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Analysis of the Systematic Differences between the Stellar Parallaxes of the TGAS and Hipparcos Catalogues Using Spherical Harmonics

A. S. Tsvetkov* and **F. A. Amosov**

St. Petersburg State University, Bibliotechnaya pl. 2, St. Petersburg, 198504 Russia

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Abstract—The systematic differences between the trigonometric parallaxes of the Hipparcos and TGAS catalogues have been investigated using spherical harmonics. The most significant harmonics in the expansion have been determined. The distribution of the parallax difference dispersion in various regions of the celestial sphere has also been studied. The distribution of the rms deviation has the simplest form in the ecliptic coordinate system.

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INTRODUCTION

Comparing catalogues is a classical problem of fundamental astrometry. Until recently, such a study could be performed only for the stellar positions and proper motions. The appearance of the first results from the Gaia mission, in particular, its subset, the TGAS catalogue, has allowed the trigonometric parallaxes of common stars from the TGAS and Hipparcos catalogues, in the latter case, its second version of astrometric data (van Leeuwen 2007), to be compared for the first time.

On April 25, 2018, Gaia Data Release 2 was issued, but it has no connections with Hipparcos objects; moreover, its authors admit that even the connection between DR1 and DR2 numbers is unreliable. The cross-tables of Gaia DR1 and DR2 connections and the tables of Gaia DR2 connections with other catalogues will be published later.

Despite the fact that the Hipparcos stellar parallaxes have a high formal accuracy, evidence for possible systematic errors in these parallaxes has repeatedly appeared. For example, Soderblom et al. (2007) showed disagreement with the Hubble Space Telescope data for Pleiades stars. Tsvetkov et al. (2008) provided a long list of stars for which the spectroscopic parallaxes based on two-dimensional spectral classification differed significantly from the Hipparcos trigonometric parallaxes.

The TGAS catalogue contains 2 057 050 stars with trigonometric parallaxes, includes only the Hipparcos and Tycho-2 stars, and uses the stellar positions in these catalogues as the first epoch. Lindegren et al. (2016) performed a primary comparison of the parallaxes in the TGAS and Hipparcos catalogues. They gave estimates of the differences and constructed the diagrams describing the general behavior of the parallax differences as a function of various stellar parameters, for example, the color index. The apparatus of scalar or vector harmonics is traditionally used to compare the stellar positions and proper motions of astrometric catalogues. This approach was first used by Brosche (1966) and is described in detail in the monograph by Vityazev (2018). In this paper we apply the apparatus of scalar spherical harmonics to analyze the systematic parallax differences.

CALCULATING THE DIFFERENCES BETWEEN THE PARALLAXES OF INDIVIDUAL STARS

The number of common stars in the Hipparcos and TGAS catalogues is 93 635. It was easy to combine the data from these catalogues, because TGAS has Hipparcos star identifiers. The distribution of stars from the combined catalogue over the celestial sphere is presented in Fig. 1. In all illustrations we use data pixelization based on the HealPix algorithm (Górski et al. 2005) with the parameter $n = 8$, which gives 768 fields. From 57 to 273 stars of the combined catalogue fell into these fields.

We left the following data in the combined catalogue:

- *hip*—the star identifier in the Hipparcos catalogue;

*E-mail: A.S.Tsvetkov@inbox.ru

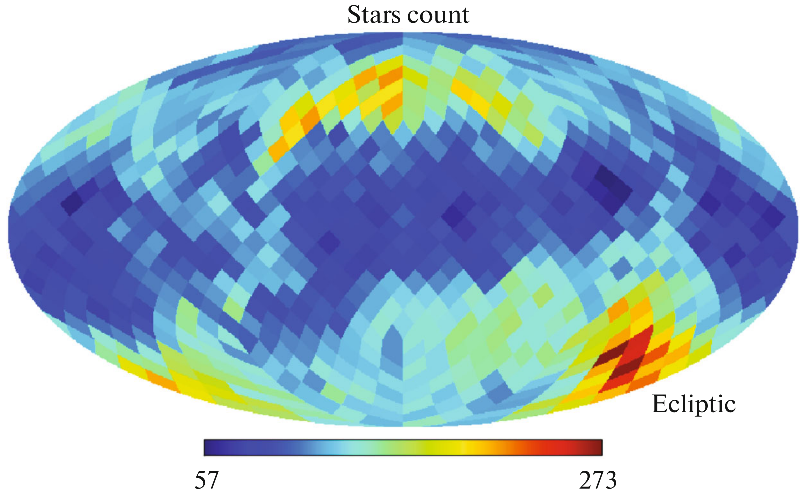

Fig. 1. (Color online) The distribution of stars from the combined catalogue over the celestial sphere in ecliptic coordinates.

- π_{tgas}—the absolute barycentric stellar parallax in TGAS;
- $\sigma_{\pi_{tgas}}$—the rms stellar parallax error in TGAS;
- l—the Galactic longitude in TGAS;
- b—the Galactic latitude in TGAS;
- π_{hip}—the trigonometric parallax in Hipparcos;
- $\sigma_{\pi_{hip}}$—the rms stellar parallax error in Hipparcos.

For each star of the combined catalogue we calculated the difference between its Hipparcos and TGAS parallaxes: $\pi_{hip} - \pi_{tgas}$.

ANALYSIS OF OUTLIERS

First we will perform an analysis for the presence of outliers in the parallax differences in order to detect single objects that could distort significantly the average result. Consider the stars for which the difference between the TGAS and Hipparcos parallaxes exceeds three rms errors of this difference $\sqrt{\sigma^2_{\pi_{hip}} + \sigma^2_{\pi_{tgas}}}$. There are 2148 such stars (Fig. 2). The correlation coefficient of the absolute value of the parallax difference with the Hipparcos and TGAS parallax errors for these stars is 0.87 and only 0.1, respectively. Thus, it can be argued that the large difference between the parallaxes is attributable to large parallax errors precisely in Hipparcos. Furthermore, the negative parallaxes significant according to the 3σ criterion, i.e., such that $\pi < -3\sigma_\pi$, are clearly erroneous. However, the number of such stars is small: a total of 6 in TGAS and 17 in Hipparcos.

In order not to distort the overall picture by the preliminary rejection of some of the observations, which could shift the systematic differences, we decided to use all 93 635 stars.

ANALYSIS OF THE TRIGONOMETRIC PARALLAX DIFFERENCES USING SPHERICAL HARMONICS

The expansions in terms of spherical harmonics can be done in different coordinate systems. The study by Lindegren et al. (2016) and our preliminary investigation showed that the parallax differences and the distribution of rms deviations over the celestial sphere have a pronounced concentration of the largest and smallest values in the regions of the ecliptic and the ecliptic poles. This fact is most clearly seen when visualizing not the parallax differences themselves (Fig. 3), but their rms deviations (Fig. 4). This makes the expansion of the systematic differences in terms of spherical harmonics in the ecliptic coordinate system appropriate.

Following the standard approach, let us represent the parallax differences as

$$\Delta_\pi(\lambda, \beta) = \sum_{nkp} \delta_{nkp} K_{nkp}(\lambda, \beta), \qquad (1)$$

where the spherical harmonics are (Arfken 1970)

$$K_{nkp}(\lambda, \beta) \qquad (2)$$
$$= R_{nk} \begin{cases} P_{n,0}(\beta), & k = 0, \quad p = 1 \\ P_{nk}(\beta)\sin k\lambda, & k \neq 0, \quad p = 0 \\ P_{nk}(\beta)\cos k\lambda, & k \neq 0, \quad p = 1, \end{cases}$$

$$R_{nk} = \sqrt{\frac{2n+1}{4\pi}} \begin{cases} \sqrt{\frac{2(n-k)!}{(n+k)!}}, & k > 0 \\ 1, & k = 0. \end{cases} \qquad (3)$$

