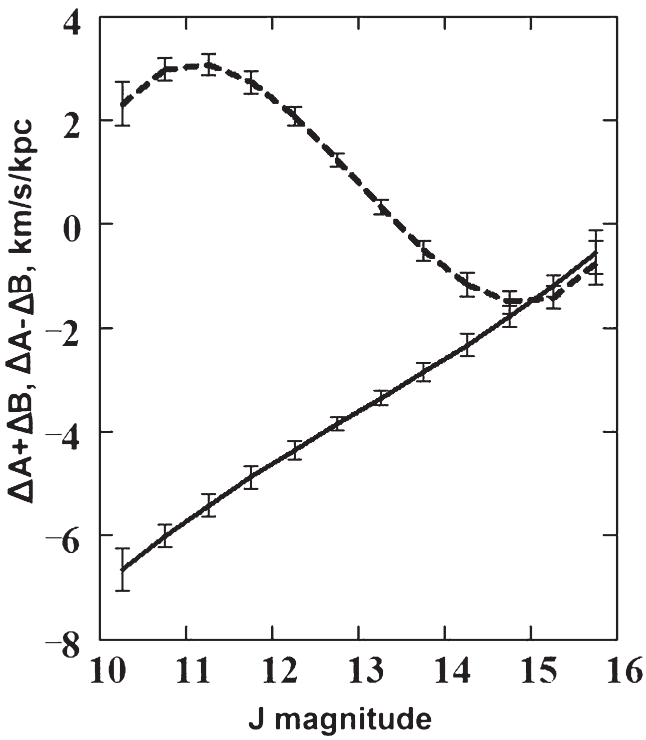

FIGURE 12 $(\Delta A + \Delta B)$ (dashed line) and $(\Delta A - \Delta B)$ (solid line) versus the magnitudes for XPM_p−UCAC4 differences.

For this purpose, we selected the stars with the available J, B, and R magnitudes, and for all the stars in narrow (0.1^m) zones of J magnitudes, the average values of B and R as well as their standard deviations from the mean values were found. After that, the mean values obtained were smoothed by the moving five-point averaging filter, and from the resulting series, the values over 0.5^m intervals were extracted. These were approximated by linear functions, which allowed us to link the magnitudes in the B, R, and J bands.

The equation derived from the UCAC4 turned out to be

$$B = (0.726 \pm 0.015)J + (6.300 \pm 0.190),\quad 10.25 \leq J \leq 15.75; \tag{26}$$

$$J = (1.378 \pm 0.028)B - (8.680 \pm 0.316),\quad 13.74 \leq B \leq 17.74; \tag{27}$$

$$R = (0.833 \pm 0.011)J + (3.725 \pm 0.144),\quad 10.25 \leq J \leq 15.75; \tag{28}$$

$$J = (1.200 \pm 0.016)R - (4.471 \pm 0.183),\quad 12.26 \leq R \leq 16.85. \tag{29}$$

The analogous equations for the catalogue XPM look as follows:

$$B = (0.778 \pm 0.005)J + (6.233 \pm 0.063),\quad 10.25 \leq J \leq 15.75; \tag{30}$$

$$J = (1.285 \pm 0.008)B - (8.011 \pm 0.094),\quad 14.21 \leq B \leq 18.49; \tag{31}$$

$$R = (0.896 \pm 0.005)J + (2.889 \pm 0.068),\quad 10.25 \leq J \leq 15.75; \tag{32}$$

$$J = (1.117 \pm 0.006)R - (3.226 \pm 0.178),\quad 12.07 \leq R \leq 17.00. \tag{33}$$

Thus, to reduce the UCAC4 positions and proper motions into system of the XPM catalogue on the B or R bands, one must determine the corresponding J magnitudes using formulae (27) and (29), and substitute the received J values in Equation 3. Obviously, when solving the inverse problem, that is, transition from XPM to UCAC4 system, it is necessary to calculate the J from Equations 31 and 33.

6 | CONCLUSIONS

As mentioned above, we have already carried out the comparison of the catalogues XPM and UCAC4 in the equatorial coordinate system (Vityazev & Tsvetkov, 2015b). For this reason, it is appropriate to state explicitly the correlation and difference with current work. In fact, the papers are correlated because similar mathematical methods of analysis based on combination of VSH and Legendre polynomials have been used. At the same time, the papers are different because, in contrast to metrological interpretation of the VSH coefficients (orientation and rotation), in case of equatorial coordinate system the kinematical interpretation of the VSH coefficients (glide parameters, Oort constants) was done in case of galactic coordinate system.

