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Kinematic Parameters of the Stellar Velocity Field within 3 kpc of the Sun Based on the Gaia Data Release 2 with Radial Velocities Catalogue[1]

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Abstract—The paper is devoted to determining the parameters of the three-dimensional Ogorodnikov–Milne kinematic model based on the Gaia DR2 star catalogue (the subset of seven million stars containing radial velocities). The behavior of the parameters as a function of the distance to stars is studied. The parameters derived from the stellar proper motions and radial velocities agree well with one another. It is noted that the angular velocity of rotation retains its stability up to distances of ~ 3 kpc, while the parameter responsible for the translational motion along the Y axis abruptly begins to change from distances starting from 1 kpc.

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INTRODUCTION

The Gaia mission is now in the active stage of a space experiment. The final catalogue is expected no earlier than 2021, but two preliminary versions, Gaia DR1 in 2016 and Gaia DR2 in April 2018, have already been published (Gaia 2018). The last version based on 22 months of observations already contains data on 1 331 909 727 stars with five determined parameters (three spatial coordinates and two proper motions). The parallax accuracy is estimated to be ~ 0.1 mas (milliarcseconds). The photometric characteristics of stars are given in the intrinsic G system and the catalogue is virtually complete in the range from $G = 12^m$ to $G = 17^m$.

The special Gaia DR2 with RV subcatalogue and, apparently, with the most accurate data contains 7 224 631 entries not only on the right ascensions, declinations, and parallaxes, but also on all three velocity components: the proper motions and radial velocities. This subset is of particular interest for stellar–kinematic studies. The distributions of these stars in distance and G magnitude are presented in Figs. 1 and 2, respectively.

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[1] In memory of Veniamin Vladimirovich Vityazev, a professor of the St. Petersburg State University, a Doctor of physical and mathematical sciences.

Gaia DR2 gives an extensive material to perform a kinematic analysis of the proper motions. We are going to carry out a series of studies in this direction using data from the second and subsequent Gaia data releases. This paper is only the first step of this investigation, a classical kinematic analysis of the stellar velocity field within the widespread standard model of proper motions and radial velocities. The succeeding stages are an analysis of the extra-model kinematic components and an analysis of the stellar motions at considerable distances, while a classical analysis implies a kinematic study of the stellar field in some bounded solar neighborhood.

OGORODNIKOV–MILNE MODEL EQUATIONS

As the first model we use the widely known Ogorodnikov–Milne model (Ogorodnikov 1965); a detailed form of the equations of this model is also presented in du Mont (1977) and Rybka (2004). In this model the stellar velocity field is represented by the linear expression

$$\mathbf{V} = \mathbf{V}_0 + \Omega \times \mathbf{r} + \mathbf{M}^{+} \times \mathbf{r}, \qquad (1)$$

where $\mathbf{V}$ is the stellar velocity, $\mathbf{V}_0$ is the influence of the translational solar motion, Ω is the angular velocity of rigid-body rotation of the stellar system, and $\mathbf{M}^{+}$ is the symmetric deformation tensor of the velocity field.

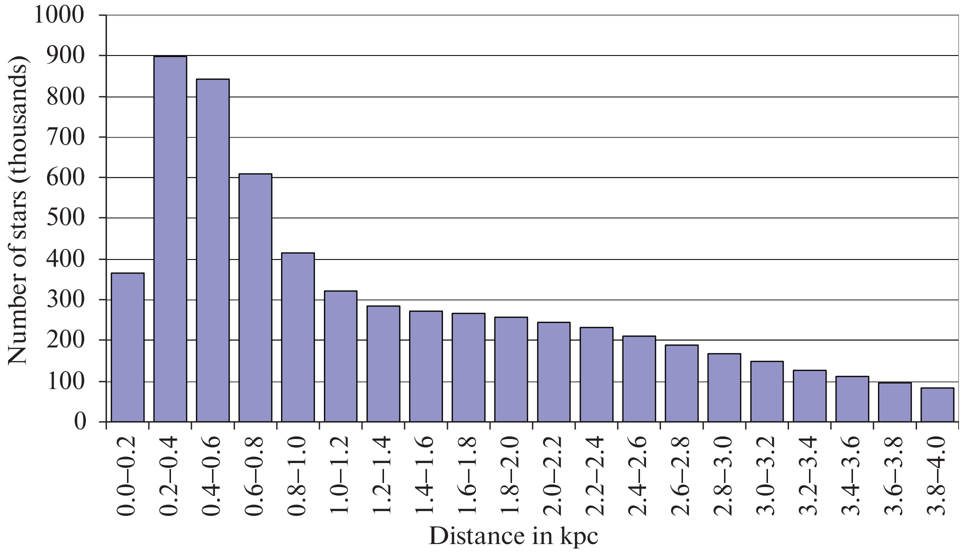

Fig. 1. Distribution of stars from the Gaia DR2 with RV catalogue in distance.

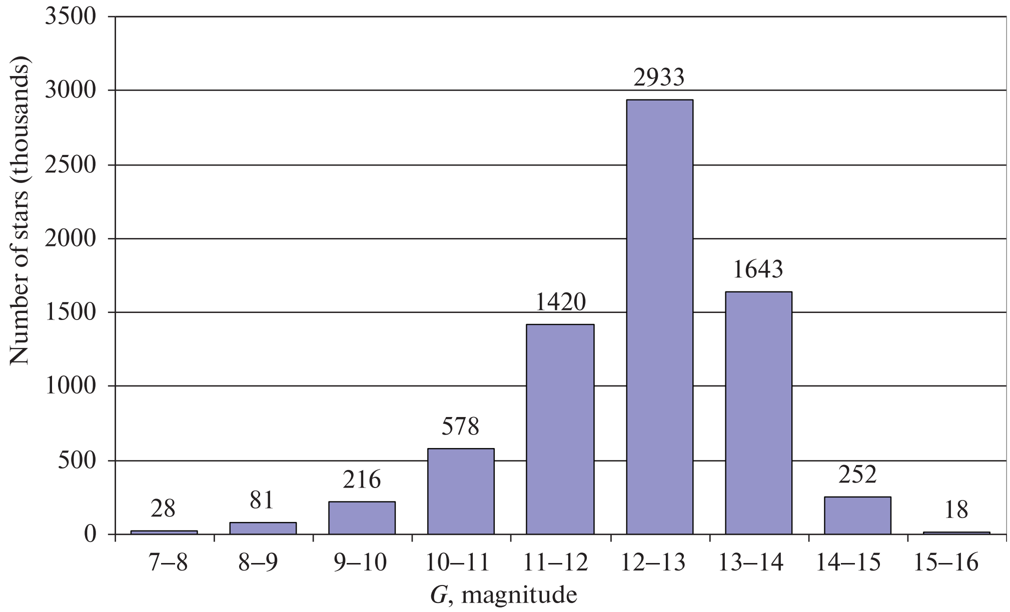

Fig. 2. Distribution of stars from the Gaia DR2 with RV catalogue in G magnitude.

The model contains only 12 parameters, but not all of them can be independently determined from the proper motions and not al of the parameters enter into the equations for the radial velocities.

U, V, W are the components of the translational solar motion vector $\mathbf{V}_0$ among the stars;

ω_1, ω_2, ω_3 are the components of the angular velocity vector Ω;

M^+_{11}, M^+_{22}, M^+_{33} are the parameters of the deformation tensor describing the contraction—expansion along the principal axes of the Galactic coordinate system;

M^+_{12}, M^+_{13}, M^+_{23} are the parameters of the tensor $\mathbf{M}^+$ describing the velocity field deformation in the principal and two perpendicular planes.

Projecting Eq. (1) onto the unit vectors of the

Table 1. Boundaries of the samples of 400 000 stellar groups in pc

Min	3	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031
Max	208	300	386	474	571	687	835	1040	1303	1594	1897	2220	2582	3031	3677

Galactic coordinate system, we get

$$\begin{aligned} V_r/r = -U/r\cos l\cos b - V/r\sin l\cos b \quad (2)\\ - W/r\sin b + M_{12}^{+}\cos^2 b\sin 2l + M_{13}^{+}\sin 2b\cos l\\ + M_{23}^{+}\sin 2b\sin l + M_{11}^{+}\cos^2 b\cos^2 l\\ + M_{22}^{+}\cos^2 b\sin^2 l + M_{33}^{+}\sin^2 b, \end{aligned}$$

$$\begin{aligned} k\mu_l\cos b = U/r\sin l - V/r\cos l \quad (3)\\ - \omega_1\sin b\cos l - \omega_2\sin b\sin l + \omega_3\cos b\\ + M_{12}^{+}\cos b\cos 2l - M_{13}^{+}\sin b\sin l\\ + M_{23}^{+}\sin b\cos l - \frac{1}{2}M_{11}^{+}\cos b\sin 2l\\ + \frac{1}{2}M_{22}^{+}\cos b\sin 2l, \end{aligned}$$

$$\begin{aligned} k\mu_b = U/r\cos l\sin b + V/r\sin l\sin b \quad (4)\\ - W/r\cos b + \omega_1\sin l - \omega_2\cos l + \omega_3\cos l\\ - \frac{1}{2}M_{12}^{+}\sin 2b\sin 2l + M_{13}^{+}\cos 2b\cos l\\ + M_{23}^{+}\cos 2b\sin l - \frac{1}{2}M_{11}^{+}\sin 2b\cos^2 l\\ - \frac{1}{2}M_{22}^{+}\sin 2b\sin^2 l + \frac{1}{2}M_{33}^{+}\sin 2b. \end{aligned}$$

In Eqs. (3) and (4) there is a linear relationship between the coefficients M_{11}^{+}, M_{22}^{+}, and M_{33}^{+}. Therefore, the substitutions $M_{11}^{*} = M_{11}^{+} - M_{22}^{+}$ and $M_{33}^{*} = M_{33}^{+} - M_{22}^{+}$ (du Mont 1977) are usually introduced when analyzing the proper motions.

Various simplifications and complications of this model are often considered, in particular, the Oort–Lindblad model of plane Galactic rotation. Assuming that the velocity field is axisymmetric, for the Oort constants we have $A = M_{12}^{+}$ and $B = \omega_3$(Miyamoto et al. 1993). The meaning of the parameters and their derivatives is considered in more detail in Vityazev et al. (2018).

Equations (2)–(4) are commonly used for a simultaneous solution from the total proper motions of a particular catalogue; if the distances to stars are unknown, then $U/\langle r\rangle$, $V/\langle r\rangle$, and $W/\langle r\rangle$, where $\langle r\rangle$ is the mean distance of the sample of stars for which the solution is produced, are determined instead of U, V, and W.

In the case of the Gaia DR2 with RV catalogue, we have a unique opportunity to find a solution for 7 million stars using the individual distances and for all three stellar velocity components or, more specifically, to find separate and simultaneous solutions from the proper motions and radial velocities. For the simultaneous solution it becomes possible to determine all 12 kinematic field components.

DATA PREPARATION

Not all stars of the catalogue, but the samples from it made according to some principle to detect a dependence of the parameters on this attribute, are traditionally used to determine the parameters of the kinematic equations. In our case, the distance to stars will be such an attribute. To ensure that all samples are equipotent, we set the same number of stars in a sample. After some experiments, this number was chosen to be 400 000. Table 1 shows what ranges of distances will correspond to these samples.

Thus, exactly six million stars are concentrated in 15 samples, which accounts for the bulk of the catalogue.

Whereas the nearby samples have a fairly uniform distribution of stars over the celestial sphere, the distant ones show, as expected, a significant concentration of stars to the Galactic equator.

Unfortunately, the photometry of stars from the catalogue is still insufficient at present and, therefore, it is so far impossible to make a sample based on stars of different spectral types or different color indices.

DETERMINING THE KINEMATIC PARAMETERS FROM THE PROPER MOTIONS

We simultaneously solved Eqs. (2) and (3) for each of the samples by the least-squares method (Statmodels Python). The results of this solution are presented in Tables 2 and 3. The solution was found from 400 000 stars of each sample without any averaging. For a convenient analysis, the distance dependence of the parameters is presented in Figs. 5–7.

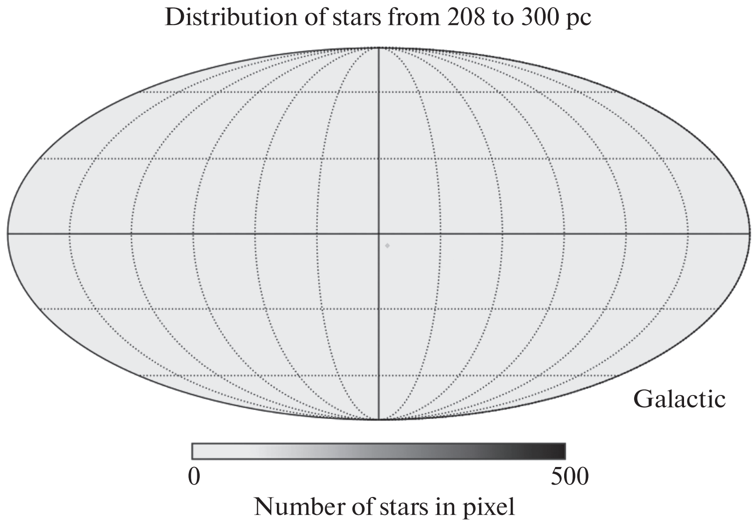

Fig. 3. Distribution of nearby stars (208–300 pc) over the celestial sphere.

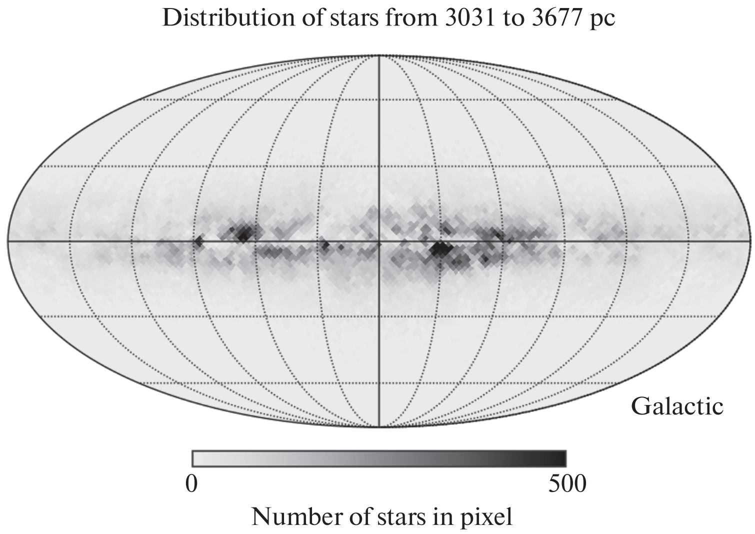

Fig. 4. Distribution of distant stars (3031–3677 pc) over the celestial sphere.

It should be said that the results are in good agreement both with the data of other researchers (see, e.g., Bobylev et al. 2006; Bobylev and Bajkova 2018) and with the data obtained from other catalogues (Vityazev and Tsvetkov 2009; Vityazev et al. 2017). The Oort parameters B (ω_3) and A (M_{12}) have traditional values, about -13 and $+15$ km s^{-1} kpc^{-1}. The remaining parameters, in particular, the solar terms also have conventional values. For nearby stars the angular velocity components ω_1 and especially ω_2 are significant, showing that the rigid-body rotation vector of nearby stars ($r < 200$ pc) is not perpendicular to the Galactic plane. This fact is also well known from Hipparcos data (Tsvetkov 2006) and can be explained by the kinematics of the Local system of stars (Tsvetkov 1995).

The main question for the linear Ogorodnikov–Milne (and Oort–Lindblad) model is the boundary of applicability. When deriving these equations, it was pointed out that the linear approximation of the expansion of the angular velocity of Galactic rotation must apparently be limited to distances of 1.5–2 kpc (Kulikovskii 1985; Binney and Merrifield 1998). However, our results show that the angular velocity

Table 2. Components of the solar velocity in km s^{-1} and the angular velocity of rigid-body rotation in km s^{-1} kpc $^{-1}$ deduced for stars at various distances from the Gaia DR2 with RV catalogue from the proper motions. The size of each sample is 400 000 stars

R	U	V	W	ω_1	ω_2	ω_3
3–208	10.1 ± 0.1	21.8 ± 0.1	7.9 ± 0.1	0.4 ± 0.6	-5.1 ± 0.6	-12.9 ± 0.6
208–300	9.7 ± 0.1	22.1 ± 0.1	7.7 ± 0.1	1.4 ± 0.2	-2.4 ± 0.2	-12.5 ± 0.2
300–386	10.1 ± 0.1	22.6 ± 0.1	7.8 ± 0.1	0.6 ± 0.2	-1.2 ± 0.2	-12.3 ± 0.2
386–474	10.1 ± 0.1	22.6 ± 0.1	7.8 ± 0.1	0.7 ± 0.1	-1.0 ± 0.1	-12.8 ± 0.1
474–571	10.3 ± 0.1	22.7 ± 0.1	7.8 ± 0.1	0.5 ± 0.1	-1.0 ± 0.1	-13.0 ± 0.1
571–687	10.4 ± 0.1	22.6 ± 0.1	7.8 ± 0.1	0.3 ± 0.1	-1.1 ± 0.1	-13.0 ± 0.1
687–835	10.7 ± 0.1	23.5 ± 0.1	7.8 ± 0.1	0.1 ± 0.1	-0.7 ± 0.1	-12.9 ± 0.1
835–1040	11.1 ± 0.1	25.8 ± 0.1	8.0 ± 0.1	0.2 ± 0.1	-0.5 ± 0.1	-12.6 ± 0.1
1040–1303	11.4 ± 0.1	29.2 ± 0.1	8.1 ± 0.1	0.4 ± 0.1	-0.2 ± 0.1	-12.6 ± 0.0
1303–1594	11.7 ± 0.1	31.9 ± 0.1	8.2 ± 0.1	0.7 ± 0.1	0.0 ± 0.1	-12.9 ± 0.0
1594–1897	12.0 ± 0.1	34.4 ± 0.1	8.2 ± 0.1	0.7 ± 0.1	0.3 ± 0.1	-13.2 ± 0.0
1897–2220	12.2 ± 0.1	37.1 ± 0.1	8.3 ± 0.1	0.9 ± 0.1	0.2 ± 0.1	-13.6 ± 0.0
2220–2582	12.2 ± 0.1	40.7 ± 0.1	8.3 ± 0.1	0.9 ± 0.1	0.1 ± 0.1	-13.9 ± 0.0
2582–3031	12.6 ± 0.1	46.2 ± 0.1	8.4 ± 0.1	0.9 ± 0.1	0.1 ± 0.1	-14.2 ± 0.0
3031–3677	13.0 ± 0.1	54.2 ± 0.1	8.6 ± 0.1	0.7 ± 0.1	0.3 ± 0.1	-14.6 ± 0.0

Table 3. Components of the deformation tensor in km s^{-1} kpc $^{-1}$ deduced for stars at various distances from the Gaia DR2 with RV catalogue from the proper motions. The size of each sample is 400 000 stars

r	M_{12}	M_{13}	M_{23}	C	K
3–208	16.8 ± 0.7	-3.1 ± 0.7	0.1 ± 0.7	-1.7 ± 0.7	2.0 ± 1.3
208–300	16.2 ± 0.3	-1.4 ± 0.3	-1.0 ± 0.3	-2.7 ± 0.3	-2.0 ± 0.5
300–386	15.4 ± 0.2	-0.4 ± 0.2	0.3 ± 0.2	-3.2 ± 0.2	-1.9 ± 0.4
386–474	15.2 ± 0.2	-1.1 ± 0.2	0.3 ± 0.2	-3.3 ± 0.2	-2.6 ± 0.3
474–571	15.2 ± 0.1	-1.1 ± 0.2	0.1 ± 0.1	-3.5 ± 0.1	-2.7 ± 0.3
571–687	15.6 ± 0.1	-1.3 ± 0.1	0.5 ± 0.1	-3.5 ± 0.1	-2.5 ± 0.2
687–835	15.6 ± 0.1	-0.6 ± 0.1	0.9 ± 0.1	-2.8 ± 0.1	-2.3 ± 0.2
835–1040	15.3 ± 0.1	-0.6 ± 0.1	0.7 ± 0.1	-2.0 ± 0.1	-2.0 ± 0.2
1040–1303	14.9 ± 0.1	-0.4 ± 0.1	0.2 ± 0.1	-1.6 ± 0.1	-1.7 ± 0.2
1303–1594	14.5 ± 0.1	-0.3 ± 0.1	-0.1 ± 0.1	-1.3 ± 0.1	-1.6 ± 0.2
1594–1897	14.4 ± 0.0	0.0 ± 0.1	-0.2 ± 0.1	-1.1 ± 0.0	-1.1 ± 0.1
1897–2220	13.9 ± 0.0	-0.1 ± 0.1	-0.5 ± 0.1	-1.2 ± 0.0	-1.4 ± 0.1
2220–2582	13.6 ± 0.0	-0.3 ± 0.1	-0.4 ± 0.1	-1.1 ± 0.0	-1.2 ± 0.1
2582–3031	13.0 ± 0.0	-0.4 ± 0.1	-0.5 ± 0.1	-1.0 ± 0.0	-1.1 ± 0.1
3031–3677	12.3 ± 0.0	-0.2 ± 0.1	-0.3 ± 0.1	-0.7 ± 0.0	-1.1 ± 0.1