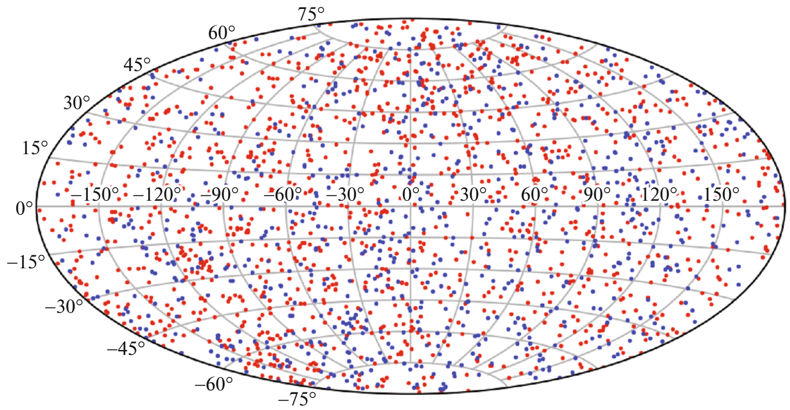

Fig. 2. (Color online) The distribution of outlier stars in parallax differences over the celestial sphere in ecliptic coordinates. The red and blue circles indicate positive and negative differences $\pi_{hip} - \pi_{tgas}$, respectively. There are a total of 2148 stars.

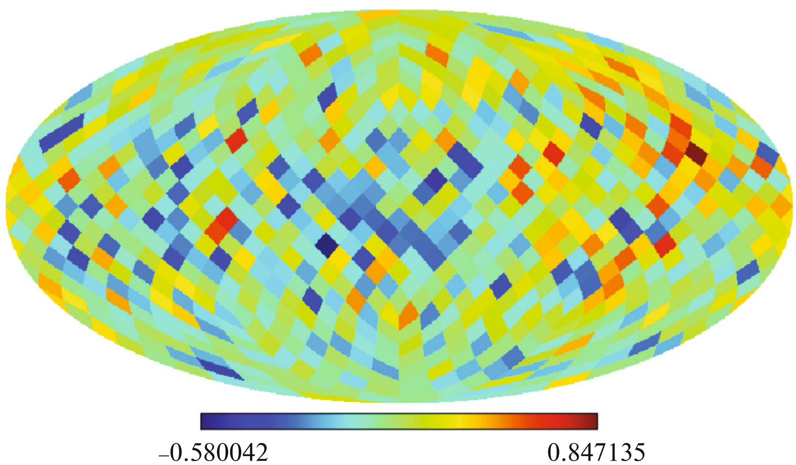

Fig. 3. (Color online) The distribution of the differences between the Hipparcos and TGAS stellar parallaxes over the celestial sphere in ecliptic coordinates (mas).

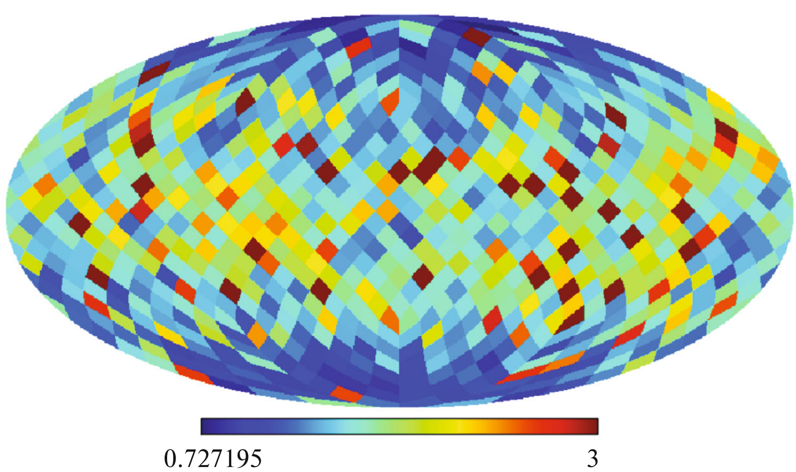

Fig. 4. (Color online) The distribution of the rms deviation of the difference between the Hipparcos and TGAS parallaxes over the celestial sphere in ecliptic coordinates, mas.

Table 1. Stars with the largest parallax differences (λ and β are in degrees; the remaining quantities are in mas)

