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Investigation of the Kinematics of Stars from the Gaia Data Release 2 with Radial Velocities Catalogue Using Scalar and Vector Spherical Harmonics

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Abstract—This paper is a continuation of our work. The proper motions and radial velocities of stars from the Gaia DR2 with RV catalogue have been decomposed. We have confirmed that the Ogorodnikov—Milne model is consistent with the observational data and found the kinematic components that are not described by this model. The beyond-the-model harmonics have been partially identified with the nonlinear terms of the extended Oort model and the parameters of the second-order kinematic model.

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INTRODUCTION

A kinematic analysis of the proper motions and radial velocities of 6 million stars from the Gaia DR2 with RV catalogue was performed by Tsvetkov and Amosov (2019). The parameters of the linear three-dimensional Ogorodnikov—Milne model (Ogorodnikov 1965) were determined from separate and simultaneous solutions. However, the traditional approach, which consists in solving the conditional equations by the least-squares method, has well-known shortcomings, because it does not allow the systematic components in the observational data that initially were not included in the model equations to be revealed. Using the apparatus of vector (when analyzing the proper motions) and scalar (when analyzing the radial velocities) spherical harmonics allows one not only to detect the disregarded systematic effects, but also to check whether the model is consistent with the observations. This technique was apparently first described by Vityazev and Tsvetkov (1989) and applied by Vityazev and Tsvetkov (1990) based data from the FK4 and earlier catalogues.

The complete form and the algorithm of calculating the vector spherical harmonics as well as the relations of the spherical harmonic decomposition coefficients to the Ogorodnikov—Milne model parameters (and the equations themselves) are given in Vityazev and Tsvetkov (2009, 2013). Similar information, but for the radial velocities, and the relations for the scalar spherical harmonic decomposition coefficients are presented in Vityazev and Tsvetkov (2014).

For the convenience of readers, in this paper we present only the tables of relations between the kinematic parameters and decomposition coefficients. The standard notation for the kinematic parameters is adopted in Tables 1 and 2:

U, V, W are the components of the translational velocity vector of the Sun V_0 among the stars, $\langle r\rangle$ is the mean distance of the group of stars under consideration;

ω_1, ω_2, ω_3 are the components of the angular velocity vector Ω;

M^+_{11}, M^+_{22}, M^+_{33} are the parameters of the deformation tensor describing the contraction—expansion along the principal axes of the Galactic coordinate system;

M^+_{12}, M^+_{13}, M^+_{23} are the parameters of the tensor $\mathbf{M}^+$ describing the velocity field deformation in the principal plane and two perpendicular planes.

The inverse relations, which we will not provide here, are also presented in Vityazev and Tsvetkov (2009).

DETERMINING THE DECOMPOSITION COEFFICIENTS FROM OBSERVATIONAL DATA

To maintain continuity and to be able to properly compare the results, we decomposed the stellar proper motions and radial velocities into vector and

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Table 1. Relations of the kinematic Ogorodnikov–Milne model parameters to the vector spherical decomposition coefficients of the stellar proper motions

Coefficient t_{nkp} or s_{nkp}	Value
t_{101}	$2.89\omega_3$
t_{110}	$2.89\omega_2$
t_{111}	$2.89\omega_1$
s_{101}	$-2.89W/\langle r\rangle$
s_{110}	$-2.89V/\langle r\rangle$
s_{111}	$-2.89U/\langle r\rangle$
s_{201}	$-0.65M_{11}^{+} - 0.65M_{22}^{+} + 1.29M_{33}^{+}$
s_{210}	$2.24M_{23}^{+}$
s_{211}	$2.24M_{13}^{+}$
s_{220}	$2.24M_{12}^{+}$
s_{221}	$1.12M_{11}^{+} - 1.12M_{22}^{+}$

Table 2. Relation of the kinematic Ogorodnikov–Milne model parameters to the scalar spherical harmonic decomposition coefficients of the stellar radial velocities

Coefficient t_{nkp} or s_{nkp}	Value
v_{001}	$1.18M_{11}^{+} + 1.18M_{22}^{+} + 1.18M_{33}^{+}$
v_{101}	$-2.05W/\langle r\rangle$
v_{110}	$-2.05V/\langle r\rangle$
v_{111}	$-2.05U/\langle r\rangle$
v_{201}	$-0.53M_{11}^{+} - 0.53M_{22}^{+} + 1.06M_{33}^{+}$
v_{210}	$1.83M_{23}^{+}$
v_{211}	$1.83M_{13}^{+}$
v_{220}	$1.83M_{12}^{+}$
v_{221}	$0.92M_{11}^{+} - 0.92M_{22}^{+}$

scalar spherical harmonics, respectively, based on distance samples of 400 000 stars, as was done in our previous paper (Tsvetkov and Amosov 2019). Since there is no multiband photometry in Gaia DR2, we did not separate the stars by any parameters except the distance. Here we will give only the boundaries of the samples of stars and the mean distance of the sample stars in Table 3.

For each sample we calculated the coefficients of the decomposition of the stellar proper motions into vector spherical harmonics (Tables 4 and 5) and of the radial velocities into scalar spherical harmonics (Table 6). We performed our calculations by directly using individual stars without any averaging. Since all spherical harmonics are orthonormal on the sphere, the root-mean-square (rms) errors of all coefficients are the same for each sample and we provide only one value. For the reader's convenience, the coefficients whose absolute value exceeds three rms errors (the so-called 3σ criterion) are highlighted in boldface in the tables.

ANALYSIS OF THE DECOMPOSITION COEFFICIENTS

Comparison of Tables 4 and 5 with Table 1 shows the presence of significant coefficients t_{101}, t_{110}, and t_{111} responsible for the solid-body rotation of the group of stars. The coefficient t_{111} showing the presence of rotation around the X axis is rather large. The remaining toroidal harmonics should be zero. However, we see that t_{301}, t_{411}, t_{321}, and some other harmonics are significant, but the main beyond-the-model component is the harmonic t_{211}; it is only slightly smaller than the main effect—the harmonic t_{101} due to the Galactic rotation around the Z axis.

A similar picture is observed when analyzing the spheroidal harmonics. The harmonics describing the translational motion of the Sun among the stars (s_{101}, s_{110}, and s_{111}) are very significant. As it must be, their values decrease with increasing distance. The harmonic s_{220} generated by the Oort parameter A is also large and does not depend on distance. The model harmonics s_{201} and s_{210} are small. Only the harmonic s_{211} responsible for the difference of the stellar sample contraction–expansion along the X and Y axes is fairly significant. Out of the beyond-the-model harmonics, s_{310} has a large value. This effect is also of the order of the Galactic rotation. The remaining beyond-the-model harmonics, though formally significant, are small.

To illustrate the data from Tables 4 and 5, we propose a technique that may be called the "spectrum" of proper motions. Figure 1 presents this spectrum for distances 835–1040 pc. For more compact data presentation we used the linear numbering of

Table 3. Boundaries of the samples of 400 000 star groups in pc

Min	3	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031
Max	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031	3677
Avr	144	255	343	430	522	627	757	933	1168	1447	1745	2056	2396	2796	3328

Table 4. Toroidal coefficients (in km s^{-1} kpc^{-1}) of the decomposition of the proper motions into vector spherical harmonics. The coefficients that are significant according to the 3σ criterion are highlighted in boldface