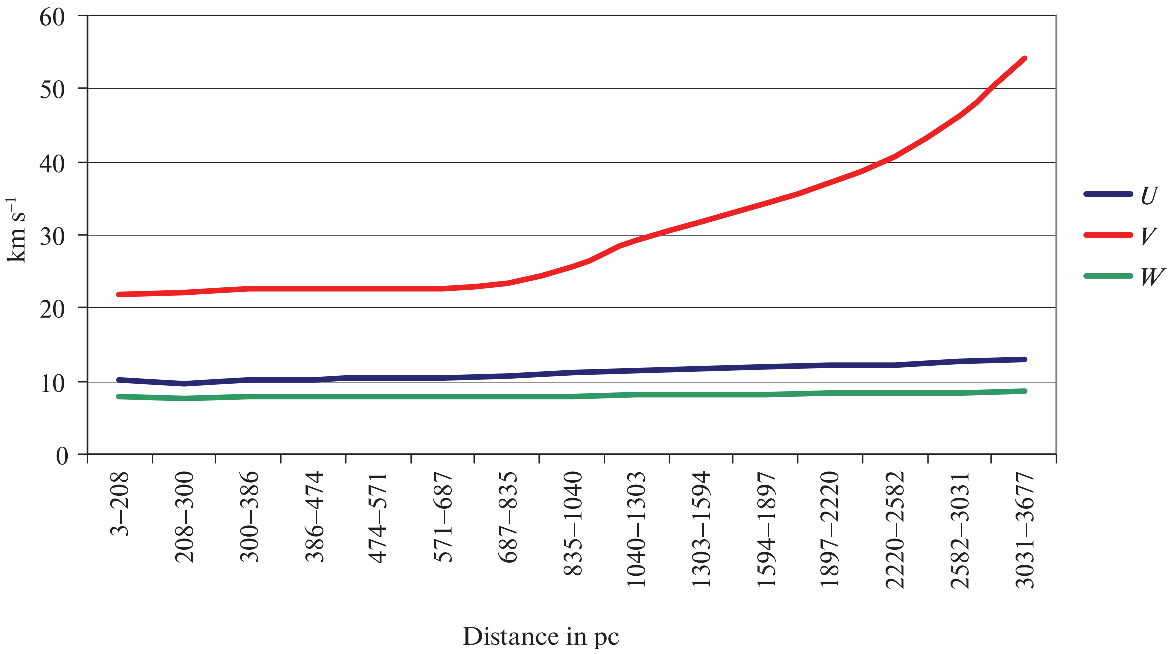

Fig. 5. Solar motion components U, V, W versus distance.

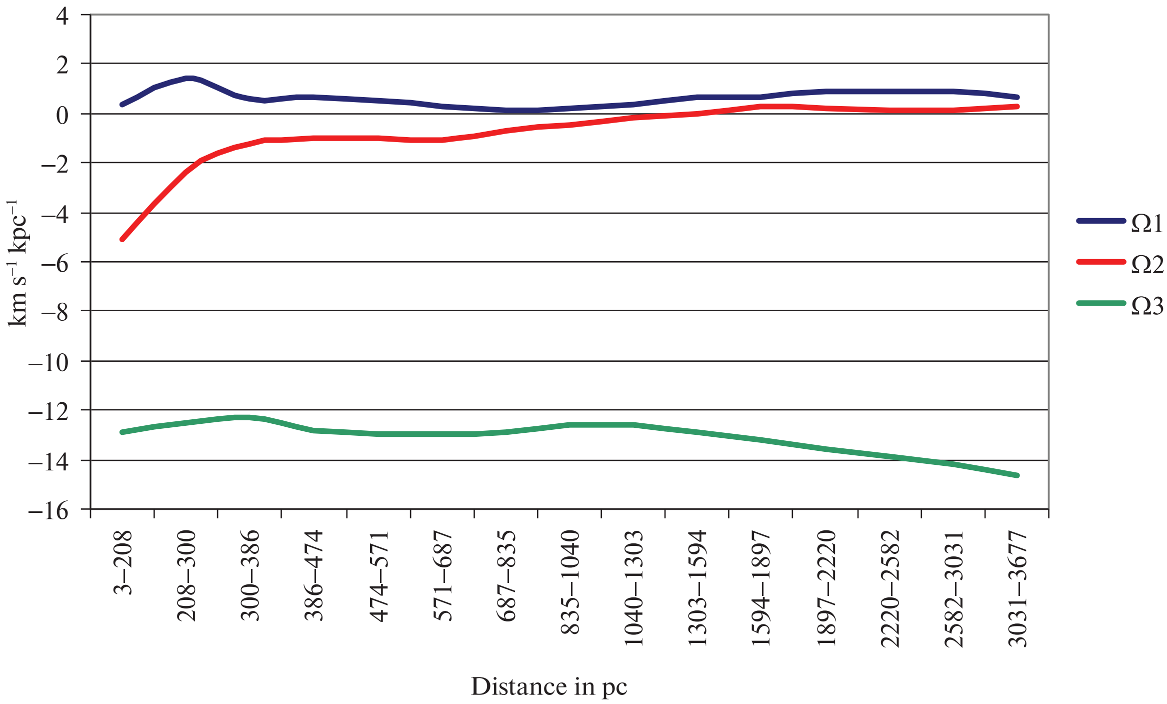

Fig. 6. Angular velocity components ω_1, ω_2, ω_3 versus distance.

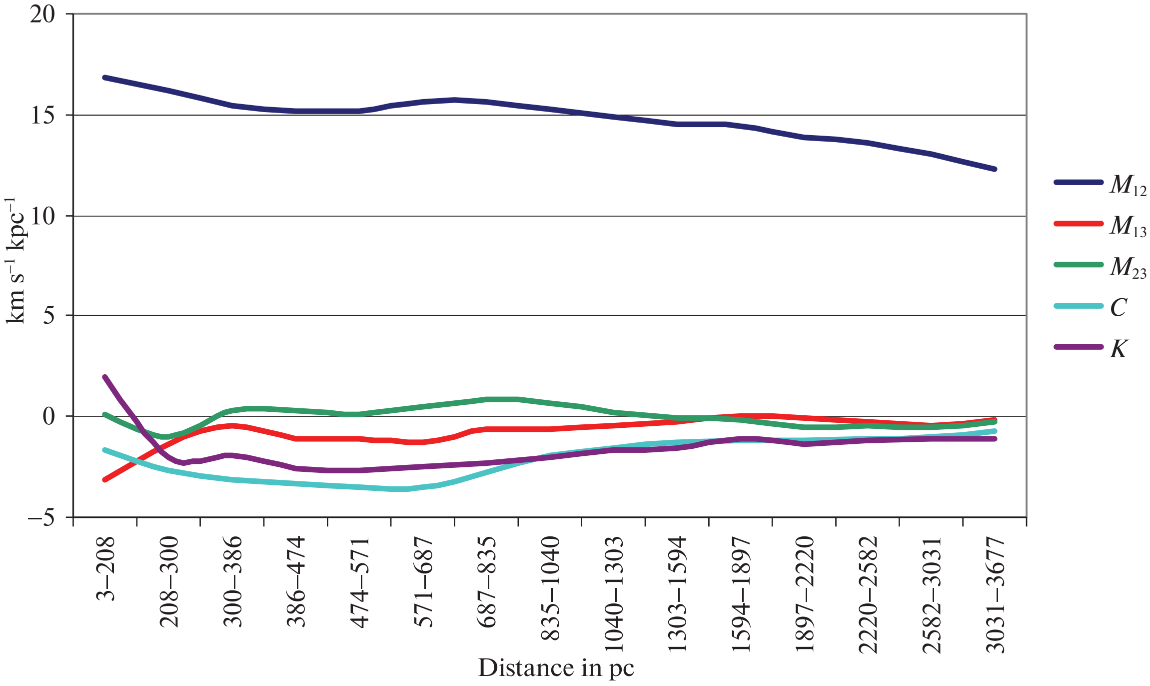

Fig. 7. Deformation tensor components M_{12}, M_{13}, M_{23}, C, K versus distance.

parameters and the coefficient M_{12} have stable values for stars at great distances as well. Other researchers also reach the same conclusion by analyzing the kinematics of open star clusters in the PPMXL catalogue (Popova and Loktin 2005).

However, there is one parameter, the solar velocity component V whose positive direction corresponds to the direction of Galactic rotation (along the Y axis of the Galactic coordinate system), that begins to change starting from a distance of 800 pc. This is clearly illustrated by Fig. 5.

The models including the nonlinear terms in the velocity field expansion should apparently be used for catalogues containing stars at such significant distances. To construct the stellar velocity field in the solar neighborhood, we tried to implement such approaches based on data from the Hipparcos catalogue (Tsvetkov 2001). A simpler method that would allow one to assess whether it is proper to apply the linear model at great distances can be to consider the stellar proper motions in a narrow field toward the Galactic center, for example, with a radius of 1° at various distances and at the opposite point. This will lead to the extraction of a cone (Fig. 8) containing $\sim 377\,000$ stars in space. We divided the distance from 5 (toward the Galactic anticenter) to 8 kpc (toward the Galactic center) into 500 zones, constructed the distribution of proper motions $\mu_l \cos b$ for each zone, and calculated the median of this distribution. We also calculated the linear stellar velocity in the Galactic plane based on these data. The results obtained are presented in Fig. 9. Its analysis shows the stability of the angular velocity and a linear growth in the stellar velocity relative to the Sun with increasing distance at distances up to 6 kpc toward the Galactic center and 4 kpc toward the Galactic anticenter. This means that the applicability of the linear Galactic rotation model spans a distance range of at least 10 kpc!

DETERMINING THE KINEMATIC PARAMETERS FROM THE RADIAL VELOCITIES AND THE SIMULTANEOUS SOLUTION

The Gaia DR2 with RV catalogue allows the kinematic parameters of the deformation tensor to be estimated from the stellar radial velocities that are deduced by a method independent of the proper motion determination. The mean radial velocity error is ~ 1 km s^{-1}, while 3/4 of the stars from the Gaia DR2 with RV catalogue have an error of less than 2 km s^{-1}. In contrast, the mean total proper motion error is 0.1 mas, which gives an error in determining the tangential velocity of 1 km s^{-1} at a distance

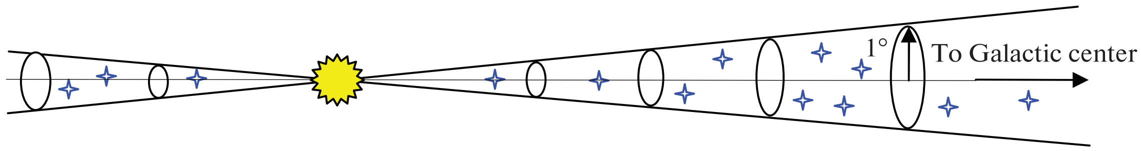

Fig. 8. Sample of stars to determine the distance dependence of the angular and linear velocities.

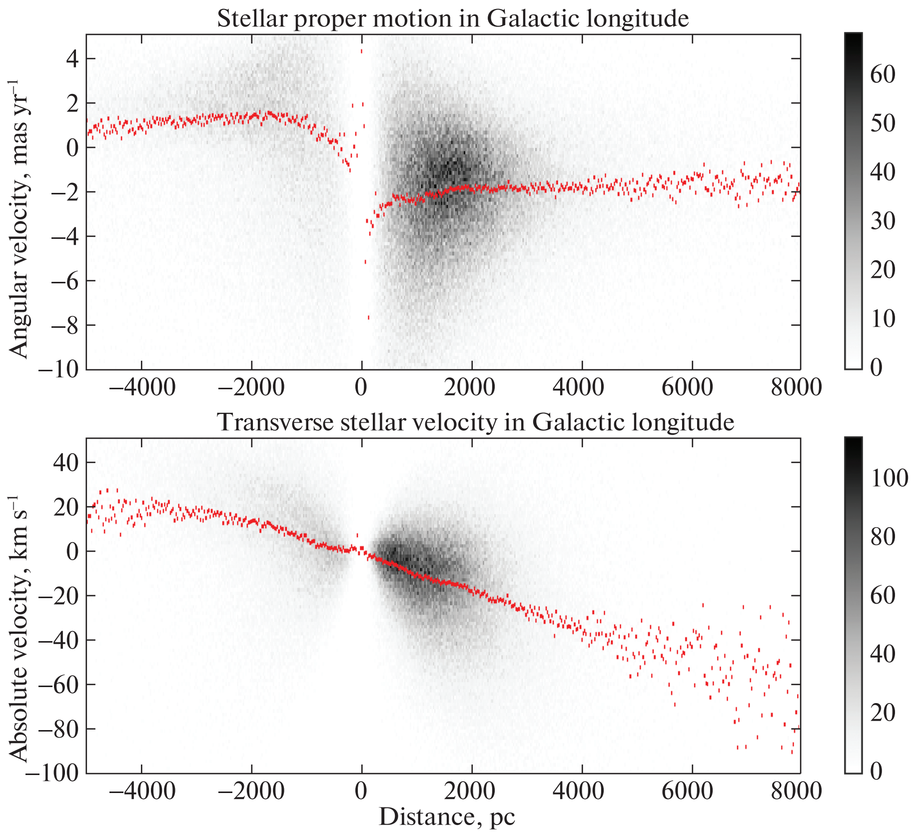

Fig. 9. Stellar proper motions $\mu_l \cos b$ in a narrow zone (see Fig. 8) and linear velocity calculated on their basis versus linear distance from the Sun (the direction toward the Galactic center is on the right). The red dots mark the median of the velocity at a given distance.

of ~ 2 kpc. Thus, on the subset of stars under study the proper motions and radial velocities turn out to be approximately equally accurate in random terms.