HIP	λ	β	$\pi_{hip}-\pi_{tgas}$	$\sigma_{\pi_{hip}-\pi_{tgas}}$	π_{hip}	$\sigma_{\pi_{hip}}$	π_{tgas}	$\sigma_{\pi_{tgas}}$
21000	66.66	−16.34	81.15	4.76	84.76	4.74	3.61	0.43
68549	200.68	29.70	−71.15	9.10	−56.16	9.09	14.99	0.38
42525	121.46	22.12	62.60	15.52	68.54	15.51	5.94	0.50
92059	279.62	−11.28	54.46	13.48	55.49	13.48	1.03	0.26
81496	245.01	37.91	−42.18	10.19	−38.04	10.19	4.14	0.25
90368	283.25	69.94	41.76	10.37	51.00	10.37	9.24	0.24
87784	269.50	−47.09	40.40	8.36	41.30	8.36	0.90	0.26
81594	255.96	−29.53	−39.09	6.69	−6.97	6.69	32.12	0.25
98679	291.96	−28.93	36.88	11.17	84.75	11.17	47.87	0.31
63028	156.49	56.02	34.74	10.01	41.33	10.00	6.59	0.38
43650	162.33	−61.41	34.53	8.77	36.40	8.77	1.87	0.26
71922	228.36	−16.14	−32.12	10.58	−31.80	10.58	0.32	0.24
116869	1.74	15.39	−31.51	9.40	−24.10	9.39	7.41	0.40
109335	31.57	66.81	30.56	8.86	34.06	8.86	3.50	0.24
26111	86.75	46.30	29.36	1.96	30.22	1.94	0.86	0.27
47696	146.02	−7.05	28.54	1.51	4.84	1.35	−23.70	0.68
39939	117.96	9.99	−28.05	9.13	−16.34	9.13	11.71	0.26
91557	280.15	12.62	−25.99	7.89	30.49	7.89	56.48	0.27
114994	8.32	38.70	−25.80	5.16	−21.35	5.14	4.45	0.47
100625	298.76	−19.84	25.38	7.08	27.99	7.06	2.61	0.48

In Eq. (2) λ and β denote, respectively, the ecliptic longitude and latitude ($0 \leq \lambda \leq 2\pi$; $-\pi/2 \leq \beta \leq \pi/2$); $P_{nk}(\beta)$ denote the Legendre polynomials (at $k=0$) and associated Legendre functions (at $k>0$), which can be calculated using the following recurrence relations:

$$P_{nk}(\beta) = \sin\beta \frac{2n-1}{n-k} P_{n-1,k}(\beta) \tag{4}$$

$$- \frac{n+k-1}{n-k} P_{n-2,k}(\beta), \qquad {\scriptstyle k=0,1,\ldots \atop n=k+1,k+2,\ldots}$$

$$P_{kk}(\beta) = \frac{(2k)!}{2^k k!} \cos^k\beta,$$

$$P_{k+1,k}(\beta) = \frac{(2k+2)!}{2^{k+1}(k+1)!} \cos^k\beta \sin\beta.$$

For convenience, the linear numbering of harmonics K_{nkp} and coefficients δ_{nkp} by one index j is often introduced, where

$$j = n^2 + 2k + p - 1. \tag{5}$$

The introduced functions satisfy the following relations:

$$\iint_{\Omega} (K_i \cdot K_j)\, d\omega = \begin{cases} 0, & i \neq j \\ 1, & i = j. \end{cases} \tag{6}$$

In other words, the set of functions K_{nkp} forms an orthonormal system of functions on the sphere.

We solve the system generated by Eq. (1) by the least-squares method for the averaged data of all

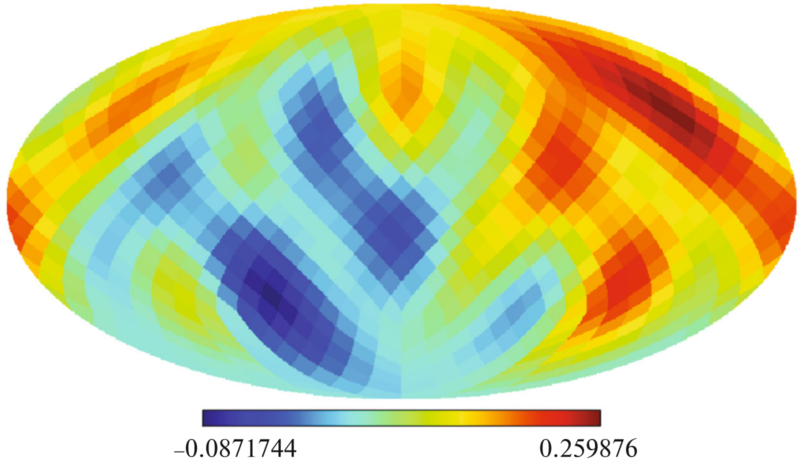

Fig. 5. The systematic parallax differences based on Table 2 (mas).

Healpix fields and for the first 49 ($n \leq 6$) expansion coefficients δ_j, because we are interested only in the low-frequency coefficients. As we will see below, the number of significant harmonics is much smaller. The data obtained have an F-statistic of 3.338 according to Fisher's test, i.e., the model is significant at a significance level of 1.3×10^{-12}. Thus, the inferred coefficients describe completely the model of systematic differences. These expansion coefficients are presented in Table 2. There are only six statistically significant coefficients at the 3σ level. The systematic parallax differences based on this table are displayed in Fig. 5. The amplitude of this quantity turned out to be very small: from -0.09 to $+0.26$ mas. Thus, the TGAS parallaxes systematically differ little from the Hipparcos parallaxes.

Table 2. Statistically significant coefficients of the expansion of the parallax difference in terms of spherical harmonics in ecliptic coordinates

j	δ_j	σ_{δ_j}	$\frac{\lvert\delta_j\rvert}{\sigma_{\delta_j}}$
0	0.34	0.02	13.66
1	0.11	0.02	4.25
2	−0.14	0.02	5.76
3	−0.11	0.02	4.51
31	0.09	0.02	3.68
46	0.08	0.02	3.43

ANALYSIS OF THE RMS DEVIATIONS OF TRIGONOMETRIC PARALLAXES USING SPHERICAL HARMONICS

For a comprehensive study of the differences between the stellar parallaxes of both catalogues we decided to investigate the pattern in the distribution of the rms deviation of parallax differences, because this will allow us to reveal the regions of the celestial sphere where the scatter of Hipparcos and TGAS parallaxes is greatest and where it is small.

The coefficients of the spherical harmonic expansion of the rms deviation of stellar parallax differences at the 3σ significance level are presented in Table 3. The distribution of rms deviations based on this table is displayed in Fig. 6. The derived regression has an F-statistic of 4.282, i.e., the model is significant at a level of 3.37×10^{-18}.

The model for the rms deviations of parallax differences turned out to be surprisingly very simple and is actually described by only two coefficients, the zeros and fourth ones, at a significance level of 1.30×10^{-22}.

DISCUSSION

A positive coefficient at the zeroth harmonic in the expansion of the parallax difference suggests that the Hipparcos parallaxes are larger than the TGAS ones, on average, over the entire celestial sphere, i.e., according to the Hipparcos data, the stars are closer. This is also confirmed by other studies (Gaia Collaboration 2017).

The coefficients 1 and 2 show an asymmetry in the distribution of differences in different hemispheres (Fig. 3). The negative coefficient 3 suggests that, in contrast to other parts of the celestial sphere, the

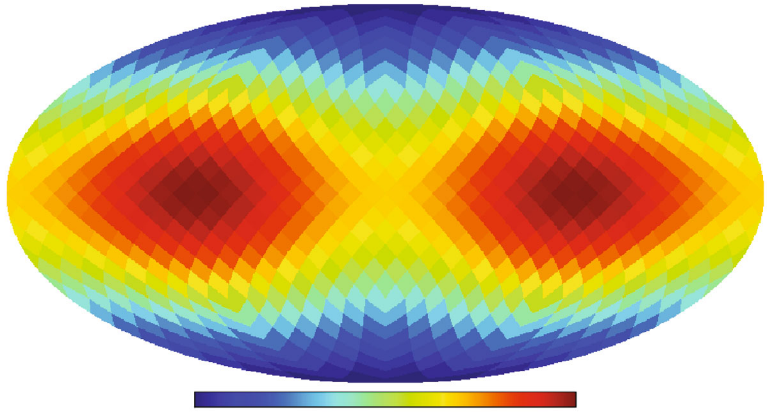

Fig. 6. The distribution of rms deviations based on Table 3 (mas).

TGAS parallaxes near the vernal equinox (zero longitude) are significantly larger than the Hipparcos ones.

Our analysis show that the statistically significant harmonics in the expansion of the rms deviation of the difference between the Hipparcos and TGAS parallaxes in terms of spherical harmonics in ecliptic coordinates have numbers 0 and 4. Thus, the statistical parallax difference depends mainly only on the ecliptic latitude. The large absolute value of the difference between the TGAS and Hipparcos parallaxes correlates sufficiently well with the Hipparcos parallax errors and the number of Hipparcos star observations (Fig. 7).

The zeroth expansion coefficient for the rms deviation of the parallax difference exceeds the expansion coefficient for the difference by an order of magnitude. This suggests that the difference between the Hipparcos and TGAS parallaxes has a *different sign* for different stars, while *this difference can also be large*. The differences between the parallaxes in the two catalogues are largely stochastic in nature.

Table 3. Statistically significant coefficients of the spherical harmonic expansion of the rms deviation of parallax differences in ecliptic coordinates

j	δ_j	σ_{δ_j}	$\frac{\lvert\delta_j\rvert}{\sigma_{\delta_j}}$
0	6.05	0.07	80.96
4	−0.83	0.07	11.07
8	−0.27	0.07	3.56

CONCLUSIONS

On the whole, this study showed that the Hipparcos and TGAS parallaxes are systematically similar, although the individual stellar parallaxes can differ significantly. Our study of the distribution of the rms deviations of the parallaxes in one catalogue from the other one showed that their amplitude in some regions of the celestial sphere could reach 2 mas (Fig. 6), which exceeds the formal declared accuracy even for Hipparcos. The differences for individual stars can reach tens of mas. If the TGAS parallaxes are assumed to be more reliable than the Hipparcos ones, then, in this case, the TGAS system of parallaxes is an improvement of the Hipparcos system precisely in terms of random errors.

Curiously, the systematic differences of the proper motions or positions in even similar (in construction) ground-based catalogues (Vityazev and Tsvetkov 2015) have a considerably more complex structure, including the systematic differences between the Tycho-2 and TGAS catalogues (Vityazev and Tsvetkov 2017).

A possible simple model of systematic differences is that the parallaxes of both catalogues were derived through space experiments, which led to highly homogeneous data.

However, there is also another hypothesis. The closeness of the TGAS and Hipparcos parallaxes from the systematic standpoint suggests that using the Hipparcos and Tycho-2 stellar coordinates as the first epoch allows the TGAS parallaxes to be deemed not completely independent of the data from the previous space mission. The description of the Gaia DR2 catalogue, where it is explicitly said that this version is, at last, independent of the Tycho-2 data, can serve as a confirmation of this fact. Publication of the

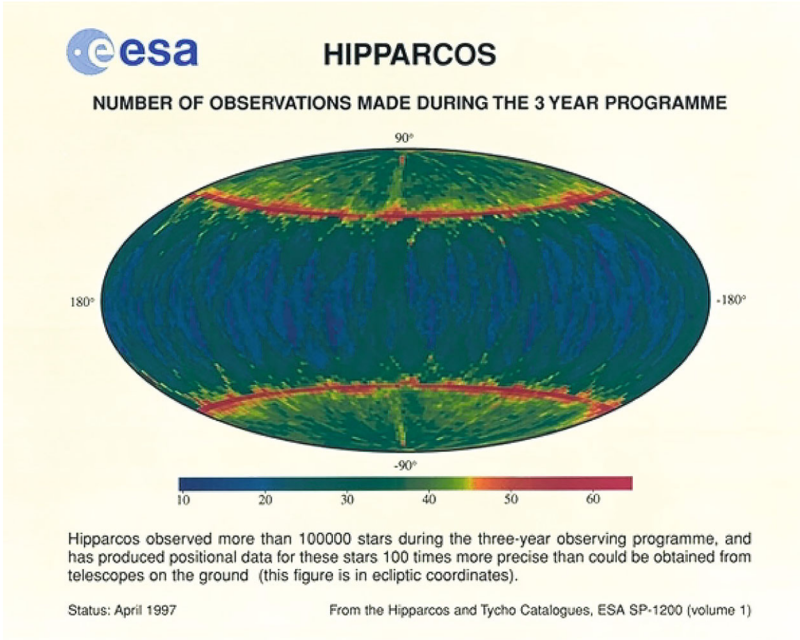

Fig. 7. The distribution of the number of Hipparcos observations of stars in ecliptic coordinates.

cross-tables of connections of DR2 with DR1 and Hipparcos will allow one to perform an appropriate study and to confirm or reject this hypothesis.

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