ISSN 1063-7737, Astronomy Letters, 2015, Vol. 41, No. 10, pp. 575–592.
Original Russian Text © V.V. Vityazev, A.S. Tsvetkov, 2015, published in Pis'ma v Astronomicheskiĭ Zhurnal, 2015, Vol. 41, No. 10, pp. 624–641.

Comparison of the XPM and UCAC4 Catalogs

V. V. Vityazev* **and A. S. Tsvetkov****

St. Petersburg State University, Bibliotechnaya pl. 2, St. Petersburg, 198504 Russia
Received April 27, 2015

Abstract—The systematic differences between the stellar positions and proper motions of the XPM and UCAC4 catalogs have been obtained in the form of decompositions into vector spherical harmonics by taking into account the magnitude equation. The systematic components have been extracted with a probability of at least 0.98 by dividing 41 316 676 stars into groups corresponding to 12 J magnitude bins with a width of 0.5^m for mean values from $10\overset{m}{.}25$ to $15\overset{m}{.}75$. A study of the systematic differences between the equatorial coordinates suggests that the range of systematic differences between the XPM and UCAC4 positions exceeds the corresponding range of differences between PPMXL and UCAC4 by a factor of 5, especially for bright stars in the range being investigated. Analysis of the orientation of the XPM and UCAC4 reference frames has shown that their mutual rotation is 2–4 mas around the X axis and 7–10 mas around the Z axis. These angles depend on the magnitude of stars. Since two systems of proper motions are given in the XPM catalog, XPMx and XPMp, we have decomposed the proper motion differences XPMx–XPMp into vector spherical harmonics and found these differences to be free from the magnitude equation. An important fact is that the first-order zonal coefficients have turned out to be greatest in absolute value. The toroidal coefficient $t_{1,0,1,0}$ found has shown that the XPMx and XPMp reference frames of proper motions rotate relative to each other around the Z axis with an angular velocity of 0.45 mas yr^{-1}. It should be added that the range of systematic differences XPMx–XPMp is 2.1 mas yr^{-1} in right ascension and 1.7 mas yr^{-1} in declination. The angular velocities of mutual rotation of the XPMp and UCAC4 reference frames change within the range from 0.6 to 2.2 mas yr^{-1}, while the analogous range for the XPMx and UCAC4 catalogs is 0.3–1.8 mas yr^{-1}. The angular velocity and coordinates of the pole of the mutual rotation axis depend on the magnitude of stars. The parameters of the mutual rotation around the Z axis derived from the differences XPMx–UCAC4 and XPMp–UCAC4 change from 0.03 ± 0.06 to 1.73 ± 0.06 mas yr^{-1} and from 0.49 ± 0.06 to 2.19 ± 0.06 mas yr^{-1}, respectively. Based on our analysis, we have shown that the XPM catalog actually comprises two catalogs, XPM(XSC) and XPM(PSC), in which the stellar positions coincide at the standard epoch J2000 and differ at any other epoch. The decomposition coefficients of the systematic differences XPM–UCAC4 we obtained allow the stellar positions and proper motions from one catalog to be reduced to the system of the other catalog by taking into account this duality.

DOI: 10.1134/S1063773715100072

Keywords: *astrometry, comparison of catalogs, spherical harmonics, magnitude equations.*

INTRODUCTION

At present, the UCAC4 (Zacharias et al. 2013) and XPM (Fedorov et al. 2009, 2010) catalogs are widely used to solve various astronomical problems (Gontcharov et al. 2011; Damljanovich et al. 2012). The UCAC4 catalog contains 113 million stars covering the entire sky with magnitudes from 8 to 16 in a nonstandard photometric band between V and R. The positional accuracy at the mean epoch is estimated to be within the range 15–100 mas, while the formal errors of the proper motions lie within the range 1–10 mas yr^{-1}. The systematic errors of the proper motions lie within the range 1–4 mas yr^{-1}. The catalog contains the positions and proper motions and is deemed complete to $R = 16$. UCAC4 is the last catalog in the UCAC (USNO CCD Astrograph Catalog) project. No photographic observations were used in this project, and all measurements were made between 1998 and 2004 using only CCD detectors.

The reference frame for UCAC4 is known to have been the HCRF, which, in turn, is based on the stellar positions and proper motions of the Hipparcos catalog. The stellar positions in this catalog were referred to the positions of quasars, while the stellar proper motions were tied to the ICRF using the following observational programs (Perryman

*E-mail: vityazev@list.ru
**E-mail: a.s.tsvetkov@inbox.ru

et al. 1997): (1) VLBI-1999 (radio stars), (2) HST 78 (HST observations), (3) EOP (Earth orientation parameters), (4) KSZ, Lick NPM, and SPM (stellar proper motions absolutized relative to galaxies). The stellar proper motions relative to quasars were used in the first three programs; the proper motions relative to galaxies were calculated in the fourth program. Thus, the HCRF stellar proper motions were determined relative to some combined inertial reference frame realized with quasars and galaxies (Kovalevsky et al. 1997). The accuracy of the corrections applied to the stellar proper motions of the preliminary H37 catalog to tie it to the ICRF is estimated to be ± 0.25 mas yr^{-1}. Obviously, it characterizes a measure of HCRF inertiality relative to the above quasar-galaxy reference frame. Despite the fact that both quasars and galaxies can theoretically serve as a basis for constructing inertial reference frames, nevertheless, the difference in the specifics of observations of these objects can lead to systematic stellar proper motion differences (especially with regard to the magnitude equation). It should be added that using all of the currently available observational data suggests that the residual HCRF rotation (a measure of its inertiality) is determined with an error of 0.1 mas yr^{-1} (Bobylev 2015).

In contrast to UCAC4, whose proper motions are specified in the reference frame realized with quasars and galaxies, the main goal of the authors of the XPM catalog was to obtain the absolute proper motions of stars using an extragalactic reference frame realized only with galaxies. The idea of using galaxies as an inertial reference frame was initiated in Pulkovo by Dneprovsky and Gerasimovic (1932). A list of the most significant works in this direction can be found in Fedorov et al. (2009). The XPM catalog was constructed by taking data from the 2MASS (Skrutskie et al. 2006) and USN0-A2.0 (Monet 1998) catalogs. It contains $\sim$314 million stars in the range $12^m < B < 19^m$ with uniform coverage of the entire sky. The resulting catalog contains the stellar positions in the ICRS at epoch J2000.0 and the original proper motions obtained by comparing the stellar positions in the 2MASS and USN0-A2 catalogs. The mean epoch differences are 45 and 17 years for the northern and southern hemispheres, respectively. The proper motion errors lie within the range from 3 to 10 mas yr^{-1}. The zero point of the system of absolute proper motions (absolute calibration) was determined from more than one million galaxies contained in the 2MASS and USNO-A2.0 catalogs. The XPM catalog is an independent (from the HCRF) realization of an extragalactic reference frame. When tying the stellar proper motions of the XPM catalog to galaxies, its authors encountered the fact that there are two sets (PSC and XSC) of extended sources (galaxies) in the 2MASS catalog, but the positions of common objects in these subcatalogs differ systematically by up to 25 mas. For this reason, the authors provide two systems of absolute proper motions obtained from the PSC and XSC catalogs, which below we will designate as XPMp and XPMx.

The XPM and UCAC4 catalogs realize the reference frames constructed in the optical band using hundreds of millions of stars. In accordance with the requirements of astrometry, it is necessary to compare the XPM catalog with other catalogs to be able to pass from the XPM system to the system of another catalog. Fedorov et al. (2011a) presented the systematic differences between the stellar proper motions in the equatorial coordinate system as functions of magnitude in graphical form. Such differences were obtained for XPM–UCAC2, XPM–UCAC3, XPM–PPMXL, and a number of other catalogs separately for the northern and southern hemispheres. At the same time, the dependence of the differences on the positions of stars in each of the hemispheres was completely ignored. Fedorov et al. (2011b) compared the absolute proper motions of the XPM catalog with the proper motions of the same stars from the PPMXL, UCAC3, Tycho-2, and XC1 catalogs. The authors did not study all components of the systematic differences but restricted themselves only to obtaining the components of the mutual rotation of the axes of the equatorial reference frames for these catalogs. The authors of the UCAC4 catalog (Zacharias et al. 2013) compared the XPM and UCAC4 stellar proper motions in a narrow RA zone from 6.0 to 6.1 h in the declination range from -60° to -30°.

A proper solution of the problem of comparing catalogs (Bien et al. 1978; Mignard and Froeschle 2000) suggests representing the systematic differences between positions and proper motions of stars by the systems of orthogonal harmonics describing their dependence on the coordinates and magnitudes of stars. To all appearances, the XPM and UCAC4 catalogs have not yet subjected to comparison with such a degree of completeness, and this paper is devoted to remedying this shortcoming. We solve three problems. The first is to represent the systematic stellar position and proper motion differences XPM–UCAC4 as decompositions into vector orthogonal harmonics. In contrast to previous similar works, here we propose a new statistical criterion that allows one to estimate the significance of all the harmonics in the decomposition of the individual differences into vector spherical harmonics for the chosen HealPix pixelization scheme (Gorski et al. 2005). Normalized Legendre polynomials are used to approximate the decomposition coefficients derived from groups of stars with different magnitudes. The constructed models of systematic differences are used to analyze the systematic differences as functions of three variables (α, δ, m). The second problem is to estimate the result of passing from the "quasar—galaxy" reference frame to the "galaxy" one, because, as has already been said, the UCAC4 and XPM stellar proper motions were determined in different reference frames. The third problem is to compare the two versions of the XPM stellar proper motions obtained by ambiguously tying the stellar proper motions to galaxies based on data from two catalogs, XSC and PSC.

REPRESENTATION OF THE SYSTEMATIC DIFFERENCES XPM-UCAC4 BY VECTOR SPHERICAL HARMONICS

Brosche (1966) was the first to represent the systematic differences between the positions and proper motions of stars by scalar spherical harmonics. The modification of this method proposed by Bien et al. (1978) became a standard tool for a separate comparison of the systems of right ascensions and declinations of the astrometric catalogs preceding Hipparcos. The joint use of both systems is based on the application of vector spherical harmonics (hereafter VSHs). This approach was proposed by Mignard and Morando (1990) and Mignard and Froeschle (2000) and was applied to compare the Hipparcos and FK5 catalogs. A further development of this method pursuing the goals of the GAIA project can be found in Mignard and Klioner (2012). In this paper, we used the method based on the application of VSHs including the magnitude equation that was first proposed by Vityazev and Tsvetkov (2015) to study the systematic differences between the PPMXL and UCAC4 catalogs. Below, we describe its main steps to obtain the systematic positions and proper motion differences XPM−UCAC4.