Min	3	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031
Max	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031	3677
t_{101}	**−35.8**	**−36.5**	**−35.9**	**−37.3**	**−37.9**	**−38.2**	**−38.4**	**−38.8**	**−39.8**	**−40.7**	**−41.2**	**−41.6**	**−41.7**	**−41.6**	**−41.2**
t_{110}	**−15.6**	**−6.4**	**−3.1**	**−3.0**	**−3.4**	**−3.6**	**−1.5**	−0.4	0.5	**1.3**	**1.5**	**1.4**	**0.8**	**1.1**	**0.9**
t_{111}	2.2	**4.3**	**2.0**	**3.1**	**2.5**	**1.2**	−0.3	**0.7**	**1.9**	**3.7**	**3.5**	**4.9**	**3.3**	**3.8**	**2.6**
t_{201}	0.2	0.5	0.3	−0.8	**−1.3**	**−1.4**	**−2.1**	**−1.3**	**−1.0**	**−0.7**	0.3	**0.7**	**1.4**	**1.2**	**1.4**
t_{210}	−1.2	−0.4	−1.3	**−2.3**	**−3.0**	**−3.5**	**−4.6**	**−3.6**	**−3.6**	**−2.9**	**−2.6**	**−1.9**	**−1.6**	−0.3	−0.2
t_{211}	**19.2**	**11.5**	**9.9**	**10.6**	**9.2**	**8.9**	**10.5**	**14.8**	**17.9**	**20.4**	**20.8**	**21.0**	**21.5**	**22.7**	**22.1**
t_{220}	−0.3	0.3	0.0	−0.2	−0.6	0.1	−0.4	−0.4	−0.3	**−0.8**	**−1.0**	**−0.9**	−0.5	0.0	−0.2
t_{221}	1.8	0.5	−0.3	−0.5	0.1	0.9	**1.0**	**1.3**	**1.2**	0.0	0.4	−0.5	−0.2	**−0.8**	−0.1
t_{301}	**−5.4**	**−2.0**	−1.3	−0.8	−0.2	−0.8	**−1.9**	**−3.1**	**−3.9**	**−3.4**	**−2.7**	**−1.7**	−0.6	0.1	**1.6**
t_{310}	2.0	**4.9**	**2.5**	**1.4**	−0.3	0.0	**1.6**	**2.3**	**2.1**	**1.8**	**0.7**	0.5	0.3	0.5	0.2
t_{311}	−0.8	**−1.9**	**−2.1**	0.0	−0.2	**−1.5**	**−2.1**	−0.3	0.4	**1.6**	**1.2**	**1.4**	0.3	0.6	−0.1
t_{320}	0.0	0.2	−0.6	0.5	0.6	**1.4**	**1.0**	−0.3	0.3	−0.3	−0.1	−0.2	0.0	−0.5	0.1
t_{321}	−1.2	0.8	−0.6	0.4	0.4	**1.0**	**1.3**	**2.5**	**2.4**	**3.1**	**3.6**	**3.5**	**3.9**	**4.1**	**4.3**
t_{330}	−1.0	0.2	−0.3	−0.6	0.1	−0.3	−0.6	−0.3	0.0	−0.3	0.2	0.0	0.5	0.0	0.2
t_{331}	2.0	−0.9	0.0	0.0	−0.4	−0.3	0.1	0.3	0.1	0.3	0.2	0.3	0.3	0.0	0.0
t_{401}	4.5	0.1	**−2.0**	−0.9	**−1.2**	**−1.2**	−0.4	0.5	**1.3**	0.4	**1.0**	**1.4**	**1.4**	**1.2**	**1.0**
t_{410}	−0.5	**−2.1**	−1.3	−0.5	0.0	−0.5	**−0.9**	−0.2	−0.3	0.6	**1.0**	**0.7**	**0.8**	**0.9**	**0.8**
t_{411}	−3.3	**−2.7**	**−2.2**	−1.0	**−2.2**	**−1.0**	**1.4**	**1.5**	−0.2	**−1.0**	**−2.4**	**−3.3**	**−3.4**	**−2.4**	**−2.4**
t_{420}	1.8	−0.4	−0.5	0.3	−0.4	0.2	−0.4	0.4	−0.3	−0.2	−0.1	0.0	0.4	0.7	0.3
t_{421}	−3.2	1.1	−1.1	1.1	0.5	0.7	0.5	0.4	**0.7**	**−0.9**	−0.3	**−0.8**	−0.5	**−0.8**	−0.1
t_{430}	−1.1	−0.6	0.6	0.2	−0.3	−0.7	−0.1	0.3	0.3	0.0	0.1	0.3	0.3	0.5	0.4
t_{431}	−1.5	0.1	−0.6	−1.0	−0.3	−0.1	0.0	0.6	**0.9**	0.5	0.6	**0.7**	0.0	0.0	−0.4
t_{440}	0.7	0.1	−0.2	−0.2	−0.2	0.0	−0.1	0.0	−0.2	0.0	0.0	0.0	−0.1	0.0	0.0
t_{441}	1.2	−0.8	−0.1	0.0	0.4	0.0	−0.2	−0.1	0.1	0.0	0.1	0.1	0.0	0.1	0.0
σ	1.7	0.6	0.5	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2

Table 5. Spheroidal coefficients (in km s^{-1} kpc^{-1}) of the decomposition of the proper motions into vector spherical harmonics. The coefficients that are significant according to the 3σ criterion are highlighted in boldface

Min	3	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031
Max	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031	3677
s_{101}	**−179**	**−88.2**	**−66.4**	**−52.8**	**−43.7**	**−36.7**	**−30.9**	**−25.9**	**−21.1**	**−16.7**	**−14.3**	**−11.9**	**−10.5**	**−9.2**	**−8.5**
s_{110}	**−517**	**−255**	**−194**	**−156**	**−130**	**−110**	**−98.3**	**−93.8**	**−91.1**	**−86.9**	**−82.6**	**−79.6**	**−77.7**	**−77.6**	**−77**
s_{111}	**−229**	**−110**	**−85.5**	**−69.4**	**−59.1**	**−50.1**	**−44.7**	**−38.1**	**−31.9**	**−27**	**−23.5**	**−19.9**	**−17.4**	**−14.3**	**−12.5**
s_{201}	−2.9	**2.4**	**2.6**	**3.3**	**3.2**	**2.9**	**2.8**	**2.2**	**2.5**	**2**	**1.4**	**2**	**1.4**	**1.8**	**1.9**
s_{210}	−1.5	**−2.5**	0.5	−0.8	**−1**	0.9	**2.8**	**1.5**	−0.5	**−2.1**	**−2.3**	**−3.2**	**−1.5**	**−1.8**	**−1**
s_{211}	**−6.1**	**−2.5**	−0.4	**−2.5**	**−2.9**	**−2.8**	**−0.7**	−0.4	−0.4	−0.3	−0.3	−0.4	**−1.2**	**−0.8**	**−1**
s_{220}	**38.7**	**36.2**	**34.6**	**33.7**	**33.6**	**34.1**	**33.5**	**31.5**	**30.1**	**28**	**26.9**	**25.6**	**24.4**	**23.3**	**21.3**
s_{221}	−3.5	**−6.1**	**−7.3**	**−7.4**	**−7.8**	**−7.2**	**−5.8**	**−4.7**	**−3.6**	**−3.3**	**−2.8**	**−3.2**	**−2.4**	**−2.6**	**−1.2**
s_{301}	**−5.8**	−0.9	−1.1	−0.6	**−1.6**	**−1.9**	**−2.4**	**−2**	**−1.7**	−0.5	**−1.1**	−0.5	−0.6	**−0.7**	**−1.4**
s_{310}	**−15.4**	**−7.5**	**−7.2**	**−8**	**−7.7**	**−7.6**	**−8.8**	**−12.1**	**−14.4**	**−15.5**	**−15.3**	**−15.8**	**−16.4**	**−17.2**	**−17.5**
s_{311}	−2.5	−0.1	−0.2	−1.2	**−1**	**−1.3**	**−2.5**	**−2.3**	**−1.7**	**−1.2**	**−0.9**	−0.6	−0.5	−0.1	0.1
s_{320}	−3.4	0	−0.3	0.6	−0.2	−0.7	**−1.5**	**−1.8**	**−1.3**	−0.1	−0.3	0.6	0.2	**0.7**	0.3
s_{321}	−1.3	−0.4	−0.8	−0.6	−0.7	−0.3	−0.3	−0.4	−0.2	−0.6	−0.6	−0.5	−0.2	0.2	0.2
s_{330}	2	**1.9**	**2.1**	**2.1**	**2**	**2**	**1.5**	**1.6**	**2.1**	**3.2**	**3.3**	**3.5**	**4**	**3.9**	**4.3**
s_{331}	−0.3	−1.3	−1.4	**−1.9**	−0.9	**−1.1**	**−1.3**	**−1**	**−1**	**−1.6**	**−1.6**	**−1.3**	**−1.2**	**−1.1**	**−1.1**
s_{401}	0.4	−1.3	0.7	−0.3	−0.3	−0.1	−0.2	−0.4	**0.8**	0.5	0.6	0.6	0.4	**0.7**	**0.9**
s_{410}	0.2	−0.3	1.2	**−1.3**	−0.6	0.7	**1.3**	0.4	**−0.7**	**−1**	**−0.9**	−0.6	0	0.2	0.4
s_{411}	**5.6**	**3**	1.4	−0.3	−0.5	0.7	**1.1**	0.6	−0.3	−0.6	**−0.9**	−0.6	**−0.9**	−0.5	**−0.7**
s_{420}	0.5	−0.1	0	−0.7	**−1.3**	−0.8	**−1**	**−2**	**−2.2**	**−2.9**	**−3.1**	**−3**	**−3.4**	**−3.3**	**−3.9**
s_{421}	−4.1	−1.1	−0.4	−0.2	**1.3**	**1.3**	**1**	0.3	0	−0.2	−0.3	**−0.7**	0	−0.3	0.1
s_{430}	−0.4	**1.9**	0.9	0.6	0.6	0.6	−0.1	−0.4	−0.2	−0.5	0	−0.2	−0.3	0.2	0
s_{431}	−1.3	0.2	0.1	−0.7	−0.2	−0.2	0.1	−0.3	−0.1	−0.3	0	0	0.4	0	0.3
s_{440}	0	0.2	0	0.7	0.7	−0.1	**−0.8**	**−0.7**	**−0.9**	0	0.1	0	0.3	0.6	**0.9**
s_{441}	4.1	0.5	−0.5	0	−0.4	0.7	**0.7**	**0.8**	**0.7**	0.6	0.4	0.1	0	−0.2	−0.1
σ	1.7	0.6	0.5	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2

