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THE VERTICAL GRADIENTS IN THE GALACTIC ROTATION DERIVED FROM THE PROPER MOTIONS OF THE UCAC4, PPMXL AND PPM CATALOGUES

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Статья посвящена кинематическому анализу собственных движений звезд массовых астрометрических каталогов UCAC4, PPMXL и XPM, содержащих положения, собственные движения и фотометрию сотен миллионов звезд до 17^m. В противоположность традиционному методу кинематического анализа собственных движений звезд, основанному на оценивании параметров кинематических моделей методом наименьших квадратов, наш подход основан на разложении собственных движений в ряды по векторным сферическим функциям. В силу полноты системы базисных функций это позволяет произвести полный анализ кинематических компонентов в наблюдаемых данных. Обилие звезд в современных каталогах позволяет проводить кинематические исследования в отдельных подсистемах Галактики, в частности, в ее северном и южном полушариях. С этой целью была построена система векторных сферических функций, ортонормированных на произвольной зоне сферы. Такой прием позволил избежать значительных корреляций между неизвестными параметрами при выполнении раздельных решений по полушариям. В настоящей статье изложен метод кинематического анализа собственных движений звезд в северном и южном полушариях Галактики, а также описаны результаты «северного» и «южного» решений, показывающие, что кинематический анализ собственных движений звезд каталогов UCAC4, PPMXL и XPM позволил обнаружить замедление скорости вращения Галактики и ускорение скорости расширения звездной системы с увеличением расстояния от основной плоскости Галактики. Для модуля вертикального градиента скорости вращения Галактики были получены следующие значения: UCAC4: 40.1±0.2; PPMXL: 36.2±0.4; XPM: 37.7 ± 0.1 km $\cdot$ s^{-1} $\cdot$ kpc^{-1}, в то время как для вертикального градиента скорости расширения соответствующие оценки оказались следующими: UCAC4: 11.9 ± 0.2; PPMXL: 19.0 ± 1.1; XPM: 10.9 ± 0.3 km $\cdot$ s^{-1} $\cdot$ kpc^{-1}. Библиогр. 16 назв. Ил. 1. Табл. 4.

Ключевые слова: астрометрия, звездная кинематика, структура Галактики, собственные движения звезд, сферические функции.

1. Introduction. Modern astrometric catalogues UCAC4 [1], PPMXL [2] and XPM [3] with full coverage of the sky contain millions of entries and provide a qualitatively new material, in particular, for investigating the kinematics of nearby stars in both Galalactic hemispheres separately. Unfortunately, due to high correlations of the parameters the standard LS solutions of the Ogorodnikov—Milne equations on hemispheres are hardly to be trusted. To remedy this we use an approach the first step of which is the expansion of proper motions on a system of vector spherical harmonics which are orthonormal on a hemisphere. At the second step the kinematical parameters are derived from the coefficients of the expansion. For the first time, this approach was used by Vityazev and Tsvetkov [4] for the kinematic analysis of the proper motions listed in astrometric catalogues Hipparcos [5], Tycho-2 [6] and UCAC3 [7]. In this paper this technique is applied to the huge present day catalogues UCAC4, PPMXL and XPM containing up to 10^8 stars.

2. Ogorodnikov—Milne equations. The kinematics of nearby stars (up to 1000 pc) may be derived from the linear 3-D Ogorodnikov—Milne model [8]. In this model we have the following parameters: π — the parallax of a star; U, V, W are the components of the solar motion vector relative to the stellar centroid; Ω_1, Ω_2, Ω_3 are the components of

the rigid-body rotation vector of the stellar centroid; M_{11}^+, M_{22}^+, M_{33}^+ are the parameters describing the contraction or expansion of the velocity field along the principal axes of the coordinate system; $M_{12}^+ = M_{21}^+$, $M_{13}^+ = M_{31}^+$, $M_{23}^+ = M_{32}^+$ — parameters describing the velocity field deformation in the principal plane and the two planes perpendicular to it. Introducing the factor $\mathcal{K} = 4.74$ for converting the dimensions of stellar proper motions $\mathrm{mas} \cdot \mathrm{y}^{-1}$ to $\mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{kpc}^{-1}$, for the proper motions in longitude and latitude $\Delta\mu_l \cos b$ and $\Delta\mu_b$ we have

$$\begin{aligned}\mathcal{K}\mu_l \cos b = \quad & U\pi \, \sin l - V\pi \, \cos l - \Omega_1 \sin b \cos l - \Omega_2 \sin b \sin l + \Omega_3 \cos b - \\ & - M_{13}^+ \sin b \sin l + M_{23}^+ \sin b \cos l + M_{12}^+ \cos b \cos 2l - \frac{1}{2} M_{11}^* \cos b \sin 2l, \quad (1)\end{aligned}$$

$$\begin{aligned}\mathcal{K}\mu_b = \quad & U\pi \, \cos l \sin b + V\pi \, \sin l \sin b - W\pi \, \cos b + \Omega_1 \sin l - \Omega_2 \cos l - \\ & - \frac{1}{2} M_{12}^+ \sin 2b \sin 2l + M_{13}^+ \cos 2b \cos l + M_{23}^+ \cos 2b \sin l - \\ & - \frac{1}{2} M_{11}^* \sin 2b \cos^2 l + \frac{1}{2} M_{33}^* \sin 2b \,. \qquad (2)\end{aligned}$$

In these equations the values $M_{11}^* = M_{11}^+ - M_{22}^+$, $M_{33}^* = M_{33}^+ - M_{22}^+$ are introduced since the catalogues we are dealing with provide us neither with radial velocities nor with parallaxes of the stars. That is why, the value M_{22}^+ can not be determined when only the PM are available [9]. In addition, if parallaxes are unknown, only the mean values of the products $\bar{U} = \langle U\pi \rangle$, $\bar{V} = \langle V\pi \rangle$, $\bar{W} = \langle W\pi \rangle$ can be derived instead of the velocity components U, V, W.

2. Vector spherical harmonics (VSH). In this paper we solve the Ogorodnikov—Milne equations using the spherical harmonics formalism. At the first step define the scalar spherical harmonics as [10]:

$$K_{nkp}(l,b) = R_{nk} \begin{cases} P_{n,0}(b), & k = 0, p = 1; \\ P_{nk}(b) \sin kl, & k \neq 0, p = 0; \\ P_{nk}(b) \cos kl, & k \neq 0, p = 1, \end{cases} \qquad (3)$$

$$R_{nk} = \sqrt{\frac{2n+1}{4\pi}} \begin{cases} \sqrt{\frac{2(n-k)!}{(n+k)!}}, & k > 0; \\ 1, & k = 0, \end{cases} \qquad (4)$$

where l and b are the longitude and latitude of the point on the sphere, respectively, $(0 \leq l \leq 2\pi;\ -\pi/2 \leq b \leq \pi/2)$; $P_{nk}(b)$ are the Legendre (at $k = 0$) and associated Legendre (for $k > 0$) polynomials that can be calculated using the recurrence relations

$$\begin{aligned} & P_{nk}(b) = \sin b \tfrac{2n-1}{n-k} P_{n-1,k}(b) - \tfrac{n+k-1}{n-k} P_{n-2,k}(b), \quad k = 0,\, 1,\, \ldots n = k+2,\, \ldots \\ & P_{kk}(b) = \tfrac{(2k)!}{2^k k!} \cos^k b, \qquad (5) \\ & P_{k+1,k}(b) = \tfrac{(2k+2)!}{2^{k+1}(k+1)!} \cos^k b \sin b. \end{aligned}$$

For convenience, a linear numeration of the harmonics K_{nkp} by one index j is often introduced. In this way the index j is used instead of (nkp), where

$$j = n^2 + 2k + p - 1. \qquad (6)$$

To proceed further, consider a set of mutually orthogonal unit vectors $\mathbf{e}_l, \mathbf{e}_b, \mathbf{e}_r$ in the directions of the longitude and latitude and along the line of sight, respectively, in a plane tangential to the sphere. Following the definitions of vector spherical harmonics in [11] and [12] we introduce radial, $\mathbf{V}_j$, toroidal, $\mathbf{T}_j$ and spheroidal $\mathbf{S}_j$ harmonics via the relations

$$\mathbf{V}_j(l,b) = K_j(l,b)\mathbf{e}_r, \tag{7}$$

$$\mathbf{T}_j = r_n \left(\frac{\partial K_j(l,b)}{\partial b}\mathbf{e}_l - \frac{1}{\cos b}\frac{\partial K_j(l,b)}{\partial l}\mathbf{e}_b \right), \tag{8}$$

$$\mathbf{S}_j = r_n \left(\frac{1}{\cos b}\frac{\partial K_j(l,b)}{\partial l}\mathbf{e}_l + \frac{\partial K_j(l,b)}{\partial b}\mathbf{e}_b \right), \tag{9}$$

where

$$r_n = \frac{1}{\sqrt{n(n+1)}}. \tag{10}$$

Denote the components of the unit vector $\mathbf{e}_l$ as T_j^l, S_j^l, and the components of the unit vector $\mathbf{e}_b$ — respectively T_j^b and S_j^b:

$$\mathbf{T}_j = T_j^l\mathbf{e}_l + T_j^b\mathbf{e}_b, \tag{11}$$

$$\mathbf{S}_j = S_j^l\mathbf{e}_l + S_j^b\mathbf{e}_b. \tag{12}$$

With $P_{n,k+1}(b) = 0$ if $n < k+1$, these components are defined as:

$$T_j^l = \frac{R_{nk}}{\sqrt{n(n+1)}} \begin{cases} P_{n,1}(b), & k = 0,\ p = 1, \\ (-k\tan bP_{nk}(b) + P_{n,k+1}(b))\sin kl, & k \neq 0,\ p = 0, \\ (-k\tan bP_{nk}(b) + P_{n,k+1}(b))\cos kl, & k \neq 0,\ p = 1; \end{cases} \tag{13}$$

$$T_j^b = \frac{R_{nk}}{\sqrt{n(n+1)}} \begin{cases} 0, & k \neq 0,\ p = 1, \\ -\frac{k}{\cos b}P_{nk}(b)\cos kl, & k \neq 0,\ p = 0, \\ +\frac{k}{\cos b}P_{nk}(b)\sin kl, & k \neq 0,\ p = 1; \end{cases} \tag{14}$$

$$S_j^b = \frac{R_{nk}}{\sqrt{n(n+1)}} \begin{cases} P_{n,1}(b), & k = 0,\ p = 1, \\ (-k\tan bP_{nk}(b) + P_{n,k+1}(b))\sin kl, & k \neq 0,\ p = 0, \\ (-k\tan bP_{nk}(b) + P_{n,k+1}(b))\cos kl, & k \neq 0,\ p = 1; \end{cases} \tag{15}$$

$$S_j^l = \frac{R_{nk}}{\sqrt{n(n+1)}} \begin{cases} 0, & k = 0,\ p = 1, \\ +\frac{k}{\cos b}P_{nk}(b)\cos kl, & k \neq 0,\ p = 0, \\ -\frac{k}{\cos b}P_{nk}(b)\sin kl, & k \neq 0,\ p = 1. \end{cases} \tag{16}$$

The introduced harmonics $\mathbf{V}_j$, $\mathbf{T}_j$, $\mathbf{S}_j$ are orthonormal on the sphere since the following relations are valid:

$$\int\limits_{\Omega} (\mathbf{V}_i \cdot \mathbf{V}_j)\, d\omega = \int\limits_{\Omega} (\mathbf{T}_i \cdot \mathbf{T}_j)\, d\omega = \int\limits_{\Omega} (\mathbf{S}_i \cdot \mathbf{S}_j)\, d\omega = \begin{cases} 0, i \neq j, \\ 1, i = j; \end{cases} \tag{17}$$

$$\int\limits_{\Omega} (\mathbf{V}_i \cdot \mathbf{T}_j)\, d\omega = \int\limits_{\Omega} (\mathbf{V}_i \cdot \mathbf{S}_j)\, d\omega = \int\limits_{\Omega} (\mathbf{S}_i \cdot \mathbf{T}_j)\, d\omega = 0, \forall\, i, j. \tag{18}$$

4. The zone vector spherical harmonics (ZVSH). Let the data of some zonal catalogue belong to the following domain of the celestial sphere:

$$Z = \begin{cases} 0 \leq l \leq 2\pi, \\ b_{\min} \leq b \leq b_{\max}. \end{cases} \tag{19}$$

Introduce the transformation

$$\hat{b} = \arcsin(\alpha\, \sin b + \beta), \tag{20}$$

that for

$$\alpha = \frac{2}{s_2 - s_1}, \quad \beta = -\frac{s_2 + s_1}{s_2 - s_1}, \tag{21}$$

$$s_1 = \sin b_{\min}, \quad s_2 = \sin b_{\max} \tag{22}$$

transforms the entire sphere into region Z.

Now, the zone vector spherical harmonics (ZVSH) are introduced as

$$\hat{\mathbf{V}}_j(l, \hat{b}) = \sqrt{\alpha} K_j(l, \hat{b}) \mathbf{e}_r, \tag{23}$$

$$\hat{\mathbf{T}}_j(l, \hat{b}) = \hat{T}_j^l(l, \hat{b}) \mathbf{e}_l + \hat{T}_j^b(l, \hat{b}) \mathbf{e}_b, \tag{24}$$

$$\hat{\mathbf{S}}_j(l, \hat{b}) = \hat{S}_j^l(l, \hat{b}) \mathbf{e}_l + \hat{S}_j^b(l, \hat{b}) \mathbf{e}_b, \tag{25}$$

where

$$\hat{T_j^l}(l, \hat{b}) = \sqrt{\alpha}\, T_j^l(l, \hat{b}); \;\; \hat{T_j^b}(l, \hat{b}) = \sqrt{\alpha}\, T_j^b(l, \hat{b}), \tag{26}$$

$$\hat{S_j^l}(l, \hat{b}) = \sqrt{\alpha}\, S_j^l(l, \hat{b}); \;\; \hat{S_j^b}(l, \hat{b}) = \sqrt{\alpha}\, S_j^b(l, \hat{b}). \tag{27}$$

These harmonics are orthonormal on the set Z, so the following relations are valid:

$$\int\limits_{Z} \left(\hat{\mathbf{V}}_i \cdot \hat{\mathbf{V}}_j\right) d\omega = \int\limits_{Z} \left(\hat{\mathbf{T}}_i \cdot \hat{\mathbf{T}}_j\right) d\omega = \int\limits_{Z} \left(\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j\right) d\omega = \begin{cases} 0, i \neq j, \\ 1, i = j; \end{cases} \tag{28}$$

$$\int\limits_{Z} \left(\hat{\mathbf{V}}_i \cdot \hat{\mathbf{T}}_j\right) d\omega = \int\limits_{Z} \left(\hat{\mathbf{V}}_i \cdot \hat{\mathbf{S}}_j\right) d\omega = \int\limits_{Z} \left(\hat{\mathbf{S}}_i \cdot \hat{\mathbf{T}}_j\right) d\omega = 0, \;\; \forall\, i, j. \tag{29}$$

Here, for example,

$$\int\limits_{Z} \left(\hat{\mathbf{T}}_i \cdot \hat{\mathbf{T}}_j\right) d\omega = \alpha \int\limits_{0}^{2\pi} dl \int\limits_{b_{\min}}^{b_{\max}} T_i^l(l, \hat{b})\, T_j^l(l, \hat{b}) \cos b\, db + \alpha \int\limits_{0}^{2\pi} dl \int\limits_{b_{\min}}^{b_{\max}} T^b{}_i(l, \hat{b})\, T_j^b(l, \hat{b}) \cos b\, db. \tag{30}$$

Since the radial velocities are not available in the catalogues UCAC4, PPMXL and XPM, in what follows we consider only the tangential stellar velocity field specified in region Z on the celestial sphere:

$$\mathbf{U}(l,b) = \mu_l^* \, \mathbf{e}_l + \mu_b^* \, \mathbf{e}_b, \tag{31}$$

where $\mu_l^* = \mathcal{K}\mu_l \cos b$; $\mu_b^* = \mathcal{K}\mu_b$.

We can now use the system of Zone Vector Spherical Harmonics to decompose the velocity field as

$$\mathbf{U}(l,b) = \sum_j t_j \hat{\mathbf{T}}_j(l,\hat{b}) + \sum_j s_j \hat{\mathbf{S}}_j(l,\hat{b}). \tag{32}$$

Given the orthonormality of the basis, the decomposition coefficients can be calculated by the following formulae:

$$t_j = \int\limits_Z \left(\mathbf{U}\cdot\hat{\mathbf{T}}_j\right) d\omega; \quad s_j = \int\limits_Z \left(\mathbf{U}\cdot\hat{\mathbf{S}}_j\right) d\omega. \tag{33}$$

Note that the expressions (32) and (33) are valid for all sky analysis since in this case $\alpha = 1$, $\beta = 0$ and $\hat{\mathbf{T}}_j(l,\hat{b})$ and $\hat{\mathbf{S}}_j(l,\hat{b})$ become $\mathbf{T}_j(l,b)$ and $\mathbf{S}_j(l,b)$ respectively.

5. The method in practice. Assume that we have at our disposal a catalogue of stars with galactic coordinates and proper motion components in latitude and longitude. As was mentioned before, the full and the zonal catalogue may be treated likewise, so let us describe the sequence of steps for the kinematic analysis of the stellar velocity field using ZVSH.

(1) Calculating the ZVSH decomposition coefficients t_j, s_j of the velocity field. These coefficients and their root-mean-square errors (rmse) can be derived from the equations

$$\mu_l^* = \sum_j t_j \hat{T}_j^l(l,\hat{b}) + \sum_j s_j \hat{S}_j^l(l,\hat{b}), \tag{34}$$

$$\mu_b^* = \sum_j t_j \hat{T}_j^b(l,\hat{b}) + \sum_j s_j \hat{S}_j^b(l,\hat{b}) \tag{35}$$

by the standard least-squares procedure. The total number of decomposition terms can be chosen from the condition that the residuals in the velocity field components with statistically significant harmonics subtracted from them behave as random quantities [10, 12].

When the method is applied to massive catalogues the pre-pixelization of data on the sphere is needed. As applied to our problem, the pixelization scheme should satisfy the requirement that the pixel centers are equidistant in both latitude and longitude. Two schemes satisfy this requirement. One of them is HEALPix [14], the other is the so-called Equidistant Cylindrical Projection (ECP). Regarding the VSH formalizm the pixelization algorithms were discussed in detail previously [13]. In this paper we dwelt on ECP, in which the stellar proper motions are averaged over spherical trapeziums obtained by a uniform division of the equator and the latitude circle into $M = 24$ and $N = 18$ parts, respectively. Since the cells we use have different areas the average PM values were weighted by $\cos(b_c)$ with b_c standing for the latitude of the cell's centre.

It is necessary to say that equations (1), (2) correspond to the physical model of the stellar velocity field since we know the physical meaning of each parameter of it. But this

model is not full since it does not embrace all physical content of the observed data. In contrast to that, equations (34) and (35) are full since all the information is captured by the decomposition coefficients (due to completeness of the VSH and ZVSH). But this model is not physical since we do not know what physics stands behind each decomposition coefficient.

(2) Determining the parameters of a specific kinematic model. Once the decomposition coefficients $t_j \pm \sigma_{t_j}, s_j \pm \sigma_{s_j}$ have been determined, we can write the equations relating the decomposition coefficients to the sought for model parameters. In case of the Ogorodnikov—Milne model and when the full catalogue is used, the decompositions (34)—(35) are finite and only VSF up $n \leq 2$ are needed to find the parameters of the physical model [13].

In this paper we analyse the kinematics of the northern and southern galactic hemispheres. In this case the decompositions (34)–(35) are infinite. The connections between the decomposition coefficients and kinematic parameters up to $n = 3$ are shown in Table 1 Upper signs correspond to the northern hemisphere, the lower — to the southern hemisphere. In case of one sign the signs of both hemispheres coincide. *Units*: $\mathrm{km \cdot s^{-1} \cdot kpc^{-1}}$. To determine the kinematic parameters, the number of equations must be equal to the number of parameters. Thus several (theoretically infinitely many) estimates of model parameters can be obtained. In practice, it is appropriate to construct the solutions for the lowest-order decomposition terms. In our method we will use two estimates of the parameters, which we refer to as the main and alternative solutions:

$$\begin{bmatrix} \bar{U} \\ \bar{V} \\ \bar{W} \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ M_{13}^{+} \\ M_{23}^{+} \\ M_{12}^{+} \\ M_{11}^{*} \\ M_{33}^{*} \end{bmatrix} = \mathbf{P} \begin{bmatrix} s_{101} \\ s_{110} \\ s_{111} \\ s_{201} \\ s_{210} \\ s_{211} \\ s_{220} \\ s_{221} \\ t_{101} \\ t_{110} \\ t_{111} \end{bmatrix}, \quad \begin{bmatrix} \bar{U} \\ \bar{V} \\ \bar{W} \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ M_{13}^{+} \\ M_{23}^{+} \\ M_{12}^{+} \\ M_{11}^{*} \\ M_{33}^{*} \end{bmatrix} = \mathbf{Q} \begin{bmatrix} s_{101} \\ s_{110} \\ s_{111} \\ s_{301} \\ t_{201} \\ t_{110} \\ t_{111} \\ t_{210} \\ t_{211} \\ t_{220} \\ t_{221} \end{bmatrix}. \tag{36}$$

The matrices P (main solution) and Q (alternative solution) are shown in Tables 2 and 3 The upper and lower signs correspond to the northern and southern hemispheres, respectively; if there is one sign, then the signs of the coefficient are identical for the northern and southern hemispheres. Comparison of the two solutions may be used to test the compatibility of the data to the model.

6. Numerical results. We applied our method to proper motions of stars listed in the catalogues UCAC4, PPMXL and XPM. No parallaxes are available in these catalogues. The full description of the results obtained for full sphere and separate hemispheres solutions may be found in [15]. The present paper is devoted only to one remarkable feature in the "northern" and "southern" solutions, since all the three catalogues gave evidence that the parameters Ω_1, $M_{2,3}^{+}$, Ω_2, $M_{1,3}^{+}$ have different signs in different hemispheres. A possible explanation of this fact comes from considering the corresponding values in the

Table 1. **The kinematics of the ZVSH ($n \le 3$) in the frame of Ogoridnikov—Milne model**

j	n	k	p	T_j	S_j
1	1	0	1	$+1.949\,\Omega_3$	$-1.949\bar{W} \pm 0.873(M^*_{33} - \frac{1}{2}M^*_{11})$
2	1	1	0	$\mp 0.768\bar{U} + 1.791\,\Omega_2 - 0.256M^+_{13}$	$-1.791\bar{V} \mp 0.768\,\Omega_1 \pm 1.279M^+_{23}$
3	1	1	1	$\pm 0.768\bar{V} + 1.791\,\Omega_1 + 0.256M^+_{23}$	$-1.791\bar{U} \mp 0.768\,\Omega_2 \pm 1.279M^+_{13}$
4	2	0	1	$\mp 0.453\,\Omega_3$	$\pm 0.453W + 0.274(M^*_{33} - \frac{1}{2}M^*_{11})$
5	2	1	0	$+0.332\bar{U} \mp 0.332\,\Omega_2 \pm 0.332M^+_{13}$	$\pm 0.332\bar{V} + 0.332\,\Omega_1 + 0.728M^+_{23}$
6	2	1	1	$-0.332\bar{V} \mp 0.332\,\Omega_1 \mp 0.332M^+_{23}$	$\pm 0.332\bar{U} - 0.332\,\Omega_2 + 0.728M^+_{13}$
7	2	2	0	$\pm 0.216M^*_{11}$	$+1.338M^+_{12}$
8	2	2	1	$\mp 0.433M^+_{12}$	$+0.669M^*_{11}$
9	3	0	1	$+0.270\,\Omega_3$	$-0.270\bar{W} \mp 0.017(M^*_{33} - \frac{1}{2}M^*_{11})$
10	3	1	0	$\mp 0.199\bar{U} + 0.199\,\Omega_2 - 0.199M^+_{13}$	$-0.199\bar{V} \mp 0.199\,\Omega_1 \mp 0.199M^+_{23}$
11	3	1	1	$\pm 0.199\bar{V} + 0.199\,\Omega_1 + 0.199M^+_{23}$	$-0.199\bar{U} \pm 0.199\,\Omega_2 \mp 0.199M^+_{13}$
12	3	2	0	$-0.109M^*_{11}$	$\mp 0.463M^+_{12}$
13	3	2	1	$+0.219M^+_{12}$	$\mp 0.231M^*_{11}$
14	3	3	0	0	0
15	3	3	1	0	0

Table 2. **Matrix $\boldsymbol{P}$ for calculating the main solution (36)**

0	0	−0.54	0	0	±1.10	0	0	0	±0.44	0
0	−0.54	0	0	±1.10	0	0	0	0	0	∓0.44
−0.29	0	0	±0.94	0	0	0	0	0	0	0
0	±0.21	0	0	−0.64	0	0	0	0	0	0.77
0	0	∓0.21	0	0	0.64	0	0	0	0.77	0
0	0	0	0	0	0	0	0	0.51	0	0
0	0	±0.15	0	0	1.16	0	0	0	0.15	0
0	±0.15	0	0	1.16	0	0	0	0	0	−0.15
0	0	0	0	0	0	0.75	0	0	0	0
0	0	0	0	0	0	0	1.49	0	0	0
±0.49	0	0	2.10	0	0	0	0.75	0	0	0

Table 3. **Matrix $\boldsymbol{Q}$ for calculating the alternative solution (36)**

0	0	−0.42	0	0	±0.56	0	2.04	0	0	0
0	−0.42	0	0	0	0	∓0.56	0	−2.04	0	0
−0.06	0	0	−3.25	0	0	0	0	0	0	0
0	±0.14	0	0	0	0	0.84	0	±1.18	0	0
0	0	∓0.14	0	0	0.84	0	±1.18	0	0	0
0	0	0	0	∓2.21	0	0	0	0	0	0
0	0	±0.28	0	0	0.28	0	±2.15	0	0	0
0	±0.28	0	0	0	0	−0.28	0	∓2.15	0	0
0	0	0	0	0	0	0	0	0	0	∓2.31
0	0	0	0	0	0	0	0	0	±4.63	0
±1.00	0	0	∓7.25	0	0	0	0	0	±2.32	0

Table 4. **Values $\Omega_1 - M_{32}^+$ and $\Omega_2 + M_{31}^+$ obtained from northern and southern galactic hemispheres of the UCAC4, PPMXL and XPM (Units: $\mathbf{km \cdot s^{-1} \cdot kpc^{-1}}$)**