Whereas at the epoch of ground-based catalogues the like kinematic parameters determined from the proper motions and radial velocities showed noticeable differences, for the Gaia DR2 with RV catalogue the parameters derived from the same samples turn out to be very close to one another (Tables 4 and 5). This is evidence that the systematic errors of the Gaia catalogue are extremely small. The behavior of the parameters as a function of the sample of stars turns out to be identical, as is clearly illustrated by Figs. 10 and 11.

DETERMINING THE KINEMATIC PARAMETERS FROM THE SIMULTANEOUS SOLUTION BASED ON THE STELLAR PROPER MOTIONS AND RADIAL VELOCITIES

The consistent values of the parameters make the simultaneous solution of Eqs. (2)–(4) based on both stellar proper motions and radial velocities legitimate.

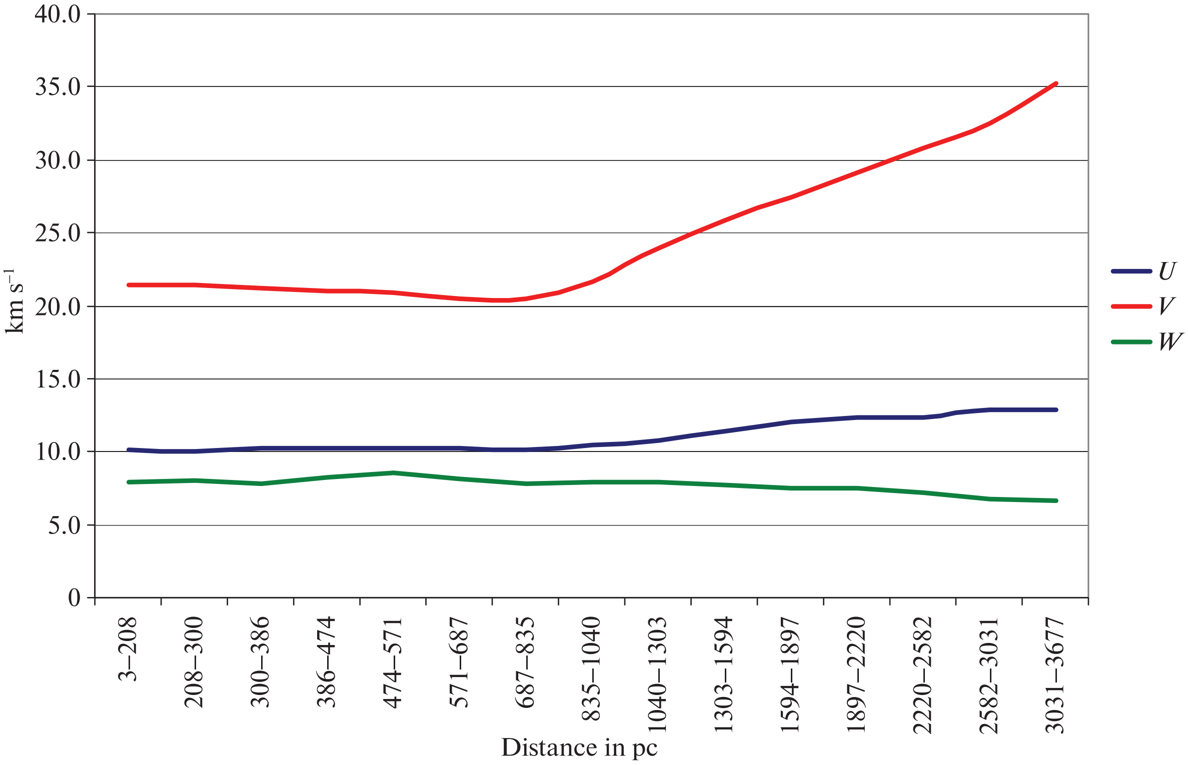

Fig. 10. Solar motion components U, V, W determined from the radial velocities versus distance.

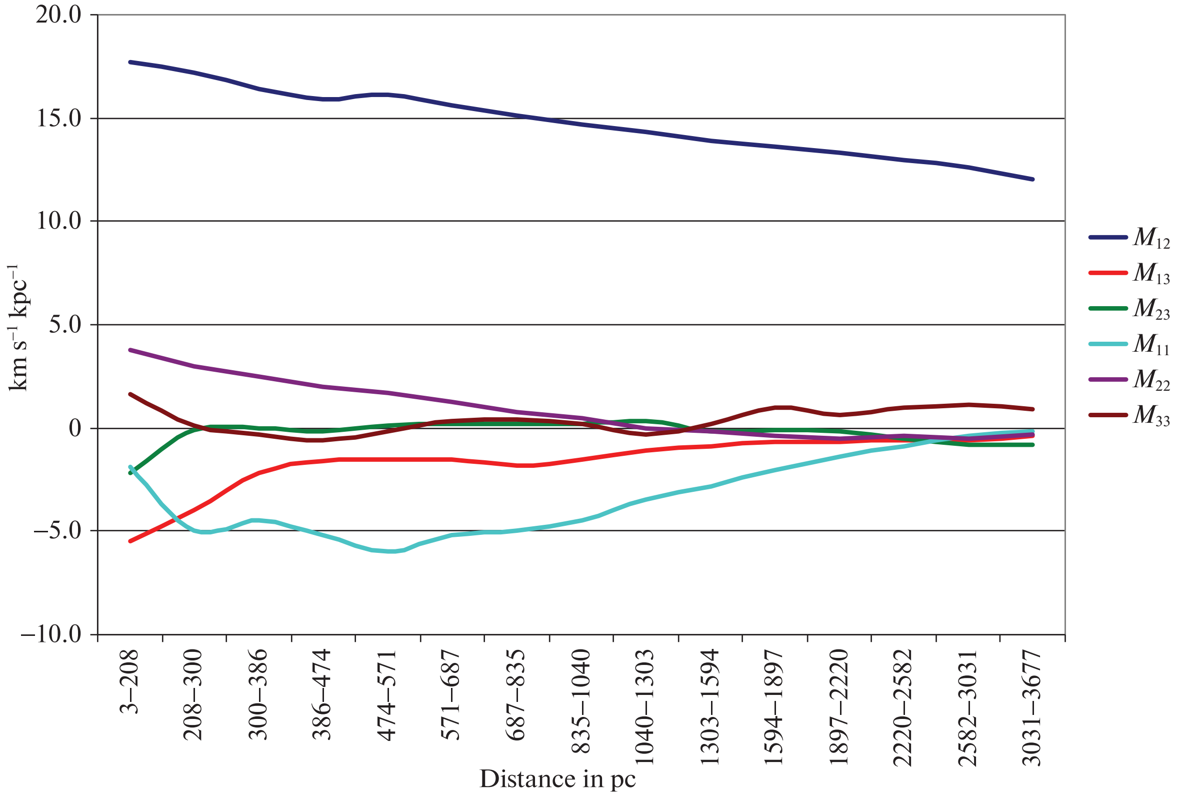

Fig. 11. Deformation tensor components determined from the radial velocities versus distance.

Table 4. Solar velocity components in km s^{-1} deduced from the stellar radial velocities of the Gaia DR2 with RV catalogue. The size of each sample is 400 000 stars

R	U	V	W
3–208	10.1 ± 0.1	21.4 ± 0.1	7.9 ± 0.1
208–300	10.0 ± 0.1	21.4 ± 0.1	8.0 ± 0.1
300–386	10.2 ± 0.1	21.2 ± 0.1	7.8 ± 0.1
386–474	10.2 ± 0.1	21.0 ± 0.1	8.2 ± 0.1
474–571	10.2 ± 0.1	20.9 ± 0.1	8.5 ± 0.1
571–687	10.2 ± 0.1	20.5 ± 0.1	8.1 ± 0.1
687–835	10.1 ± 0.1	20.5 ± 0.1	7.8 ± 0.1
835–1040	10.5 ± 0.1	21.6 ± 0.1	7.9 ± 0.1
1040–1303	10.8 ± 0.1	24.0 ± 0.1	7.9 ± 0.2
1303–1594	11.4 ± 0.1	25.9 ± 0.1	7.7 ± 0.2
1594–1897	12.0 ± 0.1	27.4 ± 0.1	7.5 ± 0.2
1897–2220	12.4 ± 0.1	29.1 ± 0.1	7.5 ± 0.2
2220–2582	12.4 ± 0.1	30.8 ± 0.1	7.2 ± 0.3
2582–3031	12.9 ± 0.1	32.5 ± 0.1	6.8 ± 0.3
3031–3677	12.9 ± 0.1	35.2 ± 0.1	6.6 ± 0.3

It is possible to obtain all 12 stellar–kinematic parameters in this solution. The results are presented in Tables 6 and 7. We see an extremely high accuracy of the results in random terms. These are apparently the most accurate estimates of the Ogorodnikov–Milne model parameters to date.

An analysis of the solution once again shows the presence of an anomaly in the kinematics of nearby stars. This is primarily because the angular velocity components ω_1 and ω_2 as well as the parameter describing the stellar field expansion along the Y axis, M_{22}, are nonzero.

The fact of a significant parameter M_{11} for all samples is interesting. Its value points to the contraction of the stellar system along the X axis (toward the Galactic center).

For stars farther than 1.5 kpc the solar motion parameter V grows rapidly, i.e., there is motion in the plane of the Galaxy in the direction of its rotation. This is so far the only evidence that the linear model is beyond the range of its applicability at great distances.

CONCLUSIONS

Our study showed that the standard Ogorodnikov–Milne stellar–kinematic model has a fairly wide spatial domain of applicability. We intentionally applied it to more distant stars than is usually done and found that the model works satisfactorily for stars at distances up to 3 kpc as well, except for the determination of the parameter V (the solar motion along the Y axis). The model parameters determined from the proper motions and radial velocities of six million stars from the Gaia DR2 with RV catalogue are in good agreement, which allowed the simultaneous solution in which all model parameters are determined to be obtained. The kinematics of the nearest (nearer than 200 pc) stars deserves separate attention, because the parameters of this sample differ from the standard ones. Despite the general stability of the model parameters, depending on the distances to stars, for distances more than 1.5–2 kpc we should think about the development of a new kinematic approach that would describe the behavior of the entire Galaxy, because the solar motion parameters begin to dramatically change for distant stars.

The rigid-body rotation component ω_1, which is definitely significant and takes a value of 0.6–1.0 km s^{-1} kpc^{-1} starting from a distance of more than 1 kpc, deserves special attention. Thus, this is not a local effect (which takes place, for example, in nearby stars for ω_2), but evidence for the rotation of the entire system of stars around the X axis directed toward the Galactic center.

Furthermore, kinematic components that are not described by the simple linear model may be contained in the proper motions and radial velocities of distant stars. The next paper will be devoted to their search.

Table 5. Deformation tensor components in km s^{-1} kpc^{-1} deduced from the stellar radial velocities of the Gaia DR2 with RV catalogue. The size of each sample is 400 000 stars

R	M_{12}	M_{13}	M_{23}	M_{11}	M_{22}	M_{33}
3–208	17.7 ± 0.6	-5.5 ± 0.6	-2.2 ± 0.6	-1.9 ± 0.7	3.8 ± 0.7	1.6 ± 0.8
208–300	17.2 ± 0.3	-4.0 ± 0.4	-0.1 ± 0.4	-5.0 ± 0.4	3.0 ± 0.4	0.1 ± 0.5
300–386	16.4 ± 0.2	-2.2 ± 0.3	0.0 ± 0.3	-4.5 ± 0.3	2.5 ± 0.3	-0.3 ± 0.4
386–474	15.9 ± 0.2	-1.6 ± 0.2	-0.2 ± 0.2	-5.2 ± 0.2	2.0 ± 0.2	-0.6 ± 0.3
474–571	16.1 ± 0.2	-1.5 ± 0.2	0.1 ± 0.2	-6 ± 0.2	1.7 ± 0.2	-0.2 ± 0.3
571–687	15.6 ± 0.1	-1.5 ± 0.2	0.2 ± 0.2	-5.2 ± 0.2	1.3 ± 0.1	0.3 ± 0.3
687–835	15.1 ± 0.1	-1.8 ± 0.2	0.2 ± 0.1	-5.0 ± 0.1	0.8 ± 0.1	0.4 ± 0.3
835–1040	14.7 ± 0.1	-1.5 ± 0.1	0.2 ± 0.1	-4.5 ± 0.1	0.5 ± 0.1	0.2 ± 0.2
1040–1303	14.3 ± 0.1	-1.1 ± 0.1	0.3 ± 0.1	-3.5 ± 0.1	0 ± 0.1	-0.3 ± 0.2
1303–1594	13.9 ± 0.1	-0.9 ± 0.1	-0.2 ± 0.1	-2.8 ± 0.1	-0.2 ± 0.1	0.2 ± 0.2
1594–1897	13.6 ± 0.1	-0.7 ± 0.1	-0.1 ± 0.1	-2.0 ± 0.1	-0.4 ± 0.1	1.0 ± 0.2
1897–2220	13.3 ± 0.0	-0.7 ± 0.1	-0.2 ± 0.1	-1.4 ± 0.1	-0.5 ± 0.1	0.6 ± 0.2
2220–2582	13.0 ± 0.0	-0.6 ± 0.1	-0.5 ± 0.1	-0.9 ± 0.1	-0.4 ± 0.0	1.0 ± 0.2
2582–3031	12.6 ± 0.0	-0.6 ± 0.1	-0.8 ± 0.1	-0.4 ± 0.0	-0.5 ± 0.0	1.1 ± 0.2
3031–3677	12.0 ± 0.0	-0.4 ± 0.1	-0.8 ± 0.1	-0.2 ± 0.0	-0.3 ± 0.0	0.9 ± 0.2

Table 6. Components of the solar velocity in km s^{-1} and the angular velocity of rigid-body rotation in km s^{-1} kpc^{-1} deduced from the simultaneous solution based on all three components

R	U	V	W	ω_1	ω_2	ω_3
3–208	10.1 ± 0.0	21.8 ± 0.0	7.9 ± 0.0	0.4 ± 0.5	-5.1 ± 0.5	-12.9 ± 0.5
208–300	9.7 ± 0.0	22.1 ± 0.0	7.7 ± 0.0	1.4 ± 0.2	-2.4 ± 0.2	-12.5 ± 0.2
300–386	10.1 ± 0.0	22.5 ± 0.0	7.8 ± 0.0	0.6 ± 0.1	-1.2 ± 0.1	-12.3 ± 0.1
386–474	10.1 ± 0.0	22.4 ± 0.0	7.8 ± 0.0	0.7 ± 0.1	-1.1 ± 0.1	-12.8 ± 0.1
474–571	10.3 ± 0.0	22.4 ± 0.0	7.8 ± 0.0	0.5 ± 0.1	-1.1 ± 0.1	-13.1 ± 0.1
571–687	10.3 ± 0.0	22.1 ± 0.0	7.8 ± 0.0	0.3 ± 0.1	-1.2 ± 0.1	-13.1 ± 0.1
687–835	10.6 ± 0.0	22.6 ± 0.0	7.8 ± 0.0	0.1 ± 0.1	-0.8 ± 0.1	-13.0 ± 0.1
835–1040	10.9 ± 0.0	24.0 ± 0.0	8.0 ± 0.0	0.2 ± 0.1	-0.7 ± 0.1	-12.9 ± 0.1
1040–1303	11.1 ± 0.1	26.4 ± 0.1	8.0 ± 0.1	0.4 ± 0.1	-0.4 ± 0.1	-13.1 ± 0.1
1303–1594	11.5 ± 0.1	28.0 ± 0.1	8.1 ± 0.1	0.7 ± 0.1	-0.2 ± 0.1	-13.6 ± 0.0
1594–1897	12.0 ± 0.1	29.3 ± 0.1	8.0 ± 0.1	0.6 ± 0.1	0.1 ± 0.1	-14.0 ± 0.0
1897–2220	12.4 ± 0.1	30.6 ± 0.1	8.1 ± 0.1	0.7 ± 0.1	0.0 ± 0.1	-14.5 ± 0.0
2220–2582	12.5 ± 0.1	32.3 ± 0.1	8.0 ± 0.1	0.9 ± 0.1	0.1 ± 0.1	-15.0 ± 0.0
2582–3031	12.9 ± 0.1	34.0 ± 0.1	7.9 ± 0.1	1.0 ± 0.1	0.2 ± 0.1	-15.6 ± 0.0
3031–3677	13.1 ± 0.1	36.7 ± 0.1	7.8 ± 0.1	1.0 ± 0.1	0.3 ± 0.1	-16.2 ± 0.0

Table 7. Deformation tensor components in km s^{-1} kpc^{-1} deduced from the simultaneous solution based on all three components

R	M_{12}	M_{13}	M_{23}	M_{11}	M_{22}	M_{33}
3–208	16.8 ± 0.6	-3.1 ± 0.6	0.1 ± 0.6	0.0 ± 2.6	3.4 ± 2.6	-0.3 ± 2.6
208–300	16.2 ± 0.2	-1.5 ± 0.2	-0.9 ± 0.2	-3.9 ± 0.6	1.5 ± 0.6	0.7 ± 0.7
300–386	15.5 ± 0.2	-0.5 ± 0.2	0.3 ± 0.2	-4.5 ± 0.4	1.9 ± 0.4	0.6 ± 0.4
386–474	15.3 ± 0.1	-1.1 ± 0.1	0.2 ± 0.1	-5.3 ± 0.3	1.4 ± 0.3	0.5 ± 0.3
474–571	15.4 ± 0.1	-1.2 ± 0.1	0.1 ± 0.1	-5.8 ± 0.2	1.4 ± 0.2	0.4 ± 0.2
571–687	15.6 ± 0.1	-1.3 ± 0.1	0.5 ± 0.1	-5.4 ± 0.1	1.5 ± 0.1	0.5 ± 0.2
687–835	15.4 ± 0.1	-1.0 ± 0.1	0.7 ± 0.1	-5.0 ± 0.1	0.9 ± 0.1	0.3 ± 0.2
835–1040	15.0 ± 0.1	-0.9 ± 0.1	0.5 ± 0.1	-4.2 ± 0.1	0.4 ± 0.1	0.1 ± 0.1
1040–1303	14.6 ± 0.0	-0.7 ± 0.1	0.2 ± 0.1	-3.5 ± 0.1	0.0 ± 0.1	-0.1 ± 0.1
1303–1594	14.2 ± 0.0	-0.6 ± 0.1	-0.2 ± 0.1	-2.8 ± 0.1	-0.1 ± 0.1	0.2 ± 0.1
1594–1897	13.8 ± 0.0	-0.4 ± 0.1	-0.1 ± 0.1	-2.1 ± 0.0	-0.2 ± 0.0	0.5 ± 0.1
1897–2220	13.4 ± 0.0	-0.5 ± 0.1	-0.3 ± 0.1	-1.6 ± 0.0	-0.3 ± 0.0	0.6 ± 0.1
2220–2582	13.1 ± 0.0	-0.5 ± 0.1	-0.5 ± 0.1	-1.1 ± 0.0	-0.3 ± 0.0	0.9 ± 0.1
2582–3031	12.7 ± 0.0	-0.5 ± 0.0	-0.7 ± 0.0	-0.6 ± 0.0	-0.3 ± 0.0	1.0 ± 0.1
3031–3677	12.0 ± 0.0	-0.3 ± 0.0	-0.7 ± 0.0	-0.3 ± 0.0	-0.2 ± 0.0	0.9 ± 0.1

REFERENCES

1. J. Binney and M. Merrifield, *Galactic Astronomy* (Princeton Univ. Press, Princeton, 1998).
2. V. V. Bobylev, G. A. Gontcharov, and A. T. Bajkova, Astron. Rep. **50**, 733 (2006).
3. V. V. Bobylev and A. T. Bajkova, Astron. Lett. **44**, 676 (2018)].
4. ESA Gaia Science Community, Data release scenario. https://www.cosmos.esa.int/web/gaia/release.
5. P. G. Kulikovskii, *Stellar Astronomy* (Nauka, Moscow, 1985) [in Russian].
6. B. A. du Mont, Astron. Astrophys. **61**, 127 (1977).
7. K. F. Ogorodnikov, *Dynamics of Stellar Systems* (Fizmatgiz, Moscow, 1965) [in Russian].
8. M. E. Popova and A. V. Loktin, Astron. Lett. **31**, 663 (2005).
9. Statmodels Python Module. http://www.statsmodels.org/.
10. J. Torra, D. Fernandez, and F. Figueras, Astron. Astrophys. **359**, 82 (2000).
11. A. Tsvetkov, Astron. Astrophys. Transact. **25**, 165 (2006).
12. A. S. Tsvetkov, Astron. Astrophys. Transact. **9**, 1 (1995).
13. A. S. Tsvetkov, Celest. Mech. Dyn. Astron. **81**, 453 (2001).
14. V. V. Vityazev, A. V. Popov, A. S. Tsvetkov, S. D. Petrov, D. A. Trofimov, and V. I. Kiyaev, Astron. Lett. **44**, 236 (2018).
15. V. V. Vityazev, A. S. Tsvetkov, V. V. Bobylev, and A. T. Baikova, Astrofizika **60**, 503 (2017).
16. V. V. Vityazev and A. S. Tsvetkov, Astron. Lett. **34**, 100 (2009).

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