Table 6. Coefficients of the decomposition of the radial velocities (km s^{-1}) into spherical harmonics. The coefficients that are significant according to the 3σ criterion are highlighted in boldface

Min	3	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031
Max	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031	3677
$v001$	**0.8**	−0.6	**−1.0**	**−2.0**	**−2.8**	**−2.7**	**−3.4**	**−4.0**	**−5.1**	**−4.7**	**−2.6**	**−2.8**	−0.7	−0.2	−0.3
$v101$	**−16.2**	**−16.3**	**−16.0**	**−16.9**	**−17.6**	**−16.7**	**−16.0**	**−16.4**	**−16.3**	**−17.0**	**−16.1**	**−16.5**	**−16.6**	**−16.2**	**−18.3**
$v110$	**−43.9**	**−44.0**	**−43.8**	**−43.7**	**−43.8**	**−43.6**	**−44.3**	**−48.6**	**−57.7**	**−65.7**	**−73.5**	**−81.6**	**−90.2**	**−99.5**	**−110.3**
$v111$	**−20.6**	**−20.3**	**−20.8**	**−21.0**	**−20.9**	**−21.0**	**−21.2**	**−22.0**	**−22.9**	**−24.4**	**−25.4**	**−26.2**	**−25.5**	**−25.5**	**−24.3**
$v201$	0.1	0.3	0.2	0.4	**1.1**	**1.6**	**2.2**	**2.1**	**1.7**	**2.0**	**3.8**	**3.0**	**4.6**	2.8	2.4
$v210$	−0.5	0.0	0.1	−0.1	−0.2	−0.2	0.1	0.4	0.9	−1.2	−0.7	**−2.2**	**−2.3**	**−3.6**	−1.7
$v211$	**−1.4**	**−1.8**	**−1.2**	**−1.3**	**−1.5**	**−1.7**	**−2.4**	**−2.2**	**−1.8**	**−1.5**	−0.9	**−1.6**	−0.2	−1.1	0.5
$v220$	**4.8**	**8.1**	**10.3**	**12.3**	**15.1**	**17.5**	**20.2**	**23.6**	**28.1**	**32.5**	**37.0**	**41.8**	**46.4**	**51.2**	**55.1**
$v221$	**−0.7**	**−1.9**	**−2.2**	**−2.7**	**−3.6**	**−3.4**	**−3.5**	**−3.5**	**−2.7**	**−2.8**	**−1.8**	−1.2	−1.0	−0.7	0.1
$v301$	0.4	0.0	**−0.8**	**−0.7**	**−0.7**	**−0.8**	−0.2	−0.6	−0.7	**−1.8**	**−1.7**	**−2.1**	**−3.0**	**−3.7**	**−5.2**
$v310$	**−0.8**	**−1.9**	**−2.5**	**−3.2**	**−3.6**	**−4.5**	**−5.3**	**−8.5**	**−15.0**	**−20.4**	**−26.1**	**−32.1**	**−38.7**	**−46.5**	**−53.4**
$v311$	0.5	0.3	0.0	−0.1	**−0.7**	−0.5	**−1.1**	**−1.4**	**−1.5**	**−2.0**	**−1.8**	**−1.6**	−0.5	0.1	1.6
$v320$	−0.2	−0.2	0.4	0.1	−0.3	−0.5	−0.2	−0.8	−0.5	−0.5	−0.6	−0.5	0.6	−0.1	0.2
$v321$	−0.2	0.0	0.0	−0.1	−0.3	0.1	0.0	−0.2	−0.5	−0.2	−0.4	−0.7	−0.4	−1.2	−1.5
$v330$	0.0	0.2	**0.7**	**0.8**	**0.9**	**1.0**	**1.1**	**1.6**	**2.4**	**3.5**	**4.4**	**5.9**	**7.6**	**9.7**	**13.0**
$v331$	−0.2	−0.1	0.0	−0.4	−0.2	−0.3	−0.5	**−1.0**	**−1.9**	**−2.4**	**−2.8**	**−2.9**	**−3.2**	**−2.8**	**−3.2**
$v401$	0.0	0.0	0.2	0.0	0.3	0.5	0.6	0.2	0.1	−0.3	0.1	0.5	2.0	1.4	2.6
$v410$	−0.1	0.2	0.5	0.5	0.0	−0.2	0.4	0.6	**1.1**	−0.1	−0.1	−0.8	−0.3	−0.4	1.1
$v411$	0.2	**0.8**	**1.1**	0.0	0.0	0.3	0.3	0.6	0.9	1.2	1.6	1.4	**2.7**	2.6	3.6
$v420$	−0.1	−0.1	−0.3	−0.5	**−0.8**	**−0.8**	**−1.6**	**−2.3**	**−3.3**	**−5.7**	**−7.8**	**−9.6**	**−12.4**	**−15.2**	**−20.3**
$v421$	0.1	0.1	0.0	0.3	−0.1	0.6	**1.0**	**1.1**	**1.1**	−0.3	−0.3	−1.0	**−2.4**	**−3.6**	**−3.0**
$v430$	−0.2	0.3	0.1	0.3	0.3	0.0	−0.4	−0.4	−0.6	−0.6	−1.1	−1.0	−0.8	−0.3	−0.4
$v431$	0.1	0.1	−0.2	−0.3	0.0	−0.2	−0.5	−0.2	0.0	0.2	0.9	0.4	0.6	0.1	−0.3
$v440$	0.0	−0.1	0.4	0.4	0.1	−0.4	−0.5	−0.8	−0.7	−0.2	0.3	1.0	**1.8**	**2.9**	**3.9**
$v441$	−0.3	0.2	−0.1	−0.1	−0.1	0.2	0.4	0.5	0.9	**1.3**	1.2	1.0	0.7	0.6	−0.7
$\sum$	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.5	0.5	0.6	0.7