	11^m	12^m	13^m	14^m	15^m	16^m
Catalogue UCAC4						
$10(\Omega_1 - M_{32}^+)_N$	424 ± 8	396 ± 7	369 ± 5	357 ± 4	352 ± 3	360 ± 3
$10(\Omega_1 - M_{32}^+)_S$	-415 ± 9	-421 ± 11	-446 ± 8	-432 ± 7	-423 ± 4	-419 ± 3
$10\left\lvert\frac{\partial V_S}{\partial z}\right\rvert$	420 ± 6	408 ± 7	408 ± 5	394 ± 4	387 ± 3	390 ± 2
$10(\Omega_2 + M_{31}^+)_N$	-156 ± 8	-197 ± 7	-163 ± 5	-109 ± 4	-78 ± 3	-66 ± 3
$10(\Omega_2 + M_{31}^+)_S$	113 ± 9	136 ± 11	118 ± 8	126 ± 7	95 ± 4	73 ± 3
$10\left\lvert\frac{\partial V_R}{\partial z}\right\rvert$	135 ± 6	167 ± 7	140 ± 5	117 ± 4	86 ± 3	68 ± 2
Catalogue PPMXL						
$10(\Omega_1 - M_{32}^+)_N$	513 ± 18	402 ± 16	451 ± 17	478 ± 10	473 ± 7	439 ± 6
$10(\Omega_1 - M_{32}^+)_S$	-456 ± 28	-470 ± 17	-405 ± 17	-354 ± 12	-332 ± 8	-294 ± 7
$10\left\lvert\frac{\partial V_S}{\partial z}\right\rvert$	484 ± 17	436 ± 12	428 ± 12	416 ± 8	403 ± 5	367 ± 5
$10(\Omega_2 + M_{31}^+)_N$	-208 ± 18	-173 ± 17	-211 ± 17	-181 ± 10	-165 ± 7	-119 ± 6
$10(\Omega_2 + M_{31}^+)_S$	162 ± 28	216 ± 17	234 ± 17	253 ± 12	189 ± 8	179 ± 7
$10\left\lvert\frac{\partial V_R}{\partial z}\right\rvert$	185 ± 16	194 ± 12	222 ± 12	$217 \pm 0,8$	177 ± 5	149 ± 4
Catalogue XPM						
$10(\Omega_1 - M_{32}^+)_N$	344 ± 14	333 ± 08	346 ± 5	377 ± 4	381 ± 3	364 ± 2
$10(\Omega_1 - M_{32}^+)_S$	-630 ± 14	-623 ± 9	-566 ± 6	-491 ± 4	-421 ± 4	-398 ± 3
$10\left\lvert\frac{\partial V_S}{\partial z}\right\rvert$	487 ± 10	478 ± 6	456 ± 4	434 ± 3	401 ± 2	381 ± 2
$10(\Omega_2 + M_{31}^+)_N$	-68 ± 14	-97 ± 8	-86 ± 5	-60 ± 3	-47 ± 03	-39 ± 2
$10(\Omega_2 + M_{31}^+)_S$	199 ± 14	196 ± 10	170 ± 7	143 ± 4	115 ± 4	86 ± 3
$10\left\lvert\frac{\partial V_R}{\partial z}\right\rvert$	133 ± 10	146 ± 6	128 ± 4	102 ± 3	81 ± 02	63 ± 2

galactocentric cylindrical coordinate system [16] where the following relation is valid:

$$\Omega_1 - M_{32}^+ = -\frac{\partial V_S}{\partial z}. \tag{37}$$

Here V_S is the circular velocity of the local reference frame around the galactic center. This quantity is identified with the Galaxy's rotational velocity in the solar neighborhood. From Table 4 which gives the numerical values for the values $\Omega_1 - M_{32}^+$ that we obtained from different samples of our catalogues, we see that the vertical gradient of the Galaxy's rotational velocity $\partial V_S/\partial z$ has different signs in the northern and southern galactic hemispheres, with the velocity itself decreasing with increasing distance from the principal galactic plane.

Again, from the Table 4 for the vertical gradient of the expansion velocity of the stellar system

$$\Omega_2 + M_{13}^+ = -\frac{\partial V_R}{\partial z} \tag{38}$$

we may conclude that the expansion velocity increases with increasing distance from the principal galactic plane. The average values of both the gradients $|\partial V_S/\partial z|$ and $|\partial V_R/\partial z|$ derived from all the catalogues under consideration are in good agreement. Nevertheless, each catalogue shows distinct dependence of the gradients on magnitude with absolute values diminishing toward the faint stars (Fig. 1). The PPMXL and UCAC4 realize

the reference frames which does not rotate with respect to quasars, whereas the XPM frame is claimed to be free of rotation with respect to galaxies since its absolute proper motion frame (the absolute calibration) was specified with more than 1 million galaxies from 2MASS and USNO-A2.0. Theoretically, both quasars and galaxies form the quasi-inertial reference systems but due to different techniques of measurement the resulting reference frames may differ systematically. In this connection, it is worth to note that the values $|\partial V_R/\partial z|$ derived from UCAC4 and XPM are practically the same. The magnitude equation visible in Fig. 1 deserves further study.

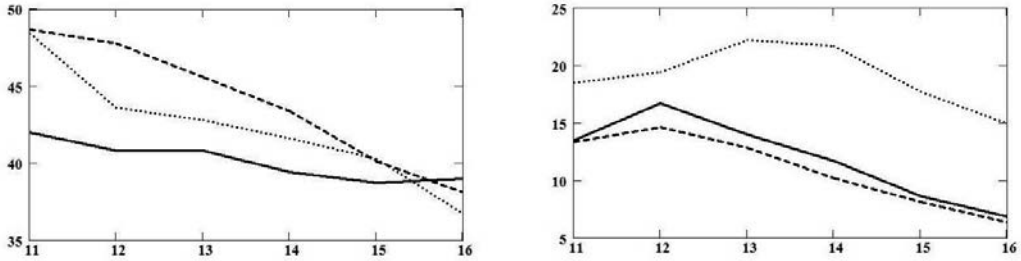

Fig. 1. Dependence of the $\left|\frac{\partial V_S}{\partial z}\right|$ (left box) and $\left|\frac{\partial V_R}{\partial z}\right|$ (right box) on magnitude. Solid line — UCAC4; dots — PPMXL; dashes — XPM. Units: $\mathrm{km \cdot s^{-1} \cdot kpc^{-1}}$.

For the first time, the values of the both gradients were derived by Vityazev and Tsvetkov [4] from the proper motions of the UCAC3, Tycho-2 and Hipparcos catalogues. Here the comparison with the all sky solution was made. It was shown that these gradients give rise to an apparent acceleration of the solar motion along the x and y axes of the rectangular Galactic coordinate system. The present paper dealing with the most rich catalogues available at present days confirms these results and shows that both the gradients though m-dependent are detected with high level of reliability.

7. Conclusions. The success of this paper is based on the vector spherical harmonics solutions of the Ogorodnikov-Milne equations on hemispheres which permitted to obtain the uncorrelated values of the kinematical parameters and to show that some of them have different signs in both hemispheres. The transition to the Galactocentrical cylinder coordinate system immediately made it clear that the change of signs is connected with the retardation of the Galaxy's rotational velocity and acceleration of the expansion velocity of the stellar system with increasing the distance from the principal Galactic plane.

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THE VERTICAL GRADIENTS IN THE GALACTIC ROTATION DERIVED FROM THE PROPER MOTIONS OF THE UCAC4, PPMXL AND PPM CATALOGUES

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The paper deals with the kinematic analysis of the proper motions listed in huge modern astrometric catalogues UCAC4, PPMXL and XPM. Our approach is based on representation of the proper motions on the set of vector spherical harmonics. To study the stellar kinematics in the northern and southern galactic hemispheres separately the set of the orthonormal vector spherical harmonics defined on the latitude zone of the sphere was constructed. In the performing the separate solutions this approach concealed the correlations between the unknown parameters and made the solution trustworthy.

It is shown that the kinematic analysis of the UCAC4, PPMXL and XPM proper motions in northern and southern Galactic hemispheres detects retardation of the Galaxy's rotational velocity and acceleration of the expansion velocity of the stellar system with increasing the distance from the principal Galactic plane. The estimates of the vertical gradient of the Galactic rotation are UCAC4: 40.1 ± 0.2; PPMXL: 36.2 ± 0.4; XPM: 37.7 ± 0.1 km $\cdot$ s^{-1} $\cdot$ kpc^{-1}, while the values of the vertical gradient of the expansion velocity turned out to be UCAC4: 11.9 ± 0.2; PPMXL: 19.0 ± 1.1; XPM: 10.9 ± 0.3 km $\cdot$ s^{-1} $\cdot$ kpc^{-1}. Refs 16. Figs 1. Tables 4.

Keywords: astrometry, stellar kinematics, proper motions, spherical harmonics.