In the first step, we used 1200 pixels with an area of 34.4 square degrees. The sphere was partitioned into pixels according to the HealPix scheme (Gorski et al. 2005). In this scheme, the number N_{pix} is the key parameter (resolution parameter) defining the partition of the sphere into equal pixels. The total number of pixels is $N = 12N_{\text{pix}}^2$. The entire sphere is divided by two parallels with declinations $\pm\arcsin(2/3)$ into three parts, the equatorial and two polar ones. $N_{\text{pix}} - 1$ parallels are chosen in each of the polar zones; the number of parallels in the equatorial zone is $(2N_{\text{pix}} + 1)$. The centers of $4N_{\text{pix}}$ pixels lie on each parallel of the equatorial region. The two parallels closest to the poles always contain four pixels each, while the number of pixels on each parallel increases by one when moving from the poles to the equator in the polar zones. The pixels are numbered $j = 0, 1, \ldots, N-1$ along the parallels from north to south.

A list of 41 316 676 stars belonging to our catalogs was compiled using the star identification procedure in the J band (2MASS photometric system). The identification procedure considered the stars in different catalogs identical if the position difference did not exceed 500 mas and if the J magnitude difference was less than 0.01. This criterion (rather than a complete coincidence, as it should actually be) was chosen for purely technical reasons: when reading one and the other catalogs and saving them in binary format, a situation where the real numbers (REAL*4) were "almost equal" could arise. Actually, the magnitudes just coincided in almost all cases, and there were no difficulties with the cross-identification of stars.

After averaging the differences between the stellar positions and proper motions of each pair of catalogs over the pixels, we formed the stellar position and proper motion differences XPM−UCAC4 in the equatorial coordinate system referred to the centers of our pixels and forming the vector fields

$$\Delta\mathbf{F}(\alpha, \delta, m) = \begin{cases} \Delta\alpha\cos\delta\mathbf{e}_\alpha + \Delta\delta\mathbf{e}_\delta, \\ \Delta\mu_\alpha\cos\delta\mathbf{e}_\alpha + \Delta\mu_\delta\mathbf{e}_\delta, \end{cases} \tag{1}$$

where $\mathbf{e}_\alpha$ and $\mathbf{e}_\delta$ are the mutually orthogonal unit vectors, respectively, in the directions of change in right ascension and declination. These fields were formed for the stars belonging to 12 J magnitude bins with a width of 0.5^m for mean values from $10^m\!.25$ to $15^m\!.75$.

In the second step, we approximated the vector fields (1) by VSHs in accordance with the formula

$$\Delta\mathbf{F}(\alpha, \delta, m) = \sum_{nkp} t_{nkp}(m)\mathbf{T}_{nkp}(\alpha, \delta) + \sum_{nkp} s_{nkp}(m)\mathbf{S}_{nkp}(\alpha, \delta). \tag{2}$$

In this formula, the toroidal, $\mathbf{T}_{nkp}(\alpha, \delta)$, and spheroidal, $\mathbf{S}_{nkp}(\alpha, \delta)$, VSHs were defined via the scalar spherical harmonics $K_{nkp}(\alpha, \delta)$ as follows (Arfken 1970):

$$\mathbf{T}_{nkp}(\alpha, \delta) = \frac{1}{\sqrt{n(n+1)}} \times \left(\frac{\partial K_{nkp}(\alpha, \delta)}{\partial\delta}\mathbf{e}_\alpha - \frac{1}{\cos\delta}\frac{\partial K_{nkp'}(\alpha, \delta)}{\partial\alpha}\mathbf{e}_\delta\right), \tag{3}$$

$$\mathbf{S}_{nkp}(\alpha, \delta) = \frac{1}{\sqrt{n(n+1)}} \times \left(\frac{1}{\cos\delta}\frac{\partial K_{nkp}(\alpha, \delta)}{\partial\alpha}\mathbf{e}_\alpha + \frac{\partial K_{nkp}(\alpha, \delta)}{\partial\delta}\mathbf{e}_\delta\right). \tag{4}$$

Expanded analytical formulas to calculate these harmonics are given in Vityazev and Tsvetkov (2015).

When specifying the initial data on the HealPix grid, the approximation coefficients t_{nkp} and s_{nkp} are calculated from the formulas

$$t_{nkp}(m_i) = \frac{4\pi}{N}\sum_{j=0}^{N-1}\Delta\mathbf{F}(\alpha_j,\delta_j,m_i)\mathbf{T}_{n,k1,p}(\alpha_j,\delta_j), \tag{5}$$

$$s_{nkp}(m_i) = \frac{4\pi}{N}\sum_{j=0}^{N-1}\Delta\mathbf{F}(\alpha_j,\delta_j,m_i)\mathbf{S}_{n,k1,p}(\alpha_j,\delta_j). \tag{6}$$

Here, the index $j = 0, 1, \ldots, N-1$ numbers the pixels in the direction from the north pole to the south one, and $m_i = 0.5^m i$, $i = 0, 1, \ldots, 11$, denote the mean magnitudes of each magnitude bins.

To separate the signal from noise, the calculation of the decomposition coefficients is accompanied by the determination of their significance level. The determination of the significance is based on the fact that the coefficients s_{nkp} and t_{nkp} for normally distributed centered noise with variance $\sigma_0^2 = 1$ are normally distributed random variables with zero mean and unit variance. Consequently, the squares of the amplitudes s^2_{nkp} and t^2_{nkp} are random variables distributed according to the χ^2 law with one degree of freedom. On this basis, we can estimate the probability q that s^2_{nkp} and t^2_{nkp} exceed a threshold X:

$$q = \int\limits_X^\infty p_k(x)dx, \tag{7}$$

where $p_k(x)$ is the density of the χ^2 distribution with k degrees of freedom.

Hence it follows that the determination of the significance of each harmonic is based on testing the a priori hypothesis that the initial data are discrete centered noise with unit variance. This hypothesis is tested for each harmonic and is rejected with a probability $p = 1 - q$ if the square of the decomposition coefficient of the centered normalized data sequence exceeds the detection threshold X determined from Eq. (7).

To determine the significance of the decomposition coefficients, it is necessary to run a check of all the harmonics that can be calculated on a chosen grid of points. This requirement is reduced to establishing the boundary values of the indices k and n. To choose the largest $k_{\max}$, we can use the fact that $k = k_{\max} = 4N_{\text{pix}}$ is the boundary value in the sense that each VSH with indices $n = k > 4N_{\text{pix}}$ will give a false value of the decomposition coefficient with indices $n = k < 4N_{\text{pix}}$. Therefore, the harmonics should be tested in index k for $k = 0, 1, \ldots, 4N_{\text{pix}} - 1$. A constraint on the index n can be derived from the condition that the sought-for decomposition coefficients are obtained with a specified accuracy. Since there exists a constraint on the accuracy of calculating the squares of the norms of the basis functions on a discrete grid of HealPix pixel centers, the limiting value of our series $n = k, k+1, \ldots, n_{\max}$ for each admissible index k is determined from the condition that a specified accuracy (for example, one percent) of calculating the squares of the norms of the basis functions breaks down:

$$\left|1 - \frac{4\pi}{N}\sum_{j=0}^{N-1}\mathbf{S}(n_{\max},k,p,\alpha_j,\delta_j) \times \mathbf{S}(n_{\max},k,p,\alpha_j,\delta_j)\right| > 0.01, \quad p = 0, 1, \tag{8}$$

$$\left|1 - \frac{4\pi}{N}\sum_{j=0}^{N-1}\mathbf{T}(n_{\max},k,p,\alpha_j,\delta_j) \times \mathbf{T}(n_{\max},k,p,\alpha_j,\delta_j)\right| > 0.01, \quad p = 0, 1. \tag{9}$$

Thus, within the ranges of admissible indices k and n, the inequalities

$$\frac{N}{4\pi}\tilde{s}^2_{nkp} > X; \quad \frac{N}{4\pi}\tilde{t}^2_{nkp} > X \tag{10}$$

suggest that the coefficients with indices n, k, and p are determined with a specified probability $p = 1 - q$ by the presence of a corresponding harmonic rather than noise. In these formulas, $\tilde{s}_{nkp}$ and $\tilde{t}_{nkp}$ are calculated from Eqs. (5) and (6) with the centered and normalized stellar position and proper motion differences.

The indices (n, k, p) of the statistically significant harmonics were determined at $X = 6.7$ in Eqs. (10), which corresponds to the detection of harmonics with a probability of 0.99 according to the χ^2 test. For our pixelization scheme ($N_{\text{pix}} = 10$), the limiting value of the index k is $k = 39$, while the highest values of the indices n were determined from conditions (8) and (9). Thereafter, the coefficients $t_{nkp}(m_i)$ and $s_{nkp}(m_i)$ themselves and their root-mean-square (rms) errors $\sigma_s(m_i)$ and $\sigma_t(m_i)$ were determined by the least-squares method (LSM) from the selected set of statistically significant harmonics for each group of stars with the mean magnitude m_i. Here, the final set of statistically significant harmonics was established using the 2–3σ criterion. Obviously, the significance level of this list is 97.7–99.9%.

Table 1. Boundaries and ranges of systematic stellar position differences XPM−UCAC4 and PPMXL−UCAC4 as a function of the J magnitude of stars, mas

J	$(\Delta\alpha\cos\delta)_{\min}$	$(\Delta\alpha\cos\delta)_{\max}$	ΔRA	$(\Delta\delta)_{\min}$	$(\Delta\delta)_{\max}$	ΔDEC
XPM−UCAC4						
11^m	−36.3	50.6	86.9	−48.3	29.3	77.6
13^m	−26.6	30.2	56.8	−7.8	18.8	26.6
15^m	−24.9	22.2	47.1	−4.8	16.7	21.5
PPMXL−UCAC4						
11^m	−9.7	6.5	17.2	−7.5	8.1	15.6
13^m	−12.9	7.2	20.1	−10.3	7.8	18.1
15^m	−18.9	10.2	29.1	−11.6	12.0	23.6

In the third step, the coefficients $t_{nkp}(m)$ and $s_{nkp}(m)$ were approximated by the expressions:

$$t_{nkp}(m) = \sum_r t_{nkpr} Q_r(\bar{m}), \qquad (11)$$

$$s_{nkp}(m) = \sum_r s_{nkpr} Q_r(\bar{m}),$$

where

$$Q_r(\bar{m}) = \sqrt{\frac{2r+1}{2}} P_r(\bar{m}), \qquad (12)$$

and $P_r(\bar{m})$ are Legendre polynomials; the following recurrence relation can be used to calculate the latter:

$$P_{r+1}(\bar{m}) = \frac{2r+1}{r+1}\bar{m}P_r(\bar{m}) - \frac{r}{r+1}P_{r-1}(\bar{m}), \qquad (13)$$

$$r = 1, 2, \ldots, \quad P_0 = 1, \quad P_1 = \bar{m}.$$

If $m_{\min} \le m \le m_{\max}$, then the argument of the Legendre polynomials belonging to the interval $[-1; +1]$ is calculated from the formula

$$\bar{m} = 2\frac{m - m_{\min}}{m_{\max} - m_{\min}} - 1. \qquad (14)$$

We established the statistically significant harmonics by taking into account the fact that the same toroidal or spheroidal coefficient with a set of indices nkp could be significant according to the χ^2 test for one J sample and insignificant for another. For this reason, the magnitude equation was determined only for those coefficients that turned out to be significant at least in three magnitude samples. In this case, the values for such a coefficient were determined for all twelve J samples. Otherwise, the harmonic was rejected. The coefficients were obtained by solving the systems of equations (11) by the LSM, with the degree of the approximating polynomial having been taken to be three to avoid strong correlations of the sought-for coefficients at our comparatively small number of J samples. In addition, in order that the rms errors of the sought-for coefficients reflect the accuracy of the initial coefficients $t_{nkp}(m)$ and $s_{nkp}(m)$ rather than the accuracy of the formal approximation of the curves $s = s_{nkp}(m)$ and $t = t_{nkp}(m)$, the rms errors of the approximation coefficients $s_{nkpr} = s_{jr}$ and $t_{nkpr} = t_{jr}$ were calculated from the formulas

$$\sigma(s_{jr}) = \sqrt{\sum_{q=0}^{3} w_{rq}^2 \sum_{i=0}^{11} L_r^2(\bar{m}_i)\sigma_s^2(m_i)}, \qquad (15)$$

$$\sigma(t_{jr}) = \sqrt{\sum_{q=0}^{3} w_{rq}^2 \sum_{i=0}^{11} L_r^2(\bar{m}_i)\sigma_t^2(m_i)},$$

where w_{rq} are the elements of the inverse matrix of the normal system of equations corresponding to the LSM solution of Eqs. (11), while $\sigma_s(m_i)$ and $\sigma_t(m_i)$ are the rms errors of the coefficients $t_{nkp}(m_i)$ and $s_{nkp}(m_i)$.

The coefficients t_{nkpr} and s_{nkpr} and their rms errors are the final result of comparing the two catalogs.

Using these coefficients, we can obtain the systematic stellar position and proper motion differences XPM−UCAC4 from the formula

$$\Delta\mathbf{F}(\alpha,\delta,m)=\sum_{nkpr} t_{nkpr}\mathbf{T}_{nkp}(\alpha,\delta)L_r(\bar{m}) \quad (16)$$
$$+\sum_{nkpr} s_{nkpr}\mathbf{S}_{nkp}(\alpha,\delta)L_r(\bar{m}).$$

ANALYSIS OF THE SYSTEMATIC DIFFERENCES XPM−UCAC4

The main purpose of the systematic stellar position and proper motion differences is the possibility of reducing the stellar positions and proper motions from the system of one catalog to the system of another catalog. In addition, the systematic differences between the positions and proper motions of the same stars allow the discrepancy between the reference frames realized by the catalogs under consideration to be studied. Froeschle and Kovalevsky (1982) showed that the mutual rotation angles of the coordinate systems and the rates of their mutual rotation could be determined by analyzing the systematic positions and proper motion differences. The same effects also manifest themselves in the coefficients of the decomposition of the systematic stellar position and proper motion differences into orthogonal harmonics. Within the model of solid-body rotation, the relationship between the rotation angles of one coordinate system relative to another and the coefficients of the decomposition of the systematic differences between the right ascensions and declinations of stars into scalar harmonics was established by Vityazev and Tsvetkov (1989) and Vityazev (1993). When using VSHs, such a relationship was found by Mignard and Morando (1990). These authors showed that the first-order toroidal coefficients in the decomposition of the systematic position differences define the mutual orientation of the reference frames associated with the catalogs under study, while the same coefficients in the decomposition of the systematic stellar proper motion differences allow the rate of mutual rotation of these frames to be calculated. In the notation of this paper, the working formulas establishing the relationships between the rotation vector components and first-order toroidal coefficients are presented, for example, in Vityazev and Tsvetkov (2009, 2013, 2014). As follows from these papers, the mutual orientation angles ϵ_x, ϵ_y, and ϵ_z and the rates of their change ω_x, ω_y, and ω_z are defined via the corresponding coefficients t_{101}, t_{110}, and t_{111} by the relations

$$\epsilon_x = t_{1,1,1}/2.89, \quad (17)$$
$$\epsilon_y = t_{1,1,0}/2.89, \quad \epsilon_z = t_{1,0,1}/2.89,$$

$$\omega_x = t_{1,1,1}/2.89, \quad (18)$$
$$\omega_y = t_{1,1,0}/2.89, \quad \omega_z = t_{1,0,1}/2.89,$$

where the coefficients t_{nkp} in Eqs. (17) and (18) correspond to the VSH decompositions of the stellar position and proper motion differences, respectively.

Bearing in mind that the XPM catalog has two versions of stellar proper motions, let us consider the systematic position and proper motion differences separately.

Systematic Position Differences

Obviously, to reduce the stellar positions from the UCAC4 system to the XPM system, the corrections $\Delta\alpha = \Delta F(\alpha,\delta,m)/\cos\delta$ and $\Delta\delta = \Delta F(\alpha,\delta,m)$ calculated from Eq. (16) with the coefficients taken from the corresponding Tables 5 and 6 should be added to the UCAC4 positions. Figures 1–3 give an idea of the form of the systematic differences $\Delta\alpha\cos\delta$ and $\Delta\delta$ in the equatorial coordinate system for three magnitude of stars, $J = 11^m, 13^m, 15^m$.

Table 1 presents the largest and smallest systematic position differences and the ranges of their variations for the differences XPM−UCAC4. For comparison, the analogous values for the differences PPMXL−UCAC4 are also given here (Vityazev and Tsvetkov 2015). Analysis of this table shows that the range of systematic position differences between XPM and UCAC4 exceeds the corresponding range of differences between PPMXL and UCAC4 by a factor of 5 for $J = 11^m$.

Let us now turn to analyzing the orientation of the XPM and UCAC4 reference frames. As has been said above, the rotation angles ϵ_x, ϵ_y, and ϵ_z of the axes of the coordinate systems realized by our catalogs are defined via the VSH decomposition coefficients t_{101}, t_{110}, and t_{111} of the stellar position differences (Table 5). The dependence of these components on the magnitude of stars is shown in Fig. 4. Here, we see that the rotation angles around the OY axis are zero; the rotation angles around the OX axis are 2–4 mas and change little with magnitude. In contrast, the OX axis of the XPM catalog is rotated relative to the OX axis of the UCAC4 catalog in the negative direction through 7–10 mas, and this rotation changes rather dramatically with the magnitude of stars. Obviously, the first-order toroidal coefficients corresponding to the systematic position differences allow the position of the pole of the mutual rotation axis on the celestial sphere to be determined. Figure 5 shows the mutual rotation angles around the pole, the coordinates of the pole, and a vector map of the systematic position differences formed by these rotations for stars of various magnitudes. We see that the rotation angles change for different magnitude

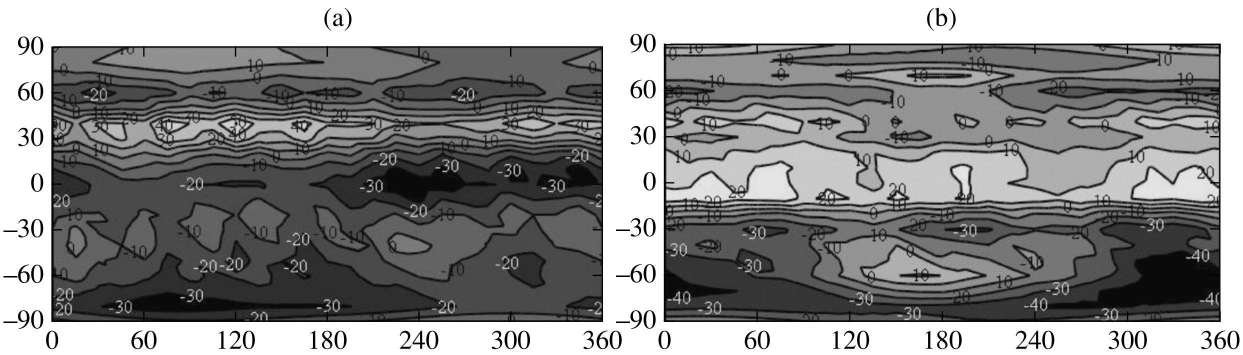

Fig. 1. Maps of systematic position differences XPM−UCAC4 for $J = 11^m$. Panels (a) and (b) show $\Delta\alpha\cos\delta$ and $\Delta\delta$, respectively. The units are mas. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

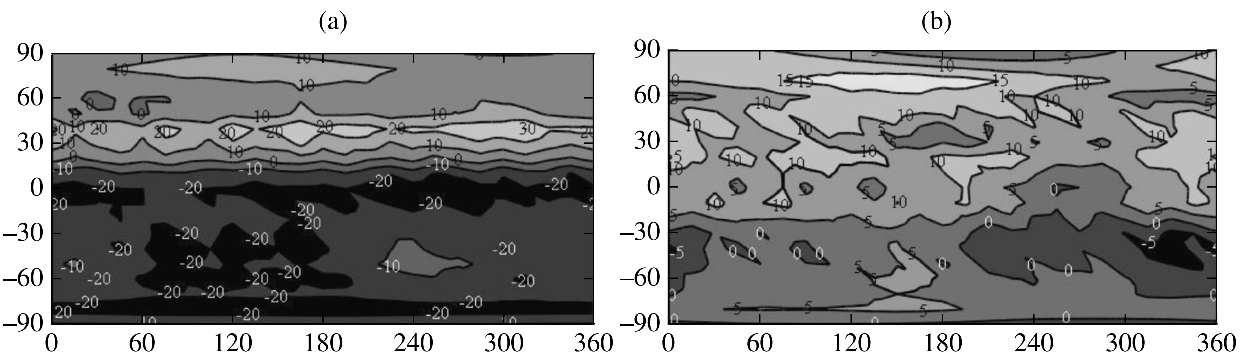

Fig. 2. Maps of systematic position differences XPM−UCAC4 for $J = 13^m$. Panels (a) and (b) show $\Delta\alpha\cos\delta$ and $\Delta\delta$, respectively. The units are mas. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

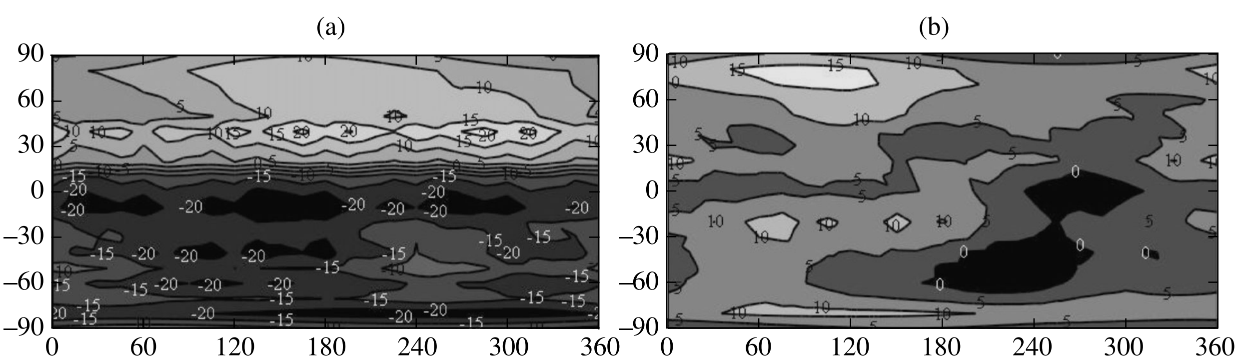

Fig. 3. Maps of systematic position differences XPM−UCAC4 for $J = 15^m$. Panels (a) and (b) show $\Delta\alpha\cos\delta$ and $\Delta\delta$, respectively. The units are mas. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