In the present work, for the first time, the systematic differences between the coordinates and the proper motions of the XPM and UCAC4 catalogues in the Galactic coordinate system were obtained. The representation of the systematic differences by vector spherical harmonics with the magnitude equation taken into consideration was made for the purpose. The detection of significant harmonics was made at the level exceeding 97.7%.

Analysis of systematic differences in the galactic coordinates allows us to find the discrepancies between the galactic coordinate frames, which are implemented by various catalogues. In this connection, we estimated that the orientation angles of the XPM and UCAC4 galactic frames did not exceed 10 mas, while the new versions of the galactic coordinate system are believed to deviate from the standard version introduced by IAU in 1958 at few minutes of arc (Liu, Zhu, & Hu, 2011; Liu, Zhu, & Zhang, 2011). For this reason, it can be said that the differences of the XPM and UCAC4 galactic coordinate frames are negligible compared to the precision of the current galactic coordinate system.

Catalogue XPM has two features. First, it is based on the galaxies and does not use a reference system implemented on quasars. Second, the XPM catalogue provides two systems of proper motions—XPM_x and XPM_p—derived from two catalogues XSC and the PSC of the 2MASS project. For this reason, we made the expansion of the XPM_x−XPM_p, XPM_x−UCAC4, and XPM_p−UCAC4 systematic differences on orthogonal functions.

Upon close examinations of the differences $XPM_x - XPM_p$, we found that they are free from the magnitude equation and do not show a distinct dependence on the galactic latitude. In addition, we have found that the systems XPM_x and XPM_p are in mutual rotation around the pole the positions of which is close to the north pole of the equatorial coordinate system. This suggests that the differences of coordinates of galaxies in the catalogues XSC and the PSC are caused not by the effects of the external galaxies avoidance along the galactic plane that would manifest themselves in the galactic coordinate system, but by the specifics of the project 2MASS methods of measurement in the equatorial coordinate system.

Study of systematic differences XPM_x−UCAC4 and XPM_p−UCAC4 shows that XPM_x and XPM_p systems rotate relative to the UCAC4 system. The rotation speed changes in different groups of magnitudes. In our opinion, the high rate of both frames XPM_x and XPM_p relative to UCAC4 is caused by the general transition from the combined “quasar-galaxy" to the pure “galactic" reference system.

Besides this, it was shown that the systematic differences between the proper motions help us to see how different the kinematical parameters derived from the catalogues under consideration can be. In particular, the differences $XPM_x - XPM_p$ with respect to the Oort constant A produce the least effect for the faintest stars $J = 15^m - 16^m$. On the other hand, with respect to Oort constant B, the XPM_x and XPM_p systems are in good agreement with UCAC4 system for faint stars in the range $J = 13^m$–16^m and in much worse agreement for the bright stars in the range of $J = 10^m$–13^m.

The results obtained in the study of systematic differences of positions and proper motions of the XPM and UCAC4 catalogues in the galactic and equatorial coordinate systems (Vityazev & Tsvetkov, 2015b) show that the transition from a quasi-inertial frame of reference, based on quasars and galaxies (UCAC4), to the reference system based on the galaxies (XPM) leads to noticeable systematic differences of coordinates and proper motions of these two catalogues.

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APPENDIX: COEFFICIENTS OF THE VSHL SYSTEMATIC DIFFERENCE DECOMPOSITIONS

TABLE A1 The values of toroidal t_{nkpr} coefficients in the VSHL expansion of the XPM−UCAC4 field $\Delta l \cos b\, \mathbf{e}_l + \Delta b\, \mathbf{e}_b$

Coeff.	Value (mas)	Coeff.	Value (mas)	Coeff.	Value (mas)
$t_{1,0,1,0}$	−26.75 ± 0.65	$t_{3,3,0,1}$	−3.44 ± 0.76	$t_{5,5,0,0}$	12.97 ± 0.65
$t_{1,1,0,0}$	−22.86 ± 0.69	$t_{3,3,0,2}$	1.71 ± 0.75	$t_{5,5,0,1}$	−4.83 ± 0.76
$t_{1,1,0,1}$	−1.30 ± 0.76	$t_{3,3,1,0}$	8.79 ± 0.66	$t_{6,3,0,0}$	5.41 ± 0.72
$t_{1,1,0,2}$	2.81 ± 0.75	$t_{4,3,1,0}$	6.86 ± 0.65	$t_{6,3,0,1}$	−1.93 ± 0.79
$t_{1,1,1,0}$	18.71 ± 0.66	$t_{4,3,1,1}$	−4.10 ± 0.76	$t_{6,3,0,2}$	2.24 ± 0.78
$t_{2,0,1,0}$	−11.05 ± 0.65	$t_{5,0,1,0}$	−9.27 ± 0.69	$t_{6,5,0,0}$	3.71 ± 0.65
$t_{2,1,0,0}$	27.92 ± 0.69	$t_{5,0,1,1}$	6.08 ± 0.76	$t_{6,5,0,1}$	−4.97 ± 0.76
$t_{2,1,0,1}$	2.18 ± 0.90	$t_{5,0,1,2}$	−1.82 ± 0.75	$t_{6,6,0,0}$	−5.66 ± 0.69
$t_{2,1,0,2}$	−4.63 ± 0.75	$t_{5,1,0,0}$	13.97 ± 0.71	$t_{6,6,0,1}$	5.46 ± 0.76
$t_{2,1,0,3}$	−2.67 ± 0.79	$t_{5,1,0,1}$	−5.58 ± 0.82	$t_{6,6,0,2}$	−2.40 ± 0.75
$t_{2,1,1,0}$	−18.42 ± 0.65	$t_{5,1,1,0}$	−6.21 ± 0.76	$t_{8,4,1,0}$	4.69 ± 0.65
$t_{2,1,1,1}$	−2.36 ± 0.76	$t_{5,2,0,0}$	−10.70 ± 0.74	$t_{8,6,1,0}$	7.46 ± 0.65
$t_{2,2,0,0}$	−35.89 ± 0.69	$t_{5,2,0,1}$	6.19 ± 0.86	$t_{8,6,1,1}$	−4.14 ± 0.76
$t_{2,2,0,2}$	2.15 ± 0.75	$t_{5,2,1,0}$	−7.66 ± 0.69	$t_{8,7,1,0}$	−6.30 ± 0.65
$t_{2,2,1,0}$	−16.97 ± 0.66	$t_{5,2,1,1}$	4.66 ± 0.76	$t_{8,7,1,1}$	2.15 ± 0.76
$t_{2,2,1,1}$	2.42 ± 0.76	$t_{5,2,1,2}$	−2.01 ± 0.75	$t_{8,8,0,0}$	−6.70 ± 0.69
$t_{3,0,1,0}$	−9.22 ± 0.65	$t_{5,3,1,0}$	−9.78 ± 0.66	$t_{8,8,0,1}$	2.07 ± 0.76
$t_{3,0,1,1}$	2.86 ± 0.76	$t_{5,3,1,1}$	3.10 ± 0.76	$t_{8,8,0,2}$	−1.58 ± 0.75
$t_{3,2,0,0}$	−14.00 ± 0.66	$t_{5,4,0,0}$	−16.43 ± 0.69	$t_{8,8,1,0}$	7.85 ± 0.65
$t_{3,2,0,1}$	3.50 ± 0.77	$t_{5,4,0,1}$	7.58 ± 0.76	$t_{8,8,1,1}$	−2.72 ± 0.76
$t_{3,2,1,1}$	−6.31 ± 0.76	$t_{5,4,0,2}$	−2.88 ± 0.75	$t_{17,17,1,0}$	6.99 ± 0.65
$t_{3,2,1,2}$	2.52 ± 0.72	$t_{5,4,1,0}$	10.46 ± 0.65		
$t_{3,3,0,0}$	6.98 ± 0.69	$t_{5,4,1,1}$	−1.69 ± 0.76		

TABLE A2 Values of spheroidal s_{nkpr} coefficients in the VSHL expansion of the XPM−UCAC4 field $\Delta l \cos b\, \mathbf{e}_l + \Delta b\, \mathbf{e}_b$

Coeff.	Value (mas)	Coeff.	Value (mas)	Coeff.	Value (mas)
$s_{1,0,1,0}$	11.01 ± 0.65	$s_{3,2,1,1}$	−9.43 ± 0.76	$s_{8,1,0,0}$	5.39 ± 0.72
$s_{1,0,1,1}$	−4.86 ± 0.76	$s_{3,2,1,2}$	4.74 ± 0.75	$s_{8,1,0,1}$	−4.23 ± 0.83
$s_{1,1,0,0}$	9.42 ± 0.69	$s_{3,3,1,0}$	−3.65 ± 0.69	$s_{8,1,1,0}$	−4.26 ± 0.73
$s_{1,1,0,1}$	12.88 ± 0.76	$s_{3,3,1,1}$	15.57 ± 0.76	$s_{8,1,1,1}$	4.01 ± 0.80
$s_{1,1,0,2}$	−11.14 ± 0.75	$s_{3,3,1,2}$	−7.50 ± 0.75	$s_{8,1,1,2}$	−2.15 ± 0.79
$s_{1,1,1,0}$	−9.60 ± 0.70	$s_{4,1,0,0}$	4.06 ± 0.73	$s_{8,6,1,0}$	3.51 ± 0.69
$s_{1,1,1,1}$	−5.05 ± 0.76	$s_{4,1,0,1}$	−7.39 ± 0.84	$s_{8,6,1,1}$	−5.70 ± 0.76
$s_{1,1,1,2}$	4.77 ± 0.75	$s_{4,4,1,0}$	6.05 ± 0.65	$s_{8,6,1,2}$	1.57 ± 0.75
$s_{2,1,0,0}$	13.06 ± 0.70	$s_{5,0,1,0}$	−3.95 ± 0.69	$s_{9,0,1,0}$	−4.91 ± 0.65
$s_{2,1,0,1}$	−7.32 ± 0.77	$s_{5,0,1,2}$	2.10 ± 0.75	$s_{9,2,1,0}$	4.14 ± 0.65
$s_{2,1,0,2}$	2.44 ± 0.76	$s_{5,4,0,0}$	9.08 ± 0.65	$s_{9,2,1,1}$	−4.52 ± 0.76
$s_{2,1,1,0}$	−11.27 ± 0.66	$s_{5,4,0,1}$	−3.76 ± 0.76	$s_{10,10,0,0}$	6.75 ± 0.65
$s_{2,1,1,1}$	5.82 ± 0.76	$s_{6,1,0,0}$	−8.13 ± 0.80	$s_{10,10,0,1}$	−2.52 ± 0.76
$s_{2,2,0,0}$	−9.59 ± 0.65	$s_{6,1,0,1}$	3.92 ± 0.92	$s_{11,0,1,0}$	4.26 ± 0.65
$s_{2,2,0,1}$	8.55 ± 0.76	$s_{6,2,0,0}$	6.84 ± 0.69	$s_{13,5,1,0}$	5.70 ± 0.69
$s_{2,2,1,0}$	−7.80 ± 0.66	$s_{6,2,0,1}$	−5.07 ± 0.76	$s_{13,5,1,1}$	−4.37 ± 0.76
$s_{2,2,1,1}$	2.91 ± 0.76	$s_{6,2,0,2}$	1.86 ± 0.75	$s_{13,5,1,2}$	2.23 ± 0.75
$s_{3,0,1,0}$	12.85 ± 0.69	$s_{7,0,1,0}$	3.54 ± 0.69	$s_{21,13,0,0}$	−6.73 ± 0.65
$s_{3,0,1,1}$	−12.05 ± 0.76	$s_{7,0,1,1}$	3.88 ± 0.76	$s_{21,13,0,1}$	3.77 ± 0.76
$s_{3,0,1,2}$	7.61 ± 0.75	$s_{7,0,1,2}$	−2.61 ± 0.75	$s_{55,38,0,0}$	−4.58 ± 0.70
$s_{3,1,0,0}$	−5.69 ± 0.65	$s_{7,1,1,0}$	2.64 ± 0.69	$s_{55,38,0,1}$	3.85 ± 0.77
$s_{3,1,0,1}$	4.60 ± 0.76	$s_{7,1,1,1}$	−6.24 ± 0.76	$s_{55,38,0,2}$	−2.12 ± 0.76
$s_{3,2,0,0}$	8.78 ± 0.70	$s_{7,1,1,2}$	3.01 ± 0.75	$s_{56,39,1,0}$	2.51 ± 0.78
$s_{3,2,0,1}$	−16.65 ± 0.77	$s_{7,6,1,0}$	−3.03 ± 0.69	$s_{56,39,1,1}$	2.35 ± 0.86
$s_{3,2,0,2}$	10.83 ± 0.76	$s_{7,6,1,1}$	7.15 ± 0.76	$s_{56,39,1,2}$	−2.40 ± 0.84
$s_{3,2,1,0}$	2.70 ± 0.69	$s_{7,6,1,2}$	−3.89 ± 0.75		

TABLE A3 Values of toroidal t_{nkpr} coefficients in the VSHL expansion of the XPM_x−UCAC4 field $\Delta\mu_l \cos b\, \mathbf{e}_l + \Delta\mu_b\, \mathbf{e}_b$

Coeff. t_{nkpr}	Value (mas/year)	Coeff. t_{nkpr}	Value (mas/year)	Coeff. t_{nkpr}	Value (mas/year)
$t_{1,0,1,0}$	0.53 ± 0.06	$t_{3,0,1,0}$	−0.15 ± 0.06	$t_{5,1,1,1}$	−0.23 ± 0.06
$t_{1,0,1,1}$	−1.13 ± 0.08	$t_{3,0,1,1}$	0.51 ± 0.06	$t_{5,2,0,0}$	−0.45 ± 0.06
$t_{1,0,1,3}$	0.16 ± 0.07	$t_{3,1,0,0}$	−0.31 ± 0.06	$t_{5,2,0,1}$	0.48 ± 0.06
$t_{1,1,0,0}$	3.26 ± 0.06	$t_{3,1,0,1}$	−0.21 ± 0.06	$t_{5,2,1,0}$	−0.81 ± 0.07
$t_{1,1,0,1}$	−1.68 ± 0.08	$t_{3,1,0,2}$	0.18 ± 0.06	$t_{5,2,1,1}$	0.45 ± 0.07
$t_{1,1,0,3}$	0.17 ± 0.07	$t_{3,2,0,0}$	−0.98 ± 0.06	$t_{5,2,1,2}$	−0.25 ± 0.07
$t_{1,1,1,0}$	−1.21 ± 0.06	$t_{3,2,0,1}$	0.41 ± 0.06	$t_{5,4,0,0}$	−0.78 ± 0.06
$t_{1,1,1,1}$	1.24 ± 0.06	$t_{3,3,0,0}$	0.77 ± 0.06	$t_{5,4,0,1}$	0.27 ± 0.06
$t_{1,1,1,2}$	−0.28 ± 0.06	$t_{3,3,0,1}$	−0.16 ± 0.06	$t_{5,4,0,2}$	−0.20 ± 0.06
$t_{2,0,1,0}$	−0.63 ± 0.06	$t_{3,3,1,0}$	0.53 ± 0.06	$t_{6,1,0,0}$	0.66 ± 0.06
$t_{2,0,1,1}$	0.19 ± 0.06	$t_{3,3,1,2}$	−0.13 ± 0.06	$t_{6,3,1,0}$	−0.60 ± 0.07
$t_{2,1,0,0}$	0.71 ± 0.06	$t_{4,1,0,0}$	−0.76 ± 0.06	$t_{6,3,1,1}$	0.22 ± 0.08
$t_{2,1,0,1}$	−0.45 ± 0.06	$t_{4,1,0,1}$	0.55 ± 0.07	$t_{7,2,1,0}$	0.30 ± 0.07
$t_{2,1,0,2}$	−0.20 ± 0.06	$t_{4,2,0,0}$	0.54 ± 0.06	$t_{7,2,1,1}$	0.32 ± 0.08
$t_{2,2,0,0}$	−0.72 ± 0.06	$t_{4,3,1,0}$	1.01 ± 0.06	$t_{8,5,1,0}$	−0.58 ± 0.06
$t_{2,2,0,1}$	0.38 ± 0.06	$t_{4,3,1,1}$	−0.50 ± 0.06	$t_{8,5,1,1}$	0.19 ± 0.06
$t_{2,2,0,2}$	0.20 ± 0.06	$t_{5,0,1,0}$	−0.80 ± 0.06	$t_{9,0,1,0}$	−0.51 ± 0.06
$t_{2,2,1,0}$	−0.78 ± 0.06	$t_{5,0,1,1}$	0.39 ± 0.06	$t_{14,12,0,0}$	−0.67 ± 0.06
$t_{2,2,1,1}$	0.42 ± 0.06	$t_{5,0,1,2}$	−0.13 ± 0.06	$t_{25,22,1,0}$	0.53 ± 0.06
$t_{2,2,1,2}$	−0.17 ± 0.06	$t_{5,1,1,0}$	0.52 ± 0.06	$t_{52,39,0,0}$	0.63 ± 0.06

TABLE A4 Values of spheroidal s_{nkpr} coefficients in the VSHL expansion of the XPM$_x$−UCAC4 field $\Delta\mu_l \cos b\, \mathbf{e}_l + \Delta\mu_b\, \mathbf{e}_b$

Coeff.	Value (mas/year)	Coeff.	Value (mas/year)	Coeff.	Value (mas/year)
$s_{1,0,1,0}$	−0.28 ± 0.06	$s_{3,2,1,0}$	1.38 ± 0.06	$s_{7,5,1,0}$	−0.55 ± 0.06
$s_{1,0,1,1}$	−1.00 ± 0.06	$s_{3,2,1,1}$	−0.31 ± 0.06	$s_{8,6,1,0}$	0.60 ± 0.06
$s_{1,0,1,2}$	0.27 ± 0.06	$s_{3,2,1,2}$	0.15 ± 0.06	$s_{8,6,1,1}$	−0.36 ± 0.06
$s_{1,1,0,0}$	−2.38 ± 0.06	$s_{3,3,1,0}$	−2.18 ± 0.06	$s_{9,7,1,0}$	0.75 ± 0.06
$s_{1,1,0,3}$	−0.23 ± 0.06	$s_{3,3,1,1}$	0.54 ± 0.06	$s_{10,0,1,0}$	−0.56 ± 0.06
$s_{1,1,1,0}$	0.33 ± 0.06	$s_{3,3,1,2}$	−0.13 ± 0.06	$s_{10,0,1,1}$	0.21 ± 0.06
$s_{1,1,1,1}$	0.77 ± 0.06	$s_{4,0,1,0}$	−0.98 ± 0.06	$s_{10,3,1,0}$	0.53 ± 0.06
$s_{2,0,1,0}$	−0.62 ± 0.06	$s_{4,1,0,0}$	−0.50 ± 0.06	$s_{10,6,1,0}$	−0.43 ± 0.06
$s_{2,1,0,0}$	0.94 ± 0.06	$s_{4,1,0,1}$	−0.34 ± 0.06	$s_{10,8,0,0}$	−0.71 ± 0.06
$s_{2,1,0,1}$	−0.18 ± 0.08	$s_{4,1,0,2}$	0.20 ± 0.06	$s_{10,10,0,0}$	0.83 ± 0.06
$s_{2,1,0,3}$	−0.18 ± 0.07	$s_{4,2,0,0}$	−0.89 ± 0.06	$s_{13,5,1,0}$	0.40 ± 0.06
$s_{2,1,1,0}$	−0.83 ± 0.06	$s_{4,2,0,1}$	0.22 ± 0.06	$s_{13,5,1,1}$	−0.39 ± 0.06
$s_{2,1,1,1}$	−0.30 ± 0.06	$s_{4,2,0,2}$	−0.14 ± 0.06	$s_{13,5,1,2}$	0.18 ± 0.06
$s_{2,2,0,0}$	−1.42 ± 0.06	$s_{4,3,1,0}$	1.10 ± 0.06	$s_{13,12,0,0}$	−0.48 ± 0.06
$s_{2,2,0,1}$	0.15 ± 0.08	$s_{4,4,0,0}$	0.88 ± 0.06	$s_{13,13,1,0}$	0.60 ± 0.06
$s_{2,2,0,3}$	0.15 ± 0.07	$s_{5,1,0,0}$	0.74 ± 0.06	$s_{20,12,1,0}$	0.38 ± 0.06
$s_{3,0,1,0}$	1.73 ± 0.06	$s_{5,1,0,1}$	−0.27 ± 0.07	$s_{20,12,1,1}$	−0.33 ± 0.06
$s_{3,0,1,1}$	−0.22 ± 0.06	$s_{5,1,1,0}$	−0.61 ± 0.06	$s_{20,12,1,2}$	0.19 ± 0.06
$s_{3,0,1,2}$	0.37 ± 0.06	$s_{5,4,0,0}$	−0.37 ± 0.06	$s_{52,39,0,0}$	−0.47 ± 0.06
$s_{3,1,0,0}$	−0.69 ± 0.06	$s_{5,4,0,1}$	−0.29 ± 0.06	$s_{53,38,0,0}$	0.74 ± 0.07
$s_{3,1,0,1}$	0.47 ± 0.06	$s_{6,6,1,0}$	−0.51 ± 0.06	$s_{55,38,0,0}$	−0.44 ± 0.06
$s_{3,2,0,0}$	2.44 ± 0.06	$s_{7,1,1,0}$	0.38 ± 0.06	$s_{55,38,0,1}$	0.28 ± 0.07
$s_{3,2,0,1}$	−0.87 ± 0.07	$s_{7,1,1,1}$	−0.39 ± 0.06	$s_{55,38,0,2}$	−0.17 ± 0.06
$s_{3,2,0,2}$	0.32 ± 0.06	$s_{7,1,1,2}$	0.15 ± 0.06		

TABLE A5 Values of toroidal t_{nkpr} coefficients in the VSHL expansion of the XPM$_p$−UCAC4 field $\Delta\mu_l \cos b\, \mathbf{e}_l + \Delta\mu_b\, \mathbf{e}_b$

Coeff.	Value (mas/year)	Coeff.	Value (mas/year)	Coeff.	Value (mas/year)
$t_{1,0,1,0}$	1.46 ± 0.06	$t_{3,0,1,1}$	0.51 ± 0.06	$t_{5,2,1,0}$	−0.81 ± 0.07
$t_{1,0,1,1}$	−1.13 ± 0.08	$t_{3,2,0,0}$	−1.41 ± 0.06	$t_{5,2,1,1}$	0.45 ± 0.07
$t_{1,0,1,3}$	0.16 ± 0.07	$t_{3,2,0,1}$	0.41 ± 0.06	$t_{5,2,1,2}$	−0.25 ± 0.07
$t_{1,1,0,0}$	4.57 ± 0.06	$t_{3,3,0,0}$	0.71 ± 0.06	$t_{5,4,0,0}$	−0.78 ± 0.06
$t_{1,1,0,1}$	−1.67 ± 0.08	$t_{3,3,0,1}$	−0.16 ± 0.07	$t_{5,4,0,1}$	0.27 ± 0.06
$t_{1,1,0,3}$	0.17 ± 0.07	$t_{3,3,1,0}$	0.58 ± 0.06	$t_{5,4,0,2}$	−0.20 ± 0.06
$t_{1,1,1,0}$	−2.15 ± 0.06	$t_{4,1,0,0}$	−0.89 ± 0.07	$t_{6,1,0,0}$	0.84 ± 0.07
$t_{1,1,1,1}$	1.24 ± 0.06	$t_{4,1,0,1}$	0.68 ± 0.07	$t_{6,1,0,1}$	−0.27 ± 0.07
$t_{1,1,1,2}$	−0.28 ± 0.06	$t_{4,2,0,0}$	0.65 ± 0.06	$t_{7,2,1,0}$	0.30 ± 0.07
$t_{2,1,0,0}$	−0.94 ± 0.06	$t_{4,3,1,0}$	0.93 ± 0.06	$t_{7,2,1,1}$	0.32 ± 0.08
$t_{2,1,0,1}$	−0.43 ± 0.07	$t_{4,3,1,1}$	−0.50 ± 0.07	$t_{8,5,1,0}$	−0.64 ± 0.06
$t_{2,1,0,2}$	−0.21 ± 0.06	$t_{5,0,1,0}$	−0.69 ± 0.06	$t_{8,5,1,1}$	0.20 ± 0.06
$t_{2,1,1,0}$	0.96 ± 0.06	$t_{5,0,1,1}$	0.39 ± 0.06	$t_{9,0,1,0}$	−0.52 ± 0.06
$t_{2,1,1,1}$	−0.19 ± 0.07	$t_{5,0,1,2}$	−0.13 ± 0.06	$t_{14,12,0,0}$	−0.68 ± 0.06
$t_{2,2,0,0}$	1.03 ± 0.06	$t_{5,1,1,0}$	0.62 ± 0.06	$t_{19,10,1,0}$	−0.47 ± 0.06
$t_{2,2,0,1}$	0.38 ± 0.06	$t_{5,1,1,1}$	−0.23 ± 0.07	$t_{25,22,1,0}$	0.57 ± 0.06
$t_{2,2,0,2}$	0.20 ± 0.06	$t_{5,2,0,0}$	−0.42 ± 0.06	$t_{52,39,0,0}$	0.70 ± 0.06
$t_{3,0,1,0}$	−0.47 ± 0.06	$t_{5,2,0,1}$	0.48 ± 0.06		