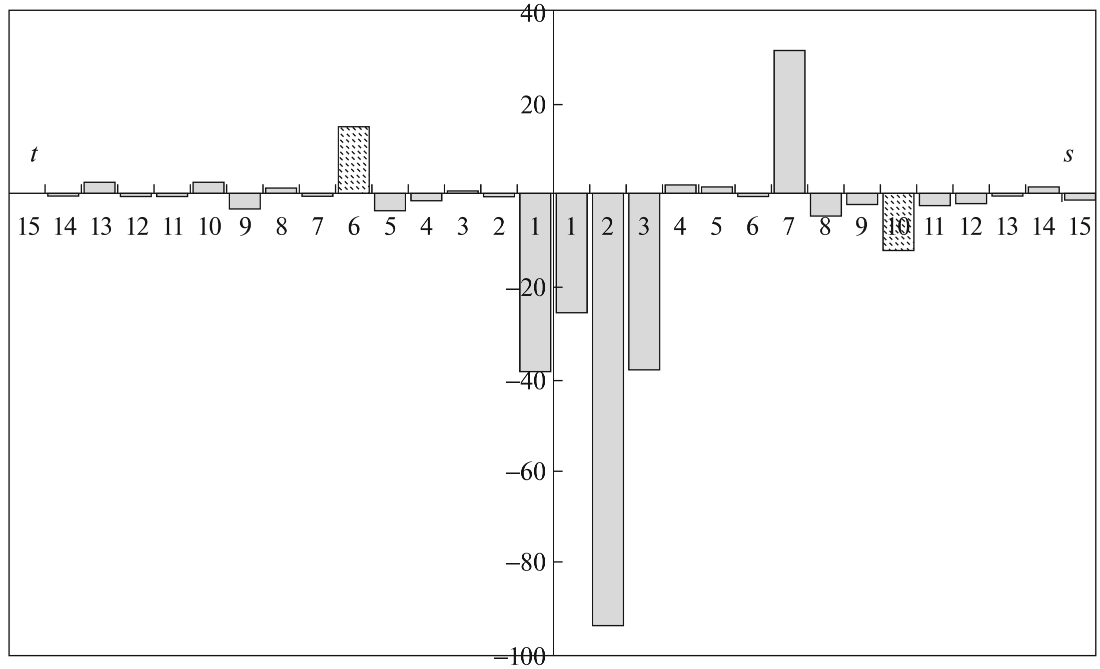

Fig. 1. Decomposition spectrum of the stellar proper motions for distances 835–1040 pc: the coefficients t_j (left) and s_j (right). The linear numbering of the coefficients is used. The coefficients in km s^{-1} kpc^{-1} are along the vertical axis.

the coefficients according to the following formula (Brosche 1966):

$$j = n^2 + 2k + p - 1, \quad (1)$$

which allows the three indices n, k, p to be transformed into one.

In this numbering the strong beyond-the-model harmonics are designated as t_6 and s_{10} and are hatched in the figure.

Let us now analyze the decomposition coefficients of the radial velocities (Table 6) and compare them with the data from Table 2. The strongest effect, this is the solar motion, corresponds to the coefficients v_{101}, v_{110}, and v_{111}. The differential Galactic rotation is clearly traceable by the coefficient v_{220} beginning from distances of several hundred parsecs. The remaining significant model harmonics are comparatively small (v_{221}, v_{001}). Again, v_{310} and, for large distances, v_{420} strongly stand out among the beyond-the-model harmonics. We observe a growth of the harmonics v_{220} and v_{310} due to their kinematic nature, because the Ogorodnikov–Milne model for the radial velocities has no distances only in the functions at the solar motion parameters, while they are present for the remaining terms. The reverse is true for the proper motions. The solar terms depend on distance, while the terms describing the Galactic kinematics do not depend on distance in the linear approximation. For this reason, the distance dependence of the beyond-the-model harmonic v_{310} serves as evidence for its kinematic nature of the disregarded stellar motion.

The decomposition spectrum of the radial velocities for stars at the same distances 835–1040 pc is presented in Fig. 2. It also uses the linear numbering of the coefficients. The strong beyond-the-model harmonic, v_{10}, is hatched.

Below we summarize the results of our analysis of the decomposition coefficients for both stellar proper motions and radial velocities.

1. The kinematics of the nearest stars differs significantly from the kinematics of more distant stars in both systematic and random terms (large errors of the coefficients). This is a well-known fact stemming from the presence of an anomaly in the kinematics of the Local System of stars (Tsvetkov 1995, 1999) and the peculiar velocities, which distort noticeably the proper motions of nearby stars.

2. There exist stable kinematic effects in the stellar motions that are not described by the model, namely the coefficients t_{211}, s_{310}, and v_{310} (or t_6, s_{10}, and v_{10}). This fact is less known, although it was revealed when analyzing the stellar proper motions from the Tycho-2 catalogue and the radial velocities from the OSACA catalogue (Vityazev and Shuksto 2004; Vityazev and Tsvetkov 2009). The latter means

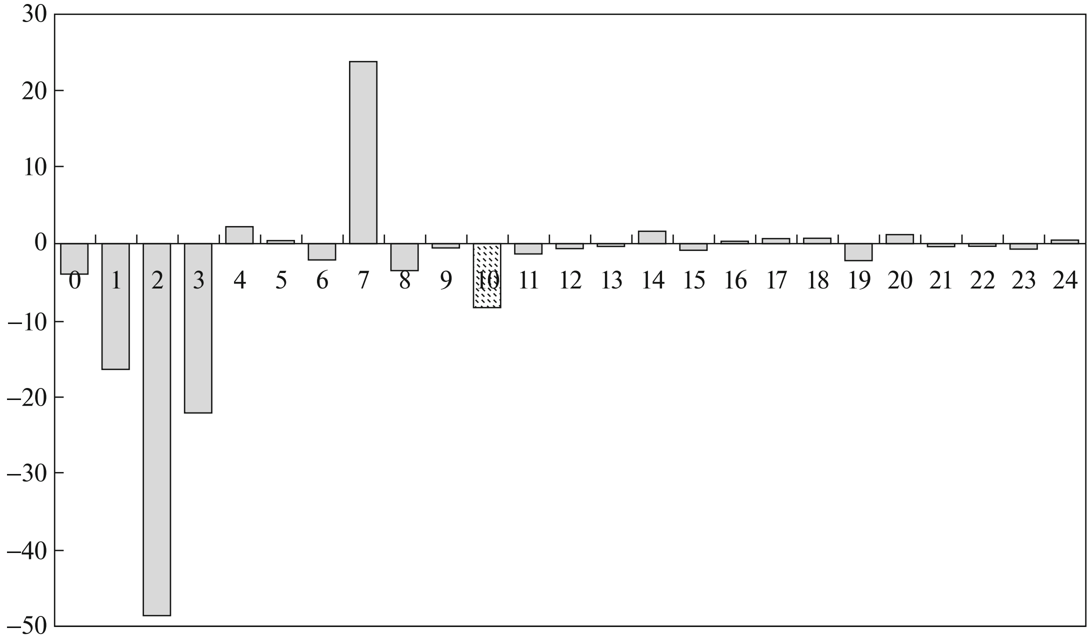

Fig. 2. Decomposition spectrum of the stellar radial velocities for distances 835–1040 pc. The linear numbering of the coefficients is used. The coefficients in km s^{-1} are along the vertical axis.

that, apparently, disregarded effects are indeed present in the kinematic picture of the stellar motions in circumsolar space. Before the appearance of the Gaia catalogue, there was a slight probability that these harmonics could result from the systematic errors in the stellar proper motions of the catalogues.

POSSIBLE NATURE OF THE BEYOND-THE-MODEL TERMS IN THE DECOMPOSITIONS

The systematic significance of the harmonics t_{211}, s_{310}, and v_{310} needs an explanation. A possible explanation is the presence of nonlinear terms in the Galactic rotation model. In the simplest case, this is a generalized Oort–Lindblad model. Generally, these equations are a special case of Bottlinger's formulas described in detail by Bobylev and Bajkova (2014, 2017). We present these equations as they are given in Vityazev and Tsvetkov (2009):

$$k\mu_l \cos b = U/r \sin l - V/r \cos l + A\cos b \cos 2l + B\cos b - rF\cos^2 b \cos^3 l - rG(3\cos^2 b \cos l - \cos^2 b \cos^3 l), \quad (2)$$

$$k\mu_b = U/r \cos l \sin b + V/r \sin l \sin b - W/r \cos b - A \sin b \cos b \sin 2l + rF\cos^2 b \sin b \sin l \cos^2 l + rG\cos^2 b \sin b \sin^3 l - K\cos b \sin b, \quad (3)$$

$$V_r/r = -U/r \cos b \cos l - V/r \cos b \sin l - W/r \sin b + A\cos^2 b \sin 2l - rF\cos^3 b \sin l \cos^2 l - rG\cos^3 b \sin^3 l + K\cos^2 b. \quad (4)$$

Here:

- $k = 4.738$ is the conversion factor from mas yr^{-1} to km s^{-1} kpc^{-1};
- l, b, r are the stellar Galactic coordinates;
- U, V, W are the components of the translational velocity vector of the Sun $\mathbf{V}_0$ among the stars;
- $A = 0.5R_0\omega_0'$ and $B = 0.5R_0\omega_0' + \omega_0$ are the Oort parameters, R_0 is the distance to the Galactic center, ω_0 is the angular velocity of Galactic rotation (recall that $A = M_{12}^+$ and $B = \omega_3$);
- K is the total contraction–expansion of the system in the XY plane;
- F and G are the second-order Oort parameters, $F = 0.5R_0\omega_0''$ and $G = A/R_0$.

If we perform a theoretical decomposition of Eqs. (2)–(4) into spherical harmonics, then, in addition to Tables 1 and 2, we will obtain the result presented in Table 7.

Table 7. Contribution of the generalized Oort model to the scalar and vector spherical harmonic decomposition coefficients

j	n	k	p	v_{nkp}	t_{nkp}	s_{nkp}
0	0	0	1	$2.363K$		
1	1	0	1	$-2.047W/\langle r\rangle$	$2.894B$	$-2.894W/\langle r\rangle$
2	1	1	0	$-2.047V/\langle r\rangle$ $-0.409F\langle r\rangle - 1.228G\langle r\rangle$		$-2.894V/\langle r\rangle$ $-1.158F\langle r\rangle - 3.473G\langle r\rangle$
3	1	1	1	$-2.047U\langle r\rangle$		$-2.894U/\langle r\rangle$
4	2	0	1	$-1.057K$		$-1.294K$
5	2	1	0			
6	2	1	1		$-0.747F\langle r\rangle - 2.242G\langle r\rangle$	
7	2	2	0	$1.831A$		$2.242A$
8	2	2	1			
9	3	0	1			
10	3	1	0	$0.109F\langle r\rangle + 0.328G\langle r\rangle$		$0.126F\langle r\rangle + 0.379G\langle r\rangle$
11	3	1	1			
12	3	2	0			
13	3	2	1			
14	3	3	0	$-0.424F\langle r\rangle + 0.424G\langle r\rangle$		$-0.489F\langle r\rangle + 0.489G\langle r\rangle$
15	3	3	1			

We see that the presence of t_{211} and s_{310} in the proper motions and v_{310} in the radial velocities can be explained by the extended Oort model. Unfortunately, it is impossible to separately obtain F and G from these three harmonics due to the linear dependence. Indeed,

$$v_{310} = 0.109(F + 3G)\,\langle r\rangle\,, \qquad (5)$$
$$t_{211} = -0.747(F + 3G)\,\langle r\rangle\,,$$
$$s_{310} = 0.126(F + 3G)\,\langle r\rangle\,,$$

i.e., we can determine the combination $(F + 3G)\,\langle r\rangle$. For stars at 835–1040 pc (the mean distance is about 0.933 kpc) we have

$$v_{310} \to F + 3G = -8.5/0.109/0.933 = -83.6, \quad (6)$$
$$t_{211} \to F + 3G = 14.8/(-0747)/0.933 = -21.2,$$
$$s_{310} \to F + 3G = -12.1/0.126/0/933 = -83.6.$$

As we see, the values obtained from the radial velocities and spheroidal harmonics are close to one another, while the value determined from the toroidal harmonic stands out.

A further examination of Table 7 forces us to check the coefficients v_{330} and s_{330} that are also generated by the parameters F and G. An analysis of these harmonics shows that, on the whole, they are small (with the exception of v_{330} for distant stars). This

Table 8. Parameters of the extended Oort model derived from the simultaneous solution of Eqs. (2)–(4)

Min	3	208	300	386	474	571	687	835
Max	208	300	386	474	571	687	835	1040
U	9.2 ± 0.0	8.9 ± 0.0	9.3 ± 0.0	9.4 ± 0.0	9.6 ± 0.0	9.8 ± 0.0	10.1 ± 0.0	10.6 ± 0.1
V	19.5 ± 0.1	19.9 ± 0.1	20.1 ± 0.1	20.4 ± 0.1	20.4 ± 0.1	20.3 ± 0.1	20.4 ± 0.1	22.0 ± 0.1
W	7.9 ± 0.0	7.8 ± 0.0	7.8 ± 0.0	7.9 ± 0.0	7.9 ± 0.0	7.9 ± 0.0	7.8 ± 0.0	8.0 ± 0.0
A	16.5 ± 0.5	15.8 ± 0.2	15.1 ± 0.1	14.8 ± 0.1	15.0 ± 0.1	15 ± 0.1	14.8 ± 0.1	14.5 ± 0.0
B	-11.0 ± 0.5	-11.0 ± 0.2	-11.1 ± 0.1	-11.7 ± 0.1	-12.1 ± 0.1	-12.3 ± 0.1	-12.3 ± 0.1	-12.3 ± 0.0
F	-13.9 ± 11.8	-10.5 ± 2.7	-10.2 ± 1.5	-8.1 ± 0.9	-6.3 ± 0.4	-4.8 ± 0.4	-2.7 ± 0.2	-2.6 ± 0.1
G	2.9 ± 4.4	2.4 ± 1.1	3.2 ± 0.6	1.5 ± 0.4	1.6 ± 0.2	1.5 ± 0.2	1.6 ± 0.1	1.4 ± 0.1
K	2.8 ± 0.5	-1.2 ± 0.2	-1.2 ± 0.1	-1.8 ± 0.1	-2.0 ± 0.1	-1.7 ± 0.1	-1.8 ± 0.1	-1.7 ± 0.0

Min	1040	1303	1594	1897	2220	2582	3031
Max	1303	1594	1897	2220	2582	3031	3677
U	11.1 ± 0.1	11.6 ± 0.1	12.1 ± 0.1	12.5 ± 0.1	12.6 ± 0.1	13.0 ± 0.1	13.2 ± 0.1
V	25.1 ± 0.1	27.8 ± 0.1	29.8 ± 0.1	31.2 ± 0.1	32.7 ± 0.1	33.3 ± 0.1	33.6 ± 0.1
W	8.1 ± 0.0	8.2 ± 0.1	8.3 ± 0.1	8.5 ± 0.1	8.7 ± 0.1	8.8 ± 0.1	9.1 ± 0.1
A	14.0 ± 0.0	13.6 ± 0.0	13.3 ± 0.0	12.9 ± 0.0	12.6 ± 0.0	12.1 ± 0.0	11.4 ± 0.0
B	-12.5 ± 0.0	-13.0 ± 0.0	-13.3 ± 0.0	-13.7 ± 0.0	-14.1 ± 0.0	-14.3 ± 0.0	-14.6 ± 0.0
F	-2.9 ± 0.1	-3.4 ± 0.1	-2.9 ± 0.1	-2.7 ± 0.0	-2.4 ± 0.0	-2.0 ± 0.0	-1.6 ± 0.0
G	1.1 ± 0.1	1.1 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	1.0 ± 0.0	1.1 ± 0.0
K	-1.7 ± 0.0	-1.4 ± 0.0	-1.1 ± 0.0	-0.8 ± 0.0	-0.6 ± 0.0	-0.3 ± 0.0	-0.1 ± 0.0

means that either these kinematic effects are absent or the parameters F and G are approximately equal.

A comparison of these facts points to a contradiction in the direct interpretation of the beyond-the-model harmonics as a manifestation of the effects of the extended Oort model.

For ultimate clarification, we simultaneously solved Eqs. (2)–(4) based on the stellar proper motions and radial velocities using the same samples by taking into account the individual distances to the stars. The results are presented in Table 8. For the same sample of stars with a mean distance of about 0.93 kpc we have $F = -2.9$ km s^{-1} kpc^{-2} and $G = 1.4$ km s^{-1} kpc^{-2}, which is in excellent agreement with the values obtained by Bobylev and Bajkova (2014), but is in serious contradiction to Eqs. (6), from which it follows that F and G should be much greater in absolute value. Thus, the large values of the harmonics t_{211}, s_{310}, and v_{310} cannot be explained by the extended Oort model. Although the

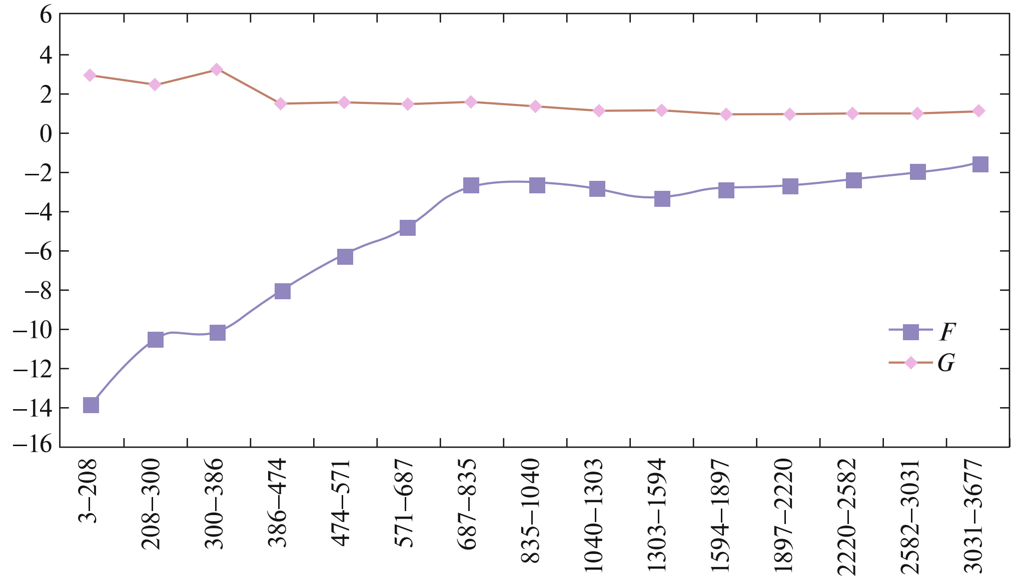

Fig. 3. (Color online) Distance dependence of the parameters F and G. The coefficients in km s^{-1} kpc^{-1} are along the vertical axis.

harmonic $s_{330} = 0.42(G-F)$ is in good agreement with the parameters F and G found, if we attempt to derive F and G, for example, from a combination of the harmonics s_{310} and s_{330}, then we will obtain unrealistically large values, $F \approx 26$ km s^{-1} kpc^{-2} and $G \approx -23$ km s^{-1} kpc^{-2}.

There is another fact from which it follows that the parameters F and G should have relatively small values. The nonlinear terms have the meaning of the derivatives of the Oort parameters with respect to the distance. However, the results of our previous paper (Tsvetkov and Amosov 2019) showed a surprising stability of the Oort parameters ($B = \omega_3$, $A = M_{12}^+$) for significant distance ranges; consequently, the derivatives of these parameters should be small. The values of many kinematic parameters are large for nearby stars, which is explained by the anomalies of the Local System of stars (Tsvetkov 1995). The parameters F and G decrease from distances greater than 500 pc and are stabilized (Fig. 3).

CONTRIBUTION OF THE SECOND-ORDER THREE-DIMENSIONAL MODEL TO THE SPHERICAL HARMONIC DECOMPOSITION COEFFICIENTS

We may consider a complete second-order three-dimensional model. Apparently, a separate paper should be devoted to a detailed derivation of the equations due to fairly cumbersome calculations. First it should be said that it apparently makes no sense to solve the second-order equations in view of the correlations, because it will be possible to obtain only linear combinations of the parameters. In this case, it seems most appropriate to apply the spherical harmonic decomposition method and to use the derived coefficients to analyze the nonlinear part of the model.

Let us introduce the partial derivatives of the kinematic parameters along the principal X, Y, Z axes of the Galactic coordinate system denoted by $\frac{\partial}{\partial r_1}$, $\frac{\partial}{\partial r_2}$, $\frac{\partial}{\partial r_3}$, respectively. The decomposition of the derived second-order equations into scalar (for the radial velocities) and vector (for the proper motions) spherical harmonics is presented in Tables 9–11, where, for compactness, we omit the factor $\langle r \rangle$ (the mean distance to the group of stars under consideration) in each partial derivative. The contribution of the first-order linear model, which has already been presented in Tables 1 and 2, is also present in these tables.

We see that a large number of derivatives of the kinematic parameters enter into the individual harmonics as a linear combination. In some cases, the second-order parameters are superimposed on the coefficients that were previously assumed to be dependent only on the first-order parameters (s_{101}, s_{110}, s_{111}, v_{101}, v_{110}, v_{111}).

Let us return, however, to the harmonics under consideration that have a large value. Let us write out the complete expressions from Tables 9–11 separately for them by grouping the terms in such a way that it is convenient to analyze them. We have (to within the factor $\langle r \rangle$)

$$s_{310} = 0.13\left(-\frac{\partial M_{11}^+}{\partial r_2} - 2\frac{\partial M_{12}^+}{\partial r_1} - 3\frac{\partial M_{22}^+}{\partial r_2} + 4\frac{\partial M_{33}^+}{\partial r_2} + 8\frac{\partial M_{23}^+}{\partial r_3}\right), \quad (7)$$

Table 9. Contribution of the second-order kinematic model to the toroidal vector spherical harmonic decomposition coefficients of the stellar radial velocities. In all partial derivatives the factor $\langle r \rangle$ was omitted

j	N	k	p	t_j
1	1	0	1	$2.89\omega_3$
2	1	1	0	$2.89\omega_2$
3	1	1	1	$2.89\omega_1$
4	2	0	1	$-0.65\frac{\partial\omega_1}{\partial r_1}-0.65\frac{\partial\omega_2}{\partial r_2}+1.30\frac{\partial\omega_3}{\partial r_3}-0.65\frac{\partial M_{13}^+}{\partial r_2}+0.65\frac{\partial M_{23}^+}{\partial r_1}$
5	2	1	0	$1.12\frac{\partial\omega_2}{\partial r_3}+1.12\frac{\partial\omega_3}{\partial r_2}-0.37\frac{\partial M_{12}^+}{\partial r_2}+0.37\frac{\partial M_{13}^+}{\partial r_3}+0.37\frac{\partial M_{22}^+}{\partial r_1}-0.37\frac{\partial M_{33}^+}{\partial r_1}$
6	2	1	1	$1.12\frac{\partial\omega_1}{\partial r_3}+1.12\frac{\partial\omega_3}{\partial r_1}-0.37\frac{\partial M_{11}^+}{\partial r_2}+0.37\frac{\partial M_{12}^+}{\partial r_1}-0.37\frac{\partial M_{23}^+}{\partial r_3}+0.37\frac{\partial M_{33}^+}{\partial r_2}$
7	2	2	0	$1.12\frac{\partial\omega_1}{\partial r_2}+1.12\frac{\partial\omega_2}{\partial r_1}+0.37\frac{\partial M_{11}^+}{\partial r_3}-0.37\frac{\partial M_{13}^+}{\partial r_1}-0.37\frac{\partial M_{22}^+}{\partial r_3}+0.37\frac{\partial M_{23}^+}{\partial r_2}$
8	2	2	1	$1.12\frac{\partial\omega_1}{\partial r_1}-1.12\frac{\partial\omega_2}{\partial r_2}-0.75\frac{\partial M_{12}^+}{\partial r_3}+0.37\frac{\partial M_{13}^+}{\partial r_2}+0.37\frac{\partial M_{23}^+}{\partial r_1}$

$$v_{310}=0.11\biggl(-\frac{\partial M_{11}^+}{\partial r_2}-2\frac{\partial M_{12}^+}{\partial r_1}-3\frac{\partial M_{22}^+}{\partial r_2}+4\frac{\partial M_{33}^+}{\partial r_2}+8\frac{\partial M_{23}^+}{\partial r_3}\biggr), \quad (8)$$

$$t_{211}=0.37\biggl(-\frac{\partial M_{11}^+}{\partial r_2}+\frac{\partial M_{12}^+}{\partial r_1}-\frac{\partial M_{23}^+}{\partial r_3}+\frac{\partial M_{33}^+}{\partial r_2}+3\biggl(\frac{\partial\omega_1}{\partial r_3}+\frac{\partial\omega_3}{\partial r_1}\biggr)\biggr). \quad (9)$$

An analysis of Eqs. (7)–(9) shows that there are four kinematic parameters, $\frac{\partial M_{11}^+}{\partial r_2}$, $\frac{\partial M_{12}^+}{\partial r_1}$, $\frac{\partial M_{23}^+}{\partial r_3}$, and $\frac{\partial M_{33}^+}{\partial r_2}$, that enter into all three coefficients. In addition, the parameter $\frac{\partial M_{22}^+}{\partial r_2}$ enters into the coefficients s_{310} and v_{310}, while $\frac{\partial\omega_1}{\partial r_3}$ and $\frac{\partial\omega_3}{\partial r_1}$ enter into t_{211}. This can probably explain the similarity of the behavior of s_{310} and v_{310} and the difference of t_{211} if our analysis is performed within the extended Oort model.

The theoretical ratio of the coefficients s_{310} and v_{310} coincides almost exactly with the ratio of the coefficients derived using the catalogue data. This means that the linear combination of parameters in Eqs. (7) and (8) has the same value when analyzing the radial velocities and proper motions.

The above reasoning does not solve the problem of identifying the beyond-the-model harmonics with some specific parameters of the kinematic model, because the system of equations specified by Tables 9–11 is underdetermined. The number of parameters to be determined exceeds the number of decomposition coefficients. Furthermore, we see that the coefficients can be proportional to one another and only some additional criteria for the model consistency with observations can be used.

Some additional information (for example, about the insignificance of some second-order parameters) that would allow us, if not to completely obtain the values of all parameters, to obtain at least the values of their less complex linear combinations, is needed for a complete description of the system within the second-order model.

We can only propose the following simplification: let us leave only the derivatives of $M_{12}^+ = A$ and $\omega_3 = B$ with respect to r_1 in Eqs. (7)–(9) by assuming the remaining values to be small; then, Eqs. (7)–(9) are reduced to

$$s_{310}=-0.26\frac{\partial M_{12}^+}{\partial r_1}\langle r\rangle, \quad (10)$$
$$v_{310}=-0.22\frac{\partial M_{12}^+}{\partial r_1}\langle r\rangle,$$
$$t_{211}=0.37\left(\frac{\partial M_{12}^+}{\partial r_1}+3\frac{\partial\omega_3}{\partial r_1}\right)\langle r\rangle.$$

Here we did not omit the factor $\langle r\rangle$. Taking the coefficients for the range 835–1040 from Tables 4–6

Table 10. Contribution of the second-order kinematic model to the spheroidal vector spherical harmonic decomposition coefficients of the stellar proper motions. In all partial derivatives the factor $\langle r\rangle$ was omitted

j	N	k	p	t_j
1	1	0	1	$-2.89W/r + 1.45\frac{\partial\omega_1}{\partial r_2} - 1.45\frac{\partial\omega_2}{\partial r_1} - 0.29\frac{\partial M^+_{11}}{\partial r_3} + 0.87\frac{\partial M^+_{13}}{\partial r_1} - 0.29\frac{\partial M^+_{22}}{\partial r_3} + 0.87\frac{\partial M^+_{23}}{\partial r_2} + 0.56\frac{\partial M^+_{33}}{\partial r_3}$
2	1	1	0	$-2.89V/r - 1.45\frac{\partial\omega_1}{\partial r_3} + 1.45\frac{\partial\omega_3}{\partial r_1} - 0.29\frac{\partial M^+_{11}}{\partial r_2} + 0.87\frac{\partial M^+_{12}}{\partial r_1} + 0.58\frac{\partial M^+_{22}}{\partial r_2} + 0.87\frac{\partial M^+_{23}}{\partial r_3} - 0.29\frac{\partial M^+_{33}}{\partial r_1}$
3	1	1	1	$-2.89U/r + 1.45\frac{\partial\omega_2}{\partial r_3} - 1.45\frac{\partial\omega_3}{\partial r_2} + 0.58\frac{\partial M^+_{11}}{\partial r_1} + 0.87\frac{\partial M^+_{12}}{\partial r_2} + 0.87\frac{\partial M^+_{13}}{\partial r_3} - 0.29\frac{\partial M^+_{22}}{\partial r_1} - 0.29\frac{\partial M^+_{33}}{\partial r_1}$
4	2	0	1	$-0.65M^+_{11} - -0.65M^+_{22} + 1.30M^+_{33}$
5	2	1	0	$2.24M^+_{23}$
6	2	1	1	$2.24M^+_{13}$
7	2	1	0	$2.24M^+_{12}$
8	2	2	1	$1.12M^+_{11} - 1.12M^+_{22}$
9	3	0	1	$-0.31\frac{\partial M^+_{11}}{\partial r_3} - 0.62\frac{\partial M^+_{13}}{\partial r_1} - 0.31\frac{\partial M^+_{12}}{\partial r_3} - 0.62\frac{\partial M^+_{23}}{\partial r_2} + 0.62\frac{\partial M^+_{33}}{\partial r_3}$
10	3	1	0	$-0.13\frac{\partial M^+_{11}}{\partial r_2} - 0.25\frac{\partial M^+_{12}}{\partial r_1} - 0.38\frac{\partial M^+_{22}}{\partial r_2} + 1.01\frac{\partial M^+_{23}}{\partial r_3} + 0.51\frac{\partial M^+_{33}}{\partial r_2}$
11	3	1	1	$-0.38\frac{\partial M^+_{11}}{\partial r_1} - 0.25\frac{\partial M^+_{12}}{\partial r_2} + 1.01\frac{\partial M^+_{13}}{\partial r_3} - 0.13\frac{\partial M^+_{22}}{\partial r_2} + 0.51\frac{\partial M^+_{33}}{\partial r_1}$
12	3	2	0	$0.80\frac{\partial M^+_{12}}{\partial r_3} + 0.80\frac{\partial M^+_{13}}{\partial r_2} + 0.80\frac{\partial M^+_{23}}{\partial r_1}$
13	3	2	1	$0.40\frac{\partial M^+_{11}}{\partial r_3} + 0.80\frac{\partial M^+_{13}}{\partial r_1} - 0.40\frac{\partial M^+_{22}}{\partial r_3} - 0.80\frac{\partial M^+_{23}}{\partial r_2}$
14	3	3	0	$0.49\frac{\partial M^+_{11}}{\partial r_2} + 0.98\frac{\partial M^+_{12}}{\partial r_1} - 0.49\frac{\partial M^+_{22}}{\partial r_2}$
15	3	3	1	$0.49\frac{\partial M^+_{11}}{\partial r_1} - 0.98\frac{\partial M^+_{12}}{\partial r_2} - 0.49\frac{\partial M^+_{22}}{\partial r_1}$

with the mean $\langle r\rangle = 0.933$ kpc:

$$t_{211} = 14.8, \quad s_{310} = -12.1, \quad v_{310} = -8.5,$$

from s_{310} and v_{310} we obtain the mean $\frac{\partial M^+_{12}}{\partial r_1} = 44$ km s^{-1} kpc^2, while $\frac{\partial\omega_3}{\partial r_1} \approx 0$!

This is a rather strange result, given that $\frac{\partial M^+_{12}}{\partial r_1} = \frac{\partial A}{\partial r_1}$, which largely coincides in its meaning with the Oort parameter F. We assumed other partial derivatives of the kinematic Ogorodnikov—Milne model parameters to be close to zero simply because these parameters themselves are usually small. However, a small value of the parameters does not imply that their derivatives are also small. Thus, the question about the nonlinear effects in the stellar velocity field needs to be investigated further. Some evidence that the beyond-the-model coefficients t_{211}, s_{310}, and v_{310} are a manifestation of the nonlinear effects is the increase in their absolute values with distance, because $\langle r\rangle$, the mean distance to the group of stars under consideration, always enters into the nonlinear parameters.

Table 11. Contribution of the second-order kinematic model to the spherical harmonic decomposition coefficients of the stellar radial velocities. In all partial derivatives the factor $\langle r\rangle$ was omitted

j	N	k	p	v_j/r
0	0	0	1	$1.18M^+_{11}+1.18M^+_{22}+1.18M^+_{33}$
1	1	0	1	$-2.05W/r+0.41\frac{\partial M^+_{11}}{\partial r_3}+0.41\frac{\partial M^+_{22}}{\partial r_3}+1.23\frac{\partial M^+_{33}}{\partial r_3}+0.82\frac{\partial M^+_{13}}{\partial r_1}+0.82\frac{\partial M^+_{23}}{\partial r_2}$
2	1	1	0	$-2.05V/r+0.41\frac{\partial M^+_{11}}{\partial r_2}+1.23\frac{\partial M^+_{22}}{\partial r_2}+0.41\frac{\partial M^+_{33}}{\partial r_2}+0.82\frac{\partial M^+_{12}}{\partial r_1}+0.82\frac{\partial M^+_{23}}{\partial r_3}$
3	1	1	1	$-2.05U/r+1.23\frac{\partial M^+_{11}}{\partial r_1}+1.41\frac{\partial M^+_{22}}{\partial r_1}+0.41\frac{\partial M^+_{33}}{\partial r_1}+0.82\frac{\partial M^+_{12}}{\partial r_2}+0.82\frac{\partial M^+_{13}}{\partial r_3}$
4	2	0	1	$-0.53M^+_{11}--0.53M^+_{22}+1.06M^+_{33}$
5	2	1	0	$1.83M^+_{23}$
6	2	1	1	$1.83M^+_{13}$
7	2	1	0	$1.831M^+_{12}$
8	2	2	1	$0.92M^+_{11}-0.92M^+_{22}$
9	3	0	1	$-0.27\frac{\partial M^+_{11}}{\partial r_3}-0.27\frac{\partial M^+_{22}}{\partial r_3}+0.54\frac{\partial M^+_{33}}{\partial r_3}-0.54\frac{\partial M^+_{23}}{\partial r_2}$
10	3	1	0	$-0.11\frac{\partial M^+_{11}}{\partial r_2}-0.33\frac{\partial M^+_{22}}{\partial r_2}+0.44\frac{\partial M^+_{33}}{\partial r_2}-0.22\frac{\partial M^+_{12}}{\partial r_1}+0.88\frac{\partial M^+_{23}}{\partial r_3}$
11	3	1	1	$-0.33\frac{\partial M^+_{11}}{\partial r_1}-0.11\frac{\partial M^+_{22}}{\partial r_1}+0.44\frac{\partial M^+_{33}}{\partial r_1}-0.22\frac{\partial M^+_{12}}{\partial r_2}+0.88\frac{\partial M^+_{13}}{\partial r_3}$
12	3	2	0	$0.69\frac{\partial M^+_{12}}{\partial r_3}+0.69\frac{\partial M^+_{13}}{\partial r_2}+0.69\frac{\partial M^+_{23}}{\partial r_1}$
13	3	2	1	$0.35\frac{\partial M^+_{11}}{\partial r_3}-0.35\frac{\partial M^+_{22}}{\partial r_3}+0.69\frac{\partial M^+_{13}}{\partial r_1}-0.69\frac{\partial M^+_{23}}{\partial r_2}$
14	3	3	0	$0.42\frac{\partial M^+_{11}}{\partial r_2}-0.42\frac{\partial M^+_{22}}{\partial r_2}+0.85\frac{\partial M^+_{12}}{\partial r_1}$
15	3	3	1	$0.42\frac{\partial M^+_{11}}{\partial r_1}-0.42\frac{\partial M^+_{22}}{\partial r_1}-0.85\frac{\partial M^+_{12}}{\partial r_2}$

There are probably local kinematic effects closer than 500 pc (Fig. 4).

The nature of the harmonics t_{211}, s_{310}, and v_{310} may be different altogether (peculiarities of the distribution of stars, stellar streams, something else). Interestingly, there exist significant coefficients (for example, v_{420} for large distances) that cannot be interpreted even within the second-order model.

CONCLUSIONS

Our study showed that such kinematic effects as the translational motion of the Sun and the solid-body rotation mainly around the Z axis are definitely present in the stellar proper motions and radial velocities, but there is also a smaller effect around the X axis. The presence of velocity field deformation in the XY plane is also beyond doubt. The remaining components of the linear model are minor. The existence of strong harmonics t_{211}, s_{310}, and v_{310} that

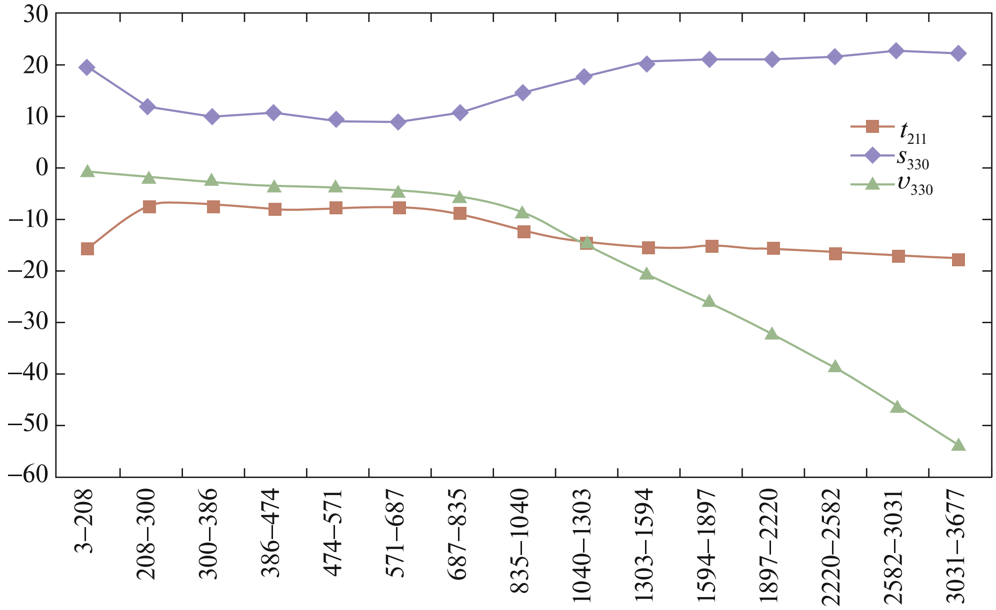

Fig. 4. (Color online) Distance dependence of the harmonics t_{211}, s_{310}, and v_{310}. The coefficients in km s^{-1} kpc^{-1} are along the vertical axis.

are not described by the linear Oort–Lindblad and Ogorodnikov–Milne stellar-kinematics models is a puzzle. An attempt to directly tie them to the extended Oort–Lindblad model allowed their existence to be explained only partly because of the emerging contradiction in determining the parameters from the harmonics s_{310}, v_{310}, and t_{211}. The key to understanding the nature of these harmonics may lie in using the complete second-order three-dimensional model. However, this is a complicated question that needs additional studies. We are planning to devote a special publication to the second-order model.

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