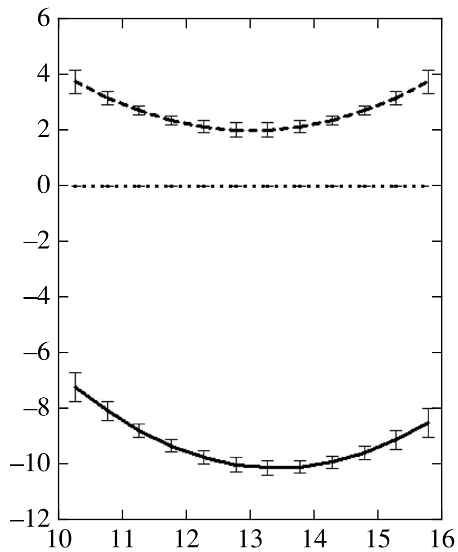

Fig. 4. Mutual orientation angles of the XPM and UCAC4 reference frames: ϵ_x (dashes), ϵ_y (dots), and ϵ_z (solid line) (mas). The J magnitudes of the samples are along the horizontal axis.

groups and can reach ∼10 mas. Interestingly, the right ascension of the rotation pole in the positive direction (counterclockwise) is virtually zero, while the declinations of this pole lie in the southern hemisphere and change for stars of different magnitudes.

Systematic Stellar Proper Motion Differences XPMx–XPMp

For each star, the XPM catalog gives two stellar proper motions obtained from two catalogs of extended sources, XSC and PSC. The authors of XPM do not give preference to any of them, reporting only that because of the differences between the positions of the same extended sources reaching 24 mas, the XPMx and XPMp proper motions can differ by 0.6 mas yr^{-1} in the northern hemisphere and by 1.5 mas yr^{-1} in the southern one. These values seem underestimated to us, because a direct comparison of even the mean values in 1200 HealPix pixels gives the range of variations from -2.7 to 2.7 mas yr^{-1} for the proper motions in right ascension and from -2.4 to 2.4 mas yr^{-1} for the proper motions in declination. Only after the approximation of these data by VSHs did the ranges of systematic differences become from -1.5 to 0.6 mas yr^{-1} in right ascension and from -1.1 to 0.6 mas yr^{-1} in declination.

To study in detail the two versions of proper motions, XPMx and XPMp, we averaged their differences in each of the 1200 HealPix pixels for the stars belonging to the same J magnitude bins that we used in processing the position differences XPM–UCAC4. Despite the fact that a different number of stars was used in each magnitude group, the mean differences in each pixel were identical for all groups of stars. The mean differences between the stellar proper motions referred to the pixel centers were then represented by the decompositions into VSHs and normalized Legendre polynomials in accordance with Eq. (16) using the technique described above. The toroidal and spheroidal coefficients of these decompositions are presented in Tables 2 and 3. Analysis of these tables allows the following facts to be established.

(1) The systematic differences XPMx–XPMp are free from the effects of the magnitude equation. This is proved by zero indices r in all toroidal and spheroidal coefficients.

(2) The coefficients of the zonal harmonics with indices $(1, 0, 1, 0)$ and $(2, 0, 1, 0)$ are largest in absolute value.

(3) The contribution to the systematic differences from all the remaining coefficients is considerably smaller than the contribution from the two lowest zonal harmonics. For this reason, the behavior of the isolines is determined mainly by the dependence on declination, as can be clearly seen from Fig. 6.

(4) The large value of the coefficient $t_{1,0,1,0} = -1.86 \pm 0.01$ mas yr^{-1} and zero value of the other first-order toroidal coefficients suggest that the reference frames corresponding to the XPMx and XPMp proper motions are in mutual rotation around the Z axis, as can be seen from Fig. 7. The relative rate of this rotation is $\omega_z = t_{1,0,1,0}L_0(0)2.89 = -0.45$ mas yr^{-1}, which exceeds the measure of HCRF inertiality (0.25 mas yr^{-1}).

(5) Nonzero first-order spheroidal coefficients determine the mutual velocity of approach (recession) of our reference frames (Fig. 7). This effect is analogous to the solar motion toward the apex.

(6) The vector map of XPMx–XPMp constructed from VSHs with an order higher than the first one shows that the proper motions are directed oppositely in the northern and southern hemispheres with passage through zero along the equator. This effect determines the general form of the velocity field and is produced by the large second zonal coefficient $t_{2010} = 2.88 \pm 0.01$ mas yr^{-1} (Fig. 8).

Systematic Stellar Proper Motion Differences XPMx–UCAC4 and XPMp–UCAC4

Similarly to the case with the positions, to reduce the stellar proper motions from the UCAC4 system to the XPMx and XPMp systems, the corrections $\Delta\mu_\alpha = \Delta F(\alpha, \delta, m)/\cos\delta$ and $\Delta\mu_\delta = \Delta F(\alpha, \delta, m)$

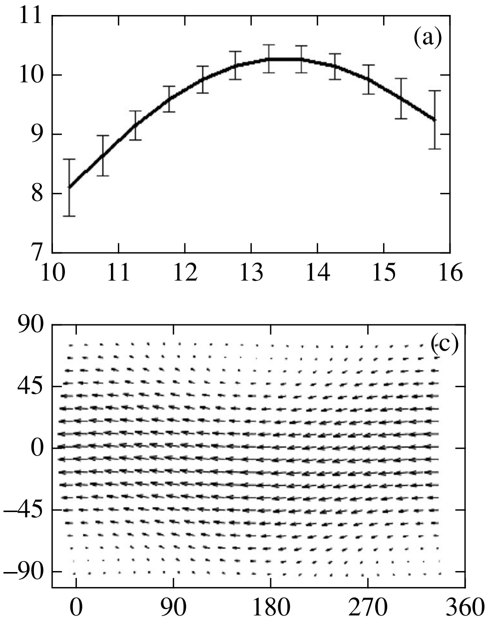

Fig. 5. (a) Mutual rotation angles of the XPM and UCAC4 reference frames around the pole; (b, d) right ascensions (deg) and declinations (deg) of the pole for stars of various magnitudes. The J magnitudes of the samples are along the horizontal axes. (c) Vector map of the rotation components around the pole corresponding to $J = 13$. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

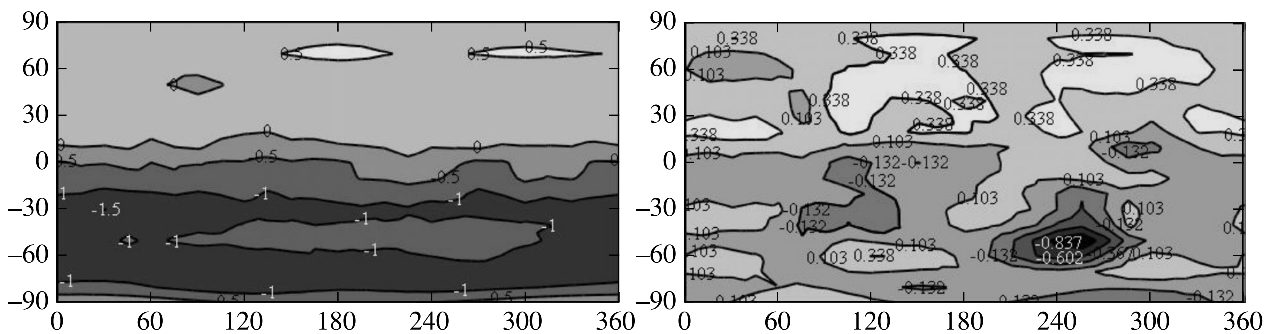

Fig. 6. Maps of differences XPMx−XPMp. Panels (a) and (b) show $\Delta\mu_\alpha \cos\delta$ and $\Delta\mu_\delta$, respectively. The units are mas yr^{-1}. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

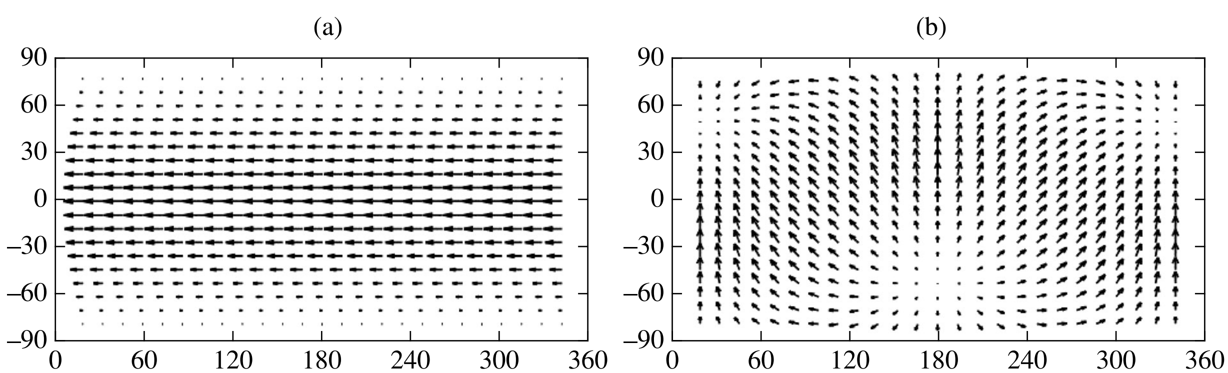

Fig. 7. Maps of stellar proper motion differences XPMx−XPMp. Panel (a) shows the effect of mutual rotation of the XPMx and XPMp reference frames around the Z axis. Panel (b) shows the apex and antiapex of mutual motion of the XPMx and XPMp reference frames. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

Table 2. Toroidal decomposition coefficients t_{nkpr} of the stellar proper motion differences XPMx–XPMp. The units of measurement are mas yr^{-1}

t_{nkpr}	Value	t_{nkpr}	Value	t_{nkpr}	Value
$t_{1,0,1,0}$	-1.86 ± 0.01	$t_{4,2,1,0}$	-0.20 ± 0.01	$t_{7,0,1,0}$	-0.14 ± 0.01
$t_{2,0,1,0}$	2.88 ± 0.01	$t_{4,3,0,0}$	-0.26 ± 0.01	$t_{8,0,1,0}$	0.52 ± 0.01
$t_{2,1,1,0}$	-0.13 ± 0.01	$t_{5,0,1,0}$	-0.21 ± 0.01	$t_{8,6,1,0}$	-0.17 ± 0.01
$t_{2,2,0,0}$	-0.18 ± 0.01	$t_{5,3,0,0}$	0.14 ± 0.01	$t_{9,0,1,0}$	-0.13 ± 0.01
$t_{3,0,1,0}$	-0.50 ± 0.01	$t_{5,4,1,0}$	-0.14 ± 0.01	$t_{9,4,1,0}$	-0.14 ± 0.01
$t_{3,1,1,0}$	0.20 ± 0.01	$t_{6,0,1,0}$	0.59 ± 0.01	$t_{10,8,1,0}$	0.18 ± 0.01
$t_{3,2,0,0}$	0.21 ± 0.01	$t_{6,4,1,0}$	0.16 ± 0.01	$t_{11,0,1,0}$	0.13 ± 0.01
$t_{4,1,1,0}$	-0.19 ± 0.01	$t_{6,5,0,0}$	0.19 ± 0.01	$t_{13,0,1,0}$	-0.23 ± 0.01

Table 3. Spheroidal decomposition coefficients s_{nkpr} of the stellar proper motion differences XPMx–XPMp. The units of measurement are mas yr^{-1}

s_{nkpr}	Value	s_{nkpr}	Value	s_{nkpr}	Value
$s_{1,0,1,0}$	0.48 ± 0.01	$s_{5,1,1,0}$	-0.14 ± 0.01	$s_{10,1,0,0}$	-0.17 ± 0.01
$s_{1,1,1,0}$	0.29 ± 0.01	$s_{5,2,1,0}$	0.17 ± 0.01	$s_{11,1,0,0}$	0.16 ± 0.01
$s_{2,0,1,0}$	0.71 ± 0.01	$s_{6,2,1,0}$	-0.14 ± 0.01	$s_{12,0,1,0}$	0.23 ± 0.01
$s_{3,0,1,0}$	0.25 ± 0.01	$s_{7,0,1,0}$	0.23 ± 0.01	$s_{13,3,0,0}$	-0.16 ± 0.01
$s_{3,2,0,0}$	0.22 ± 0.01	$s_{7,2,0,0}$	-0.15 ± 0.01	$s_{50,34,1,0}$	-0.18 ± 0.01
$s_{4,1,0,0}$	0.14 ± 0.01	$s_{8,2,0,0}$	0.14 ± 0.01		
$s_{5,1,0,0}$	-0.21 ± 0.01	$s_{9,2,0,0}$	-0.13 ± 0.01		

calculated from Eq. (16) with the coefficients taken from the corresponding Tables 7–10 should be added to the UCAC4 proper motions.

Let us now study the extent to which the XPMx and XPMp proper motions differ from UCAC4. Table 4 gives the largest and smallest systematic proper motion differences and the ranges of differences $\mathrm{XPM_x}$–UCAC4, $\mathrm{XPM_p}$–UCAC4, and PPMXL–UCAC4 as functions of magnitude as well as similar data for the differences $\mathrm{XPM_x}$–$\mathrm{XPM_p}$ that do not depend on magnitude. Analysis of this table allows two facts to be established. First, for bright stars ($J = 11^m$), the ranges of systematic differences between the proper motions of both versions of the XPM catalog and UCAC4, while they do not differ between themselves very much, nevertheless, exceed

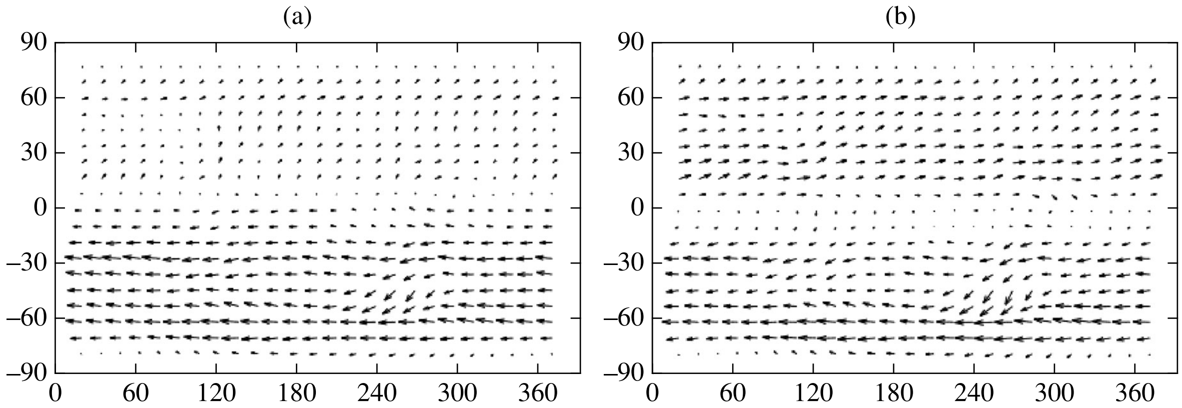

Fig. 8. Maps of stellar proper motion differences XPMx−XPMp. (a) All of the significant harmonics were taken into account. (b) Only the highest harmonics were taken into account. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

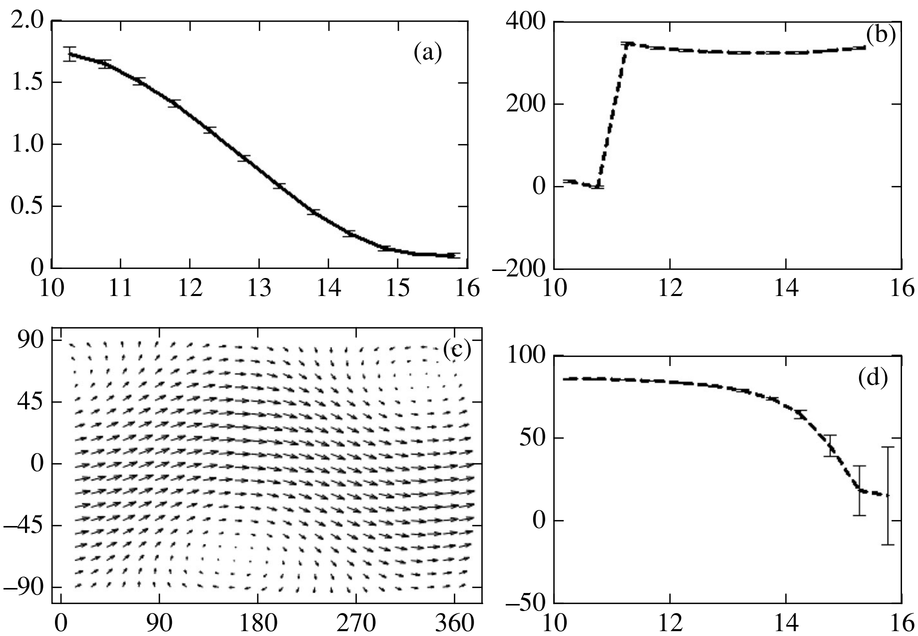

Fig. 9. (a) Angular velocity of mutual rotation of the XPMx and UCAC4 reference frames (mas yr^{-1}) around the pole; (b, d) right ascensions (deg) and declinations (deg) of the pole for stars of various magnitudes. The J magnitudes of the samples are along the horizontal axes. (c) Vector map of the rotation components around the pole corresponding to $J = 13$. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

the corresponding differences between PPMXL and UCAC4 approximately by a factor of 1.5. Second, the agreement between XPMx and XPMp in systematic terms is much better than the agreement of these versions with the UCAC4 catalog of proper motions. At the same time, the extreme values and ranges of differences XPM−UCAC4 for both versions decrease when passing to fainter stars. This confirms the conclusions reached by the authors of the XPM catalog (Fedorov et al. 2009) that faint galaxies appear as point objects in the images and, therefore, can be used as reliable astrometric references. For this reason, one would expect the magnitude equation to clearly manifest itself only for bright stars from the range being investigated, which is confirmed by our Tables 1 and 4.

Table 4. Boundaries and ranges of systematic stellar proper motion differences XPMx−UCAC4, XPMp−UCAC4, XPMx−XPMp, and PPMXL−UCAC4 as a function of the J magnitude of stars, mas yr^{-1}

J	$(\Delta\mu_\alpha\cos\delta)_{\min}$	$(\Delta\mu_\alpha\cos\delta)_{\max}$	$\Delta PM_\alpha\cos\delta$	$(\Delta\mu_\delta)_{\min}$	$(\Delta\mu_\delta)_{\max}$	ΔPM_δ
			XPMx−UCAC4			
11^m	−2.16	4.07	6.23	−7.29	2.40	9.69
13^m	−1.74	2.50	4.24	−5.14	1.51	6.65
15^m	−1.73	1.53	3.26	−4.43	1.32	5.75
			XPMp−UCAC4			
11^m	−1.04	4.44	5.48	−6.66	2.29	8.95
13^m	−0.70	2.76	3.46	−4.42	1.43	5.85
15^m	−0.86	2.21	3.07	−4.38	0.89	5.26
			XPMx−XPMp			
	−1.51	0.64	2.15	−1.07	0.57	1.64
			PPMXL−UCAC4			
11^m	−2.23	1.76	3.99	−2.41	2.34	4.75
13^m	−3.50	2.36	5.86	−2.86	2.13	4.99
15^m	−3.84	2.93	6.77	−3.45	1.43	4.88

Obviously, the first-order toroidal coefficients allow the position of the pole of the mutual rotation axis to be determined. Figures 9 and 10 show the rates of mutual rotation, the coordinates of the rotation pole, and vector maps of the rates of mutual rotation of the XPMx and XPMp reference frames around UCAC4 for stars of various magnitudes. We see that the rotation rates change for different magnitude groups. It follows from these figures that the rate of mutual rotation of the XPMp and UCAC4 reference frames changes within the range from 0.6 to 2.2 mas yr^{-1}, while the analogous rate for XPMx and UCAC4 lies within the range from 0.3 to 1.8 mas yr^{-1}. Previously (Vityazev and Tsvetkov 2015), we showed that the residual rotation rates of the UCAC4 reference frame essentially reproduce the measure of Hipparcos inertiality (0.25 mas yr^{-1}). On this basis, it can be asserted that both XPMx and XPMp catalogs have a noticeable residual rotation rate relative to UCAC4 (and, consequently, the ICRF), especially large for bright stars of our range. Obviously, this fact is attributable not to the calibration based on the data taken from XSC and PSC but to the transition from the "quasar–galaxy" reference frame to the purely "galaxy" one.

CONCLUSIONS

For the first time, we have obtained the systematic differences between the stellar positions and proper motions of the XPM and UCAC4 catalogs in the form of decompositions into vector spherical harmonics by taking into account the magnitude equation. The systematic components were extracted with a probability of at least 0.98. The derived decomposition

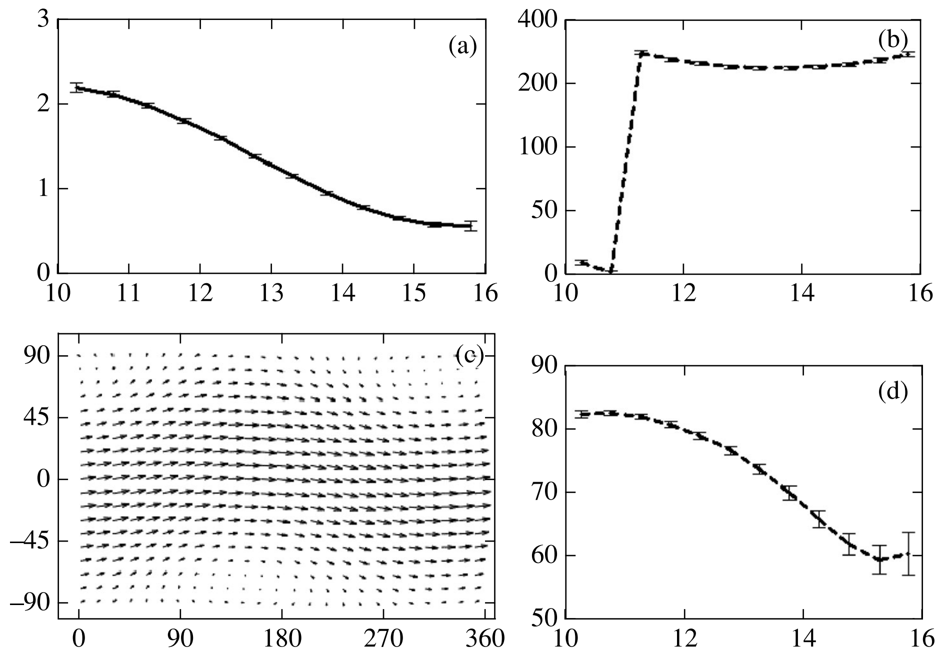

Fig. 10. (a) Angular velocity of mutual rotation of the XPMp and UCAC4 reference frames (mas yr^{-1}) around the pole; (b, d) right ascensions (deg) and declinations (deg) of the pole for stars of various magnitudes. The J magnitudes of the samples are along the horizontal axes. (c) Vector map of the rotation components around the pole corresponding to $J = 13$. The right ascension (deg) and declination (deg) are along the horizontal and vertical axes, respectively.

coefficients allow the coordinates and proper motions to be reduced from the UCAC4 system to the XPM system and vice versa.

Since the UCAC4 and XPM catalogs reproduce the reference frames constructed on quasars and galaxies, particular attention was given to searching for greatly differing components of the systematic differences to ascertain the consequences of passing from the quasi-inertial ICRF to another quasi-inertial frame constructed on galaxies. A study of the systematic differences between the equatorial coordinates showed that the range of systematic differences between the XPM and UCAC4 positions exceeds the corresponding range of differences between PPMXL and UCAC4 by a factor of 5, especially for bright stars in the range being investigated. Indeed, for the differences XPM–UCAC4 at $J = 11^m$, the ranges of differences in right ascension and declination are 86.9 and 77.6 mas, respectively. The analogous ranges for the differences PPMXL–UCAC4 are 17.2 and 15.6 mas. Analysis of the orientation of the XPM and UCAC4 reference frames showed that their mutual rotation is 2–4 mas around the X axis and 7–10 mas around the Z axis. These angles depend on the magnitude of stars. The resulting rotation of the two reference frames within 8–10 mas occurs around the rotation pole whose right ascension is zero and whose declination changes within the range from $-80°$ to $-60°$, depending on the magnitude of stars. Note that the mutual rotation of the PPMXL and UCAC4 systems for $j = 11^m$ is approximately a factor of 4 smaller.

Since two systems of proper motions are given in the XPM catalog, XPMx and XPMp, we decomposed the differences XPMx–XPMp into vector spherical harmonics. The authors of the XPM catalog did not give preference to any of the systems of proper motions XPMx and XPMp; therefore, our goal was to ascertain a measure of their proximity and difference. The derived decompositions showed the differences XPMx–XPMp to be free from the magnitude equation. This by no means implies that each of these systems is not distorted by the magnitude equation. Most likely, they both are distorted by identical errors dependent on the magnitude of stars. An important fact is that the first-order zonal toroidal and spheroidal coefficients are the greatest ones determining the final dependence of the differences on the coordinates. As a result, the systematic differences XPMx–XPMp depend weakly on the right ascensions, and their form on the sky map is determined mainly by the dependence on the declinations. The large toroidal coefficient $t_{1,0,1,0}$ found showed that the XPMx and XPMp reference frames of proper motions rotate relative to each other around the Z axis with an angular velocity of -0.45 mas yr^{-1}, which is almost

Table 5. Toroidal decomposition coefficients t_{nkpr} of the field of stellar position differences XPM−UCAC4. The units of measurement are mas yr^{-1}

t_{nkpr}	Value	t_{nkpr}	Value	t_{nkpr}	Value
$t_{1,0,1,0}$	-38.17 ± 0.65	$t_{4,3,0,2}$	2.42 ± 0.66	$t_{9,0,1,0}$	2.12 ± 0.65
$t_{1,0,1,1}$	-1.50 ± 0.71	$t_{4,3,1,0}$	-6.13 ± 0.66	$t_{9,0,1,1}$	-5.61 ± 0.71
$t_{1,0,1,2}$	2.70 ± 0.69	$t_{4,3,1,1}$	4.57 ± 0.71	$t_{9,0,1,2}$	1.90 ± 0.69
$t_{1,1,1,0}$	10.48 ± 0.65	$t_{4,3,1,2}$	-2.05 ± 0.69	$t_{10,0,1,0}$	6.77 ± 0.65
$t_{1,1,1,2}$	2.12 ± 0.69	$t_{5,0,1,0}$	-32.67 ± 0.65	$t_{10,0,1,1}$	-4.18 ± 0.70
$t_{2,0,1,0}$	53.26 ± 0.65	$t_{5,0,1,1}$	13.20 ± 0.70	$t_{10,0,1,2}$	1.73 ± 0.69
$t_{2,0,1,1}$	2.79 ± 0.83	$t_{5,0,1,2}$	-4.74 ± 0.69	$t_{13,0,1,0}$	-4.91 ± 0.62
$t_{2,0,1,2}$	-4.51 ± 0.69	$t_{6,0,1,0}$	-2.94 ± 0.65	$t_{13,0,1,1}$	2.07 ± 0.70
$t_{2,0,1,3}$	-2.76 ± 0.72	$t_{6,0,1,1}$	6.80 ± 0.70	$t_{16,0,1,0}$	5.12 ± 0.65
$t_{3,0,1,0}$	18.36 ± 0.62	$t_{6,0,1,2}$	-1.60 ± 0.69	$t_{16,0,1,1}$	2.96 ± 0.71
$t_{3,1,0,0}$	7.26 ± 0.66	$t_{6,2,0,0}$	-5.59 ± 0.65	$t_{16,0,1,2}$	-1.80 ± 0.69
$t_{3,1,0,1}$	-8.11 ± 0.71	$t_{6,2,0,1}$	5.07 ± 0.71	$t_{19,0,1,0}$	-2.79 ± 0.62
$t_{3,1,0,2}$	2.72 ± 0.69	$t_{6,2,0,2}$	-2.14 ± 0.69	$t_{19,0,1,1}$	-2.06 ± 0.71
$t_{4,1,0,0}$	-6.48 ± 0.62	$t_{7,0,1,0}$	6.29 ± 0.62	$t_{21,0,1,0}$	1.96 ± 0.62
$t_{4,1,0,1}$	2.61 ± 0.71	$t_{8,0,1,0}$	15.33 ± 0.62	$t_{21,0,1,1}$	1.75 ± 0.70
$t_{4,3,0,1}$	-5.47 ± 0.71	$t_{8,0,1,1}$	-5.43 ± 0.71		

Table 6. Spheroidal decomposition coefficients s_{nkpr} of the field of stellar position differences XPM−UCAC4. The units of measurement are mas yr^{-1}

s_{nkpr}	Value	s_{nkpr}	Value	s_{nkpr}	Value
$s_{1,0,1,0}$	16.88 ± 0.65	$s_{4,1,1,2}$	-1.67 ± 0.69	$s_{8,0,1,2}$	3.69 ± 0.69
$s_{1,0,1,1}$	10.52 ± 0.71	$s_{4,3,0,0}$	-7.01 ± 0.62	$s_{8,1,1,0}$	-4.19 ± 0.66
$s_{1,0,1,2}$	-9.99 ± 0.69	$s_{4,3,0,1}$	1.94 ± 0.71	$s_{8,1,1,1}$	4.86 ± 0.71
$s_{1,1,1,0}$	-3.93 ± 0.65	$s_{5,0,1,0}$	8.11 ± 0.62	$s_{8,1,1,2}$	-1.54 ± 0.69
$s_{1,1,1,1}$	11.25 ± 0.71	$s_{5,0,1,1}$	-2.47 ± 0.70	$s_{11,0,1,0}$	5.51 ± 0.62
$s_{1,1,1,2}$	-6.04 ± 0.69	$s_{5,1,1,0}$	1.88 ± 0.66	$s_{11,8,1,0}$	6.30 ± 0.62
$s_{2,0,1,0}$	20.77 ± 0.65	$s_{5,1,1,1}$	-6.13 ± 0.71	$s_{11,8,1,1}$	-4.27 ± 0.70
$s_{2,0,1,1}$	-13.58 ± 0.70	$s_{5,1,1,2}$	2.40 ± 0.69	$s_{13,0,1,0}$	-6.68 ± 0.65
$s_{2,0,1,2}$	2.14 ± 0.69	$s_{6,1,1,0}$	-5.28 ± 0.66	$s_{13,0,1,1}$	8.60 ± 0.70
$s_{3,0,1,0}$	-15.66 ± 0.65	$s_{6,1,1,1}$	4.61 ± 0.71	$s_{13,0,1,2}$	-4.54 ± 0.69
$s_{3,0,1,1}$	27.68 ± 0.70	$s_{6,1,1,2}$	-1.72 ± 0.69	$s_{19,9,1,0}$	-5.98 ± 0.62
$s_{3,0,1,2}$	-16.61 ± 0.69	$s_{6,3,1,0}$	4.34 ± 0.65	$s_{19,9,1,1}$	3.06 ± 0.70
$s_{3,1,0,0}$	-2.55 ± 0.62	$s_{6,3,1,1}$	-3.51 ± 0.70	$s_{22,0,1,0}$	6.13 ± 0.66
$s_{3,1,0,1}$	-2.47 ± 0.71	$s_{6,3,1,2}$	2.18 ± 0.69	$s_{22,0,1,1}$	-4.35 ± 0.71
$s_{3,1,1,0}$	6.60 ± 0.62	$s_{7,0,1,0}$	-3.78 ± 0.65	$s_{22,0,1,2}$	1.76 ± 0.69
$s_{3,1,1,1}$	-3.04 ± 0.71	$s_{7,0,1,1}$	13.24 ± 0.71	$s_{55,39,0,0}$	3.34 ± 0.67
$s_{3,2,1,0}$	6.31 ± 0.62	$s_{7,0,1,2}$	-6.72 ± 0.69	$s_{55,39,0,1}$	2.10 ± 0.76
$s_{4,0,1,0}$	1.98 ± 0.62	$s_{7,1,1,0}$	-2.71 ± 0.66	$s_{56,39,0,0}$	8.36 ± 0.77
$s_{4,0,1,1}$	3.05 ± 0.71	$s_{7,1,1,2}$	1.47 ± 0.69	$s_{56,39,0,1}$	-6.05 ± 0.88
$s_{4,1,1,0}$	-3.37 ± 0.66	$s_{8,0,1,0}$	6.66 ± 0.65		
$s_{4,1,1,1}$	4.96 ± 0.71	$s_{8,0,1,1}$	-8.69 ± 0.71		

Table 7. Toroidal decomposition coefficients t_{nkpr} of the stellar proper motion differences XPMp−UCAC4. The units of measurement are mas yr^{-1}

t_{nkpr}	Value	t_{nkpr}	Value	t_{nkpr}	Value
$t_{1,0,1,0}$	5.17 ± 0.06	$t_{4,0,1,1}$	-0.58 ± 0.06	$t_{6,2,0,0}$	-0.61 ± 0.06
$t_{1,0,1,1}$	-2.40 ± 0.07	$t_{4,0,1,2}$	0.13 ± 0.06	$t_{6,2,0,1}$	0.34 ± 0.06
$t_{1,0,1,2}$	0.14 ± 0.06	$t_{4,1,0,0}$	-0.52 ± 0.06	$t_{6,2,0,2}$	-0.13 ± 0.06
$t_{1,0,1,3}$	0.25 ± 0.06	$t_{4,1,0,1}$	0.32 ± 0.06	$t_{6,5,1,0}$	0.60 ± 0.06
$t_{1,1,0,0}$	-0.45 ± 0.06	$t_{4,1,0,2}$	-0.12 ± 0.06	$t_{7,4,0,0}$	0.62 ± 0.06
$t_{1,1,0,1}$	-0.19 ± 0.06	$t_{4,1,1,0}$	0.68 ± 0.06	$t_{7,4,0,1}$	-0.19 ± 0.06
$t_{1,1,0,2}$	0.24 ± 0.06	$t_{4,1,1,1}$	-0.23 ± 0.06	$t_{7,4,1,0}$	0.41 ± 0.06
$t_{1,1,1,0}$	1.11 ± 0.06	$t_{4,3,1,0}$	-0.58 ± 0.06	$t_{7,4,1,1}$	0.25 ± 0.06
$t_{2,0,1,0}$	-1.54 ± 0.06	$t_{4,3,1,1}$	0.33 ± 0.06	$t_{7,5,0,0}$	0.30 ± 0.06
$t_{2,0,1,1}$	-0.56 ± 0.06	$t_{4,3,1,2}$	-0.19 ± 0.06	$t_{7,5,0,1}$	0.20 ± 0.06
$t_{2,0,1,2}$	-0.16 ± 0.06	$t_{4,4,0,0}$	-0.64 ± 0.06	$t_{8,1,0,0}$	-0.52 ± 0.06
$t_{2,1,1,0}$	0.60 ± 0.06	$t_{4,4,0,1}$	0.27 ± 0.06	$t_{8,1,0,1}$	0.29 ± 0.06
$t_{2,1,1,1}$	-0.31 ± 0.07	$t_{5,0,1,0}$	-0.70 ± 0.06	$t_{9,0,1,0}$	0.69 ± 0.06
$t_{2,1,1,3}$	0.13 ± 0.06	$t_{5,0,1,1}$	0.47 ± 0.06	$t_{9,0,1,1}$	-0.37 ± 0.06
$t_{3,0,1,0}$	1.46 ± 0.06	$t_{5,0,1,2}$	-0.28 ± 0.06	$t_{10,0,1,0}$	0.38 ± 0.06
$t_{3,0,1,1}$	-0.47 ± 0.06	$t_{5,1,1,0}$	-0.83 ± 0.07	$t_{10,0,1,1}$	-0.27 ± 0.06
$t_{3,1,0,0}$	0.79 ± 0.06	$t_{5,1,1,1}$	0.48 ± 0.07	$t_{10,1,0,0}$	0.59 ± 0.06
$t_{3,1,0,1}$	-0.38 ± 0.06	$t_{5,3,1,0}$	0.67 ± 0.06	$t_{14,2,1,0}$	-0.29 ± 0.06
$t_{3,1,0,2}$	0.23 ± 0.06	$t_{5,3,1,1}$	-0.17 ± 0.06	$t_{14,2,1,1}$	-0.22 ± 0.06
$t_{3,3,1,0}$	-0.43 ± 0.06	$t_{5,3,1,2}$	0.12 ± 0.06	$t_{19,9,0,0}$	-0.51 ± 0.06
$t_{3,3,1,1}$	-0.19 ± 0.06	$t_{6,0,1,0}$	-0.60 ± 0.06	$t_{41,21,1,0}$	0.53 ± 0.06
$t_{3,3,1,2}$	0.25 ± 0.06	$t_{6,0,1,1}$	0.19 ± 0.06	$t_{41,21,1,1}$	-0.22 ± 0.06
$t_{4,0,1,0}$	0.73 ± 0.06	$t_{6,1,1,0}$	0.68 ± 0.06		

Table 8. Spheroidal decomposition coefficients s_{nkpr} of the stellar proper motion differences XPMp−UCAC4. The units of measurement are mas yr^{-1}

s_{nkpr}	Value	s_{nkpr}	Value	s_{nkpr}	Value
$s_{1,0,1,0}$	-2.59 ± 0.06	$s_{3,1,1,0}$	0.45 ± 0.06	$s_{8,0,1,0}$	1.00 ± 0.06
$s_{1,0,1,1}$	-0.83 ± 0.07	$s_{3,1,1,1}$	-0.38 ± 0.06	$s_{8,0,1,1}$	-0.52 ± 0.06
$s_{1,0,1,2}$	0.16 ± 0.06	$s_{3,2,0,0}$	-0.49 ± 0.06	$s_{8,0,1,2}$	0.28 ± 0.06
$s_{1,0,1,3}$	-0.17 ± 0.06	$s_{3,2,0,1}$	0.29 ± 0.06	$s_{9,0,1,0}$	-1.07 ± 0.06
$s_{1,1,0,0}$	0.72 ± 0.06	$s_{4,0,1,0}$	1.80 ± 0.06	$s_{9,0,1,1}$	0.15 ± 0.06
$s_{1,1,0,1}$	-0.33 ± 0.06	$s_{4,1,0,0}$	-0.42 ± 0.06	$s_{10,0,1,0}$	1.21 ± 0.06
$s_{1,1,1,0}$	-1.26 ± 0.06	$s_{4,1,1,0}$	-0.80 ± 0.06	$s_{11,0,1,0}$	-1.00 ± 0.06
$s_{1,1,1,1}$	0.75 ± 0.06	$s_{4,1,1,1}$	0.33 ± 0.06	$s_{13,0,1,0}$	-0.95 ± 0.06
$s_{1,1,1,2}$	-0.27 ± 0.06	$s_{4,1,1,2}$	-0.20 ± 0.06	$s_{13,0,1,1}$	0.38 ± 0.06
$s_{2,0,1,0}$	1.22 ± 0.06	$s_{5,0,1,0}$	-0.94 ± 0.06	$s_{13,0,1,2}$	-0.16 ± 0.06
$s_{2,0,1,3}$	-0.33 ± 0.05	$s_{5,1,1,0}$	1.06 ± 0.06	$s_{13,1,0,0}$	-0.51 ± 0.06
$s_{3,0,1,0}$	-4.25 ± 0.06	$s_{5,1,1,1}$	-0.55 ± 0.06	$s_{14,4,1,0}$	0.29 ± 0.06
$s_{3,0,1,1}$	1.01 ± 0.07	$s_{5,1,1,2}$	0.25 ± 0.06	$s_{14,4,1,1}$	-0.38 ± 0.06
$s_{3,0,1,2}$	-0.54 ± 0.06	$s_{7,0,1,0}$	-0.72 ± 0.06	$s_{14,4,1,2}$	0.13 ± 0.06
$s_{3,0,1,3}$	0.12 ± 0.06	$s_{7,0,1,1}$	0.52 ± 0.06		
$s_{3,1,0,0}$	0.57 ± 0.06	$s_{7,0,1,2}$	-0.20 ± 0.06		

Table 9. Toroidal decomposition coefficients t_{nkpr} of the stellar proper motion differences XPMx−UCAC4. The units of measurement are mas yr^{-1}

t_{nkpr}	Value	t_{nkpr}	Value	t_{nkpr}	Value
$t_{1,0,1,0}$	3.31 ± 0.06	$t_{4,0,1,0}$	0.80 ± 0.06	$t_{6,2,0,1}$	0.34 ± 0.06
$t_{1,0,1,1}$	2.40 ± 0.07	$t_{4,0,1,1}$	-0.58 ± 0.05	$t_{6,2,0,2}$	-0.13 ± 0.06
$t_{1,0,1,2}$	0.13 ± 0.06	$t_{4,0,1,2}$	0.13 ± 0.05	$t_{6,5,1,0}$	0.54 ± 0.06
$t_{1,0,1,3}$	0.25 ± 0.06	$t_{4,1,0,0}$	-0.66 ± 0.06	$t_{7,1,0,0}$	0.59 ± 0.06
$t_{1,1,0,0}$	-0.50 ± 0.06	$t_{4,1,0,1}$	0.31 ± 0.06	$t_{7,1,0,1}$	-0.19 ± 0.06
$t_{1,1,0,1}$	-0.19 ± 0.06	$t_{4,1,0,2}$	-0.13 ± 0.06	$t_{7,4,0,0}$	0.58 ± 0.06
$t_{1,1,0,2}$	0.24 ± 0.06	$t_{4,1,1,0}$	0.49 ± 0.06	$t_{7,4,0,1}$	-0.19 ± 0.06
$t_{1,1,1,0}$	1.21 ± 0.06	$t_{4,1,1,1}$	-0.23 ± 0.06	$t_{7,4,1,0}$	0.31 ± 0.06
$t_{2,0,1,0}$	1.34 ± 0.06	$t_{4,2,1,0}$	-0.54 ± 0.06	$t_{7,4,1,1}$	0.24 ± 0.06
$t_{2,0,1,1}$	-0.56 ± 0.06	$t_{4,3,1,0}$	-0.52 ± 0.06	$t_{8,0,1,0}$	0.48 ± 0.06
$t_{2,0,1,2}$	-0.16 ± 0.06	$t_{4,3,1,1}$	0.34 ± 0.06	$t_{8,0,1,1}$	-0.27 ± 0.06
$t_{2,1,1,0}$	0.46 ± 0.06	$t_{4,3,1,2}$	-0.19 ± 0.06	$t_{8,1,0,0}$	-0.47 ± 0.06
$t_{2,1,1,1}$	-0.31 ± 0.07	$t_{4,4,0,0}$	-0.65 ± 0.06	$t_{8,1,0,1}$	0.29 ± 0.06
$t_{2,1,1,3}$	0.13 ± 0.07	$t_{4,4,0,1}$	0.27 ± 0.06	$t_{8,1,1,0}$	0.28 ± 0.06
$t_{3,0,1,0}$	0.95 ± 0.07	$t_{5,0,1,0}$	-0.90 ± 0.06	$t_{8,1,1,1}$	0.27 ± 0.06
$t_{3,0,1,1}$	-0.48 ± 0.07	$t_{5,0,1,1}$	0.48 ± 0.06	$t_{9,0,1,0}$	0.56 ± 0.06
$t_{3,1,0,0}$	0.87 ± 0.06	$t_{5,0,1,2}$	-0.28 ± 0.06	$t_{9,0,1,1}$	-0.37 ± 0.06
$t_{3,1,0,1}$	-0.38 ± 0.06	$t_{5,1,1,0}$	-0.91 ± 0.07	$t_{10,1,0,0}$	0.48 ± 0.06
$t_{3,1,0,2}$	0.23 ± 0.06	$t_{5,1,1,1}$	0.51 ± 0.08	$t_{13,0,1,0}$	-0.39 ± 0.06
$t_{3,2,1,0}$	0.30 ± 0.06	$t_{5,3,1,0}$	0.59 ± 0.06	$t_{14,2,1,0}$	-0.32 ± 0.06
$t_{3,2,1,1}$	0.20 ± 0.06	$t_{5,3,1,1}$	-0.18 ± 0.06	$t_{14,2,1,1}$	-0.22 ± 0.06
$t_{3,3,1,0}$	-0.36 ± 0.06	$t_{5,3,1,2}$	0.12 ± 0.06	$t_{14,2,1,2}$	0.11 ± 0.06
$t_{3,3,1,1}$	-0.20 ± 0.06	$t_{6,1,1,0}$	0.73 ± 0.06	$t_{19,9,0,0}$	-0.58 ± 0.06
$t_{3,3,1,2}$	0.25 ± 0.06	$t_{6,2,0,0}$	-0.63 ± 0.06	$t_{52,35,1,0}$	0.74 ± 0.06

Table 10. Spheroidal decomposition coefficients s_{nkpr} of the stellar proper motion differences XPMx−UCAC4. The units of measurement are mas yr^{-1}

s_{nkpr}	Value	s_{nkpr}	Value	s_{nkpr}	Value
$s_{1,0,1,0}$	-2.10 ± 0.06	$s_{3,0,1,3}$	0.12 ± 0.06	$s_{8,0,1,0}$	0.97 ± 0.06
$s_{1,0,1,1}$	-0.82 ± 0.07	$s_{3,1,0,0}$	0.45 ± 0.07	$s_{8,0,1,1}$	-0.53 ± 0.06
$s_{1,0,1,2}$	0.17 ± 0.06	$s_{3,1,1,0}$	0.36 ± 0.06	$s_{8,0,1,2}$	0.28 ± 0.06
$s_{1,0,1,3}$	-0.17 ± 0.06	$s_{3,1,1,1}$	-0.37 ± 0.06	$s_{9,0,1,0}$	-1.03 ± 0.06
$s_{1,1,0,0}$	0.84 ± 0.06	$s_{4,0,1,0}$	1.86 ± 0.06	$s_{9,0,1,1}$	0.14 ± 0.06
$s_{1,1,0,1}$	-0.33 ± 0.06	$s_{4,1,1,0}$	-0.78 ± 0.06	$s_{10,0,1,0}$	1.26 ± 0.06
$s_{1,1,1,0}$	-0.96 ± 0.06	$s_{4,1,1,1}$	0.33 ± 0.06	$s_{10,3,0,0}$	-0.44 ± 0.06
$s_{1,1,1,1}$	0.74 ± 0.06	$s_{4,1,1,2}$	-0.20 ± 0.06	$s_{11,0,1,0}$	-0.97 ± 0.06
$s_{1,1,1,2}$	-0.27 ± 0.06	$s_{5,0,1,0}$	-0.95 ± 0.06	$s_{11,10,0,0}$	0.42 ± 0.06
$s_{2,0,1,0}$	1.93 ± 0.06	$s_{5,1,1,0}$	0.89 ± 0.05	$s_{13,0,1,0}$	-1.03 ± 0.06
$s_{2,0,1,3}$	-0.33 ± 0.06	$s_{5,1,1,1}$	-0.54 ± 0.05	$s_{13,0,1,1}$	0.39 ± 0.06
$s_{2,1,0,0}$	0.54 ± 0.06	$s_{5,1,1,2}$	0.25 ± 0.06	$s_{13,0,1,2}$	-0.15 ± 0.06
$s_{2,1,0,1}$	0.18 ± 0.06	$s_{6,1,0,0}$	0.62 ± 0.07	$s_{13,1,0,0}$	-0.53 ± 0.06
$s_{3,0,1,0}$	-4.00 ± 0.06	$s_{7,0,1,0}$	-0.49 ± 0.06	$s_{13,1,0,1}$	-0.16 ± 0.06
$s_{3,0,1,1}$	1.02 ± 0.07	$s_{7,0,1,1}$	0.52 ± 0.06		
$s_{3,0,1,2}$	-0.53 ± 0.06	$s_{7,0,1,2}$	-0.20 ± 0.06		

twice as large as the measure of HCRF inertiality, 0.25 mas yr^{-1}. It should be added that the range of systematic differences XPMx−XPMp is 2.1 mas yr^{-1} in right ascension and 1.7 mas yr^{-1} in declination. Comparison of both systems of proper motions with the data from the UCAC4 catalog showed that the ranges of differences between XPMx and UCAC4 exceed the corresponding ranges of differences between XPMp and UCAC4 approximately by 10%. At the same time, the rate of mutual rotation of XPMx and UCAC4 turned out to be lower than that of XPMp and UCAC4. Interestingly, the difference is smallest (by a factor of 1.3) for bright stars (to $J = 12^m$) and largest (by more than a factor of 3) for stars with $J > 14.5^m$. It can also be noted that the rate of mutual rotation of the XPMx and UCAC4 systems reaches a threshold of 0.25 mas yr^{-1} (the measure of HCRF inertiality) for $J = 14.5^m$, while the mutual rotation rate of XPMp and UCAC4 for all magnitude ranges exceeds this threshold by more than a factor of 2.

Based on our analysis, it can be said that because of the significant differences between the XPMx and XPMp proper motions found, the XPM catalog comprises two catalogs, XPM(XSC) and XPM(PSC), in which the stellar positions coincide at the standard epoch J2000 and differ at any other epoch. The decomposition coefficients of the systematic stellar position and proper motion differences XPM−UCAC4 we obtained allow the stellar positions and proper motions from one catalog to be reduced to the system of another catalog by taking into account this duality of the XPM catalog.

ACKNOWLEDGMENTS

This work was supported by grant no. 6.37.343.2015 of the St. Petersburg State University.

REFERENCES

1. G. B. Arfken, *Mathematical Methods for Physicists* (Academic, New York, 1995; Atomizdat, Moscow, 1970), p. 493.
2. R. Bien, W. Fricke, T. Lederle, and H. Schwn, Veroff. Heidelberg Astron. Rech.-Inst. **29**, 1 (1978).
3. V. V. Bobylev, Astron. Lett. **41**, 156 (2015).
4. P. Brosche, Verof. Astron. Rech.-Inst. Heidelberg, No. 17, 1 (1966).
5. G. Damljanovich, I. S. Mili, N. Maigurova, M. Martynov, and N. Pejovi, Publ. Astron. Observ. Belgrade **91**, 191 (2012).
6. N. Dneprovsky and B. Gerasimovic, Pulkovo Observ. Circ., No. 3, 1 (1932).
7. P. N. Fedorov, A. A. Myznikov, and V. S. Akhmetov, Mon. Not. R. Astron. Soc. **393**, 133 (2009).
8. P. N. Fedorov, V. S. Akhmetov, V. V. Bobylev, and A. T. Bajkova, Mon. Not. R. Astron. Soc. **406**, 1734 (2010).
9. P. N. Fedorov, V. S. Akhmetov, V. V. Bobylev, and G. A. Gontcharov, Mon. Not. R. Astron. Soc. **415**, 665 (2011a).
10. P. N. Fedorov, V. S. Akhmetov, and V. V. Bobylev, Mon. Not. R. Astron. Soc. **416**, 403 (2011b).
11. M. Froeschle and J. Kovalevsky, Astron. Astrophys. **116**, 89 (1982).
12. G. A. Gontcharov, A. T. Bajkova, P. N. Fedorov, and V. S. Akhmetov, Mon. Not. R. Astron. Soc. **413**, 1581 (2011).
13. K. M. Gorski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke, and M. Bartelmann, Astrophys. J. **622**, 759 (2005).
14. J. Kovalevsky, L. Lindegren, M. A. C. Perryman, P. D. Hemenway, K. J. Johnston, V. S. Kislyuk, J. F. Lestrade, L. V. Morrison, et al., Astron. Astrophys. **323**, L620 (1997).
15. F. Mignard and M. Froeschle, Astron. Astrophys. **354**, 732 (2000).
16. F. Mignard and S. Klioner, Astron. Astrophys. **547**, A59 (2012).
17. F. Mignard and B. Morando, in *Proceedings of the Colloque Andre' Danjon* (1990), p. 151.
18. D. G. Monet, Bull. Am. Astron. Soc. **30**, 1427 (1998).
19. M. Perryman et al., *The Hipparcos and Tycho Catalogues,* ESA SP-1200 (ESA, 1997), Vols. 1–17.
20. M. F. Skrutskie, R. M. Cutri, R. Stiening, M. D. Weinberg, S. Schneider, J. M. Carpenter, C. Beichman, R. Capps, et al., Astron. J. **131**, 1163 (2006).
21. V. V. Vityazev, Astron. Astrophys. Trans. **4**, 195 (1993).
22. V. V. Vityazev and A. S. Tsvetkov, Vestn. Leningr. Univ., Ser. 1, No. 2, 73 (1989).
23. V. V. Vityazev and A. S. Tsvetkov, Astron. Lett. **35**, 100 (2009).
24. V. V. Vityazev and A. S. Tsvetkov, Astron. Nachr. **334**, 760 (2013).
25. V. V. Vityazev and A. S. Tsvetkov, Mon. Not. R. Astron. Soc. **442**, 1249 (2014).
26. V. V. Vityazev and A. S. Tsvetkov, Astron. Lett. **41**, 317 (2015).
27. N. Zacharias, C. T. Finch, T. M. Girard, A. Henden, J. L. Bartlett, D. G. Monet, and M. I. Zacharias, Astron. J. **145**, 44 (2013).

Translated by V. Astakhov