TABLE A6 Values of spheroidal s_{nkpr} coefficients in the VSHL expansion of the XPM$_p$−UCAC4 field $\Delta\mu_l \cos b\, \mathbf{e}_l + \Delta\mu_b\, \mathbf{e}_b$

Coeff.	Value (mas/year)	Coeff.	Value (mas/year)	Coeff.	Value (mas/year)
$s_{1,0,1,0}$	−0.24 ± 0.06	$s_{3,2,1,0}$	1.60 ± 0.06	$s_{5,4,0,1}$	−0.29 ± 0.06
$s_{1,0,1,1}$	−1.01 ± 0.06	$s_{3,2,1,1}$	−0.31 ± 0.06	$s_{8,1,1,0}$	−0.51 ± 0.06
$s_{1,0,1,2}$	0.27 ± 0.06	$s_{3,2,1,2}$	0.15 ± 0.06	$s_{8,1,1,1}$	0.38 ± 0.07
$s_{1,1,0,0}$	−2.80 ± 0.06	$s_{3,3,1,0}$	−2.43 ± 0.06	$s_{8,1,1,2}$	−0.13 ± 0.06
$s_{1,1,0,3}$	−0.24 ± 0.06	$s_{3,3,1,1}$	0.55 ± 0.07	$s_{8,6,1,0}$	0.59 ± 0.06
$s_{1,1,1,0}$	0.69 ± 0.06	$s_{3,3,1,2}$	−0.15 ± 0.06	$s_{8,6,1,1}$	−0.36 ± 0.06
$s_{1,1,1,1}$	0.77 ± 0.07	$s_{4,0,1,0}$	−0.96 ± 0.06	$s_{9,7,1,0}$	0.75 ± 0.06
$s_{2,1,0,0}$	0.66 ± 0.06	$s_{4,1,0,0}$	−0.50 ± 0.06	$s_{10,0,1,0}$	−0.60 ± 0.06
$s_{2,1,0,1}$	−0.19 ± 0.08	$s_{4,1,0,1}$	−0.34 ± 0.07	$s_{10,0,1,1}$	0.22 ± 0.06
$s_{2,1,0,3}$	−0.18 ± 0.07	$s_{4,1,0,2}$	0.21 ± 0.06	$s_{10,6,1,0}$	−0.45 ± 0.06
$s_{2,1,1,0}$	−0.41 ± 0.06	$s_{4,2,0,0}$	−0.88 ± 0.06	$s_{10,8,0,0}$	−0.60 ± 0.06
$s_{2,1,1,1}$	−0.30 ± 0.07	$s_{4,2,0,1}$	0.23 ± 0.06	$s_{10,10,0,0}$	0.76 ± 0.06
$s_{2,2,0,0}$	−0.94 ± 0.06	$s_{4,2,0,2}$	−0.13 ± 0.06	$s_{50,39,0,0}$	0.36 ± 0.06
$s_{2,2,0,3}$	0.19 ± 0.06	$s_{4,2,1,0}$	−0.57 ± 0.06	$s_{50,39,0,1}$	0.21 ± 0.07
$s_{3,0,1,0}$	1.62 ± 0.06	$s_{4,2,1,1}$	0.25 ± 0.06	$s_{52,39,0,0}$	−0.46 ± 0.06
$s_{3,0,1,1}$	−0.21 ± 0.06	$s_{4,2,1,2}$	−0.19 ± 0.06	$s_{53,38,0,0}$	0.71 ± 0.08
$s_{3,0,1,2}$	0.38 ± 0.06	$s_{4,3,1,0}$	1.10 ± 0.06	$s_{55,37,0,0}$	0.62 ± 0.06
$s_{3,1,0,0}$	−0.85 ± 0.06	$s_{4,4,0,0}$	0.83 ± 0.06	$s_{55,38,0,0}$	−0.43 ± 0.07
$s_{3,1,0,1}$	0.47 ± 0.07	$s_{5,1,0,0}$	0.71 ± 0.06	$s_{55,38,0,1}$	0.31 ± 0.07
$s_{3,2,0,0}$	2.60 ± 0.06	$s_{5,1,0,1}$	−0.25 ± 0.07	$s_{55,38,0,2}$	−0.18 ± 0.07
$s_{3,2,0,1}$	−0.86 ± 0.07	$s_{5,3,1,0}$	−0.68 ± 0.08		
$s_{3,2,0,2}$	0.32 ± 0.06	$s_{5,4,0,0}$	−0.51 ± 0.06		