ISSN 1063-7737, Astronomy Letters, 2020, Vol. 46, No. 8, pp. 509–517. © Pleiades Publishing, Inc., 2020.
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Using Spherical Harmonics in the Galactocentric Coordinate System to Study the Kinematics of Globular Star Clusters

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Received April 3, 2020; revised June 6, 2020; accepted June 25, 2020

Abstract—The technique of spherical harmonics, both scalar and vector ones, has long been applied to analyze the astronomical data on a sphere, for example, in the representation of systematic errors, in stellar kinematics. Up to now, spherical harmonics have been used exclusively in heliocentric coordinate systems: the equatorial or Galactic one. To study the kinematics of the entire Galaxy (and not only the solar neighborhood), it is reasonable to pass to the Galactocentric coordinate system. The second release of the Gaia catalogue does not yet allow such an analysis to be performed for individual stars due to the relatively low accuracy of the parallaxes. However, such a study seems possible for globular star clusters, despite their small number. Although the kinematics of globular clusters was studied in detail in many papers, we want to test the method of analyzing the Galactocentric proper motions and radial velocities using spherical harmonics based on data from this catalogue.

DOI: 10.1134/S1063773720080058

Keywords: *Gaia, globular clusters, spherical harmonics, Galactocentric coordinate system, Galactic kinematics.*

INTRODUCTION

The final Gaia catalogue (Gaia Collaboration 2016) with accurate parallaxes of all objects on Galactic scales that will become accessible to the astronomical community in several years will allow the tasks of studying the kinematics of the entire Galaxy that have been unthinkable previously to be set. These tasks include a kinematic study of various regions in the Galaxy and not only the solar neighborhood, as has been done until now.

The use of spherical harmonics, scalar and vector ones, to analyze the kinematics of stars is a well-proven technique that has been applied for many years. The use of spherical harmonics in astrometry began with the paper by Brosche (1966). The application of the apparatus of spherical harmonics to analyze the proper motions and radial velocities is described in detail in Vityazev and Tsvetkov (2013) and Vityazev et al. (2014), respectively. The application of this technique in the study of stars in the solar neighborhood has almost always brought additional information compared to the standard Oort–Lindblad or Ogorodnikov–Milne models. The detection of outside-the-model harmonics pointed to unusual kinematic phenomena.

As a rule, a heliocentric coordinate system (the equatorial or Galactic one) is used in the works on stellar kinematics, including the kinematics of globular star clusters (Koch et al. 2018; Helmi et al. 2018). There is a reasonable explanation for this related to observational selection, because all star catalogues, except for the latest PPMXL (Roeser et al. 2010), UCAC4 (Zacharias et al. 2013), XPM (Fedorov et al. 2009, 2010), and Gaia, cover only the nearest solar neighborhoods.

In our view, the use of spherical harmonics in heliocentric coordinate systems has exhausted itself to some extent. Passing to the Galactocentric coordinate system seems promising to us. The "view" from the Galactic center will possibly allow unfamiliar and unexpected effects to be seen in the proper motions and radial velocities of stars. The revealed harmonics in them will possibly give a stimulus to constructing new rotation models for the Galaxy and its subsystems.

Unfortunately, it should be said that the Gaia Data Release 2 (Gaia Collaboration 2018) does not yet allow the kinematics of stars at large distances (for example, near the Galactic center) to be studied, primarily because the parallaxes have a low accuracy. However, this will become possible after the publication of the next Gaia data release.

Pending this event, we propose to develop and test a technique for analyzing the Galactocentric proper

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motions and radial velocities based on data from a small catalogue of globular star clusters containing only 150 entries (Vasiliev 2019). This catalogue is almost complete and includes globular clusters located "on the other side" of the Galactic center. Each entry contains the equatorial coordinates of the cluster center and its mean proper motion and radial velocity derived from a whole group of stars. We took the cluster distances from Harris (2010) and the radial velocities from Baumgardt et al. (2019). As a result of the statistical data averaging for individual stars, these quantities are quite reliable, which is reflected in an extremely low rms error, much lower than that for individual stars at such large distances.

The distribution of globular star clusters easily allows us to pass to the Galactocentric coordinate system, because the globular clusters form a system more or less symmetric relative to the Galactic center. It should be said that Bajkova and Bobylev (2019) and Budanova et al. (2019) have already used the rectangular and cylindrical coordinates anchored to the Galactic center.

It should be noted that quite a few papers are devoted to analyzing the kinematics of globular star clusters based on Gaia DR2 data. An example is the determination of the proper motions and spatial orbits in the already mentioned paper by Baumgardt et al. (2019) using 6-dimensional phase space. Binney and Wong (2017) constructed the Galactic gravitational potential and modeled the orbits of clusters. The paper by Massari et al. (2019) is devoted to the origin of the system of globular clusters as a whole. A large number of papers study the kinematic characteristics and the orbits of individual star clusters (see, e.g., Bobylev and Bajkova 2017). However, our goal more likely pursues not the results proper, but the perfection of the technique, testing its reliability, in prospect, as applied to the big data in succeeding Gaia releases.

It should be noted that the small number of clusters compared to the star catalogues is clearly insufficient for reliable statistical studies, especially with the use of spherical harmonics. Therefore, we do not hope to obtain significant and unexpected results. However, it can be recalled that the first studies of the velocity field for solar-neighborhood stars using spherical harmonics were based on a catalogue of only 512 stars (Fricke 1967).

CONVERSION TO THE GALACTOCENTRIC COORDINATE SYSTEM

The complete procedure of passing from the heliocentric (spherical or rectangular) coordinates to the Galactocentric ones is implemented in the astropy module, a library in Python (https://docs.astropy.org). Its documentation provides the complete algorithm, the calculational formulas, and the constants used.

The conversion algorithm consists in calculating the Cartesian heliocentric coordinates and the rectangular components of the Solar System velocity by taking into account the Galactic rotation and the peculiar solar motion; the coordinates are then converted the other center and the total solar velocity is subtracted from the velocities of objects. Next, the spherical Galactocentric coordinates and the proper motions and radial velocities are calculated in the new frame.

Applying this algorithm yielded a catalogue of globular star clusters containing their rectangular and spherical Galactocentric coordinates as well as the rectangular components of the Galactocentric velocities of the clusters and their spherical components: the proper motions and radial velocities relative to the Galactic center. The catalogue is given in the Appendix (https://cloud.mail.ru/public/RFRr/3kkbCxd7y).

The numerical values of the conversion parameters are given below.

To convert the coordinates of globular star clusters to the Galactocentric coordinate system, we adopted the following coordinates of the Galactic center:

$$\alpha_C = 266.4051^\circ, \quad \delta_C = -28.93617^\circ. \tag{1}$$

The distance to the Galactic center is

$$R_C = 8.122 \text{ kpc}. \tag{2}$$

These parameters were taken from the paper by the GRAVITY collaboration (Abuter et al. 2018). The algorithm also takes into account the height of the Sun above the Galactic plane. This quantity is now estimated (Bennett and Bovy 2019) as

$$Z_\odot = 20.8 \text{ pc}. \tag{3}$$

To calculate the Galactocentric velocity, it is necessary to take into account the velocity of the Sun relative to the Galactic center, which is a sum of the peculiar velocity and the rotation velocity around the Galactic center. From the combinations of data by the GRAVITY collaboration and those from Reid and Brunthaler (2004) and Drimmel and Poggio (2018), the algorithm uses the following components of the solar Galactocentric velocity:

$$V_X = 12.9, \quad V_Y = 245.6, \quad V_Z = 7.78 \text{ km s}^{-1}. \tag{4}$$

Whereas from the solar neighborhood the distribution of globular star clusters looks with a concentration to the Galactic center (Fig. 1), the distribution of these clusters in the Galactocentric coordinate system is much more uniform (Fig. 2). It should be noted

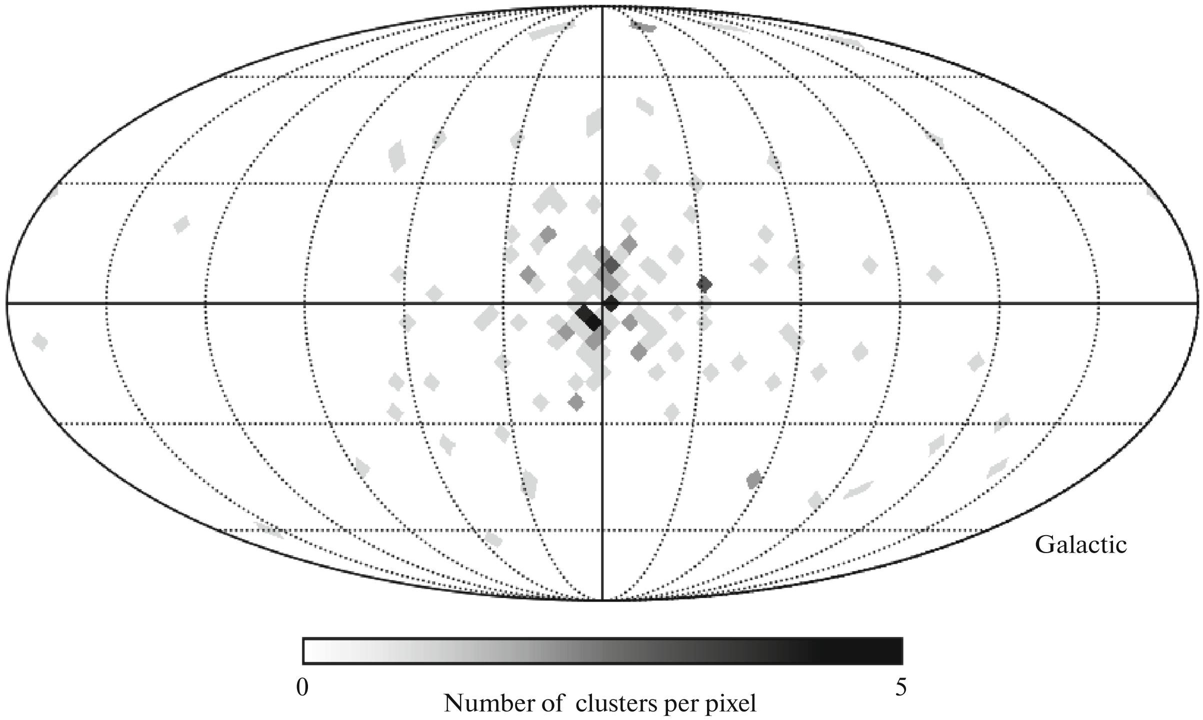

Fig. 1. Distribution of globular star clusters in the heliocentric Galactic coordinate system.

zero longitude in both figures is located at the center, i.e., the axes of both coordinate systems are parallel to each other. Thus, the direction toward the Sun in Fig. 2 corresponds to the image edges (longitude 180°).

REPRESENTING THE GALACTOCENTRIC PROPER MOTIONS AND RADIAL VELOCITIES WITH SPHERICAL HARMONICS

The use of spherical harmonics, both scalar and vector ones, to analyze the kinematics of stars is a well-known technique that has long been applied. In astrometry spherical harmonics have been used since 1966 (Brosche 1966). The application of the apparatus of spherical harmonics to analyze the proper motions and radial velocities is described in detail in Vityazev and Tsvetkov (2013) and Vityazev et al. (2014), respectively. Therefore, we will not present in detail the formulas for the calculation of spherical harmonics, but, for convenience, we will restrict ourselves to Table 1, which gives a relationship of the spherical expansion coefficient to the parameters of the linear 3D kinematic model. Traditionally, we expand not the proper motions and radial velocities themselves, but the quantities $k\mu_l \cos b$, $k\mu_b$, and V_r/r ($k = 4.738$ is the conversion factor from mas yr^{-1} to km s^{-1} kpc^{-1}) in the Galactocentric coordinate system. In this case, all expansion coefficients are expressed in the same units, km s^{-1} kpc^{-1}.

The motion of objects in the linear model is considered as a superposition of the translational motion of the system as a whole, rigid-body rotation, and the deformation tensor:

$$\mathbf{V}_* = \mathbf{V} + \mathbf{\Omega} \times \mathbf{r} + \mathbf{M}^+ \times \mathbf{r}, \qquad (5)$$

where $\mathbf{V}_*$ is the object's velocity, $\mathbf{V}$ is the system's translational motion, $\mathbf{\Omega}$ is the angular velocity of rigid-body rotation of the system as a whole, and $\mathbf{M}$ is the symmetric deformation tensor of the velocity field. This model has 12 parameters:

U, V, W are the components of the translational motion vector $\mathbf{V}$ of the entire system of objects relative to the center of the coordinate system;

$\omega_1, \omega_2, \omega_3$ are the components of the rigid-body rotation vector $\mathbf{\Omega}$ of the system as a whole;

$M^+_{11}, M^+_{22}, M^+_{33}$ are the parameters of the deformation tensor $\mathbf{M}^+$ describing the contraction—expansion along the principal axes of the coordinate system;

$M^+_{12}, M^+_{13}, M^+_{23}$ are parameters of the deformation tensor $\mathbf{M}^+$ describing the velocity field deformation in the corresponding planes.

The results of the expansion of the Galactocentric proper motions $k\mu_l \cos b$ and $k\mu_b$ in terms of vector

Table 1. Relationship of the kinematic parameters of the linear model to the coefficients of the vector (t_i, s_i) and scalar (v_i) spherical expansions of the proper motions $k\mu_l \cos b$ and $k\mu_b$ and radial velocities V_r/r

i	t_i	s_i	v_i
0	–	–	$1.18M^+_{11} + 1.18M^+_{22} + 1.18M^+_{33}$
1	$2.89\omega_3$	$2.89W/\langle r\rangle$	$2.05W/\langle r\rangle$
2	$2.89\omega_2$	$2.89V/\langle r\rangle$	$2.05V/\langle r\rangle$
3	$2.89\omega_1$	$2.89U/\langle r\rangle$	$2.05U/\langle r\rangle$
4		$-0.65M^+_{11} - 0.65M^+_{22} + 1.29M^+_{33}$	$-0.53M^+_{11} - 0.53M^+_{22} + 1.06M^+_{33}$
5		$2.24M^+_{23}$	$1.83M^+_{23}$
6		$2.24M^+_{13}$	$1.83M^+_{13}$
7		$2.24M^+_{12}$	$1.83M^+_{12}$
8		$1.12M^+_{11} - 1.12M^+_{22}$	$0.92M^+_{11} - 0.92M^+_{22}$

$\langle r\rangle$ is the mean distance of the objects under consideration.

Table 2. Coefficients of the expansion of the Galactocentric proper motions $k\mu_l \cos b$ and $k\mu_b$ and radial velocities V_r/r for globular clusters in terms of vector and scalar spherical harmonics and their rms errors (km s^{-1} kpc^{-1})

N	t_i	σ	s_i	σ	v_i	σ
0					−0.4	15.7
1	−39.4	16.5	1.6	16.5	−24.2	16.6
2	−12.2	16.7	17.6	16.7	29.2	16.1
3	12.0	18.0	28.5	18.0	−10.7	13.8
4	24.9	17.0	6.2	17.0	11.8	16.6
5	31.3	17.3	−7.7	17.3	15.8	16.5
6	20.3	17.7	4.3	17.7	15.5	14.2
7	−16.9	16.0	6.2	16.0	−6.9	15.4
8	1.9	16.8	−28.3	16.8	9.1	14.4
9	−6.8	18.1	−16.4	18.1	5.9	15.4
10	−21.3	16.9	26.7	16.9	6.3	18.1
11	13.3	16.6	27.2	16.6	−13.4	14.9
12	−22.3	16.7	11.4	16.7	3.4	15.5
13	−19.6	17.1	3.8	17.1	−2.3	14.2
14	−17.1	15.6	6.1	15.6	−2.2	14.4
15	−11.4	16.0	7.6	16.0	7.5	13.9

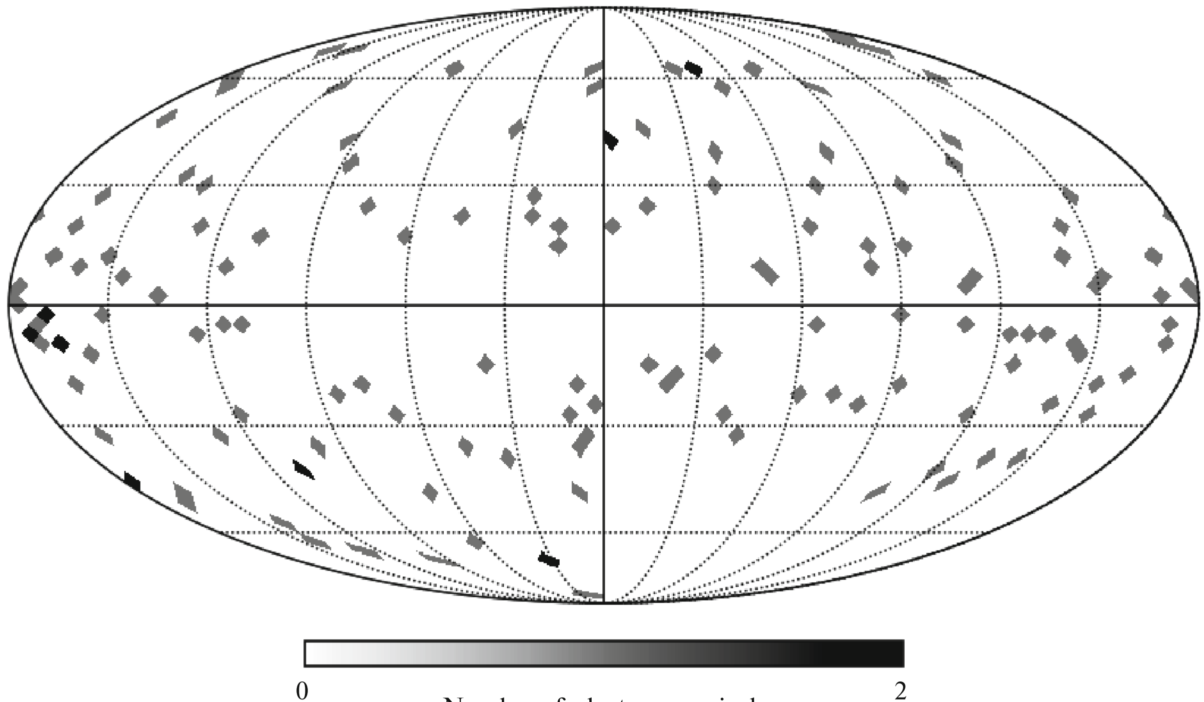

Fig. 2. Distribution of globular star clusters in the Galactocentric coordinate system.

spherical harmonics and of the Galactocentric radial velocities V_r/r in terms of scalar spherical harmonics for globular clusters are presented in the combined Table 2. They are quite unexpected. Whereas a rich and interesting result is observed when analyzing the proper motions of stars in the solar neighborhood, the kinematics of the globular star clusters as a whole system is rather poor. An analysis of Table 2 shows a significant random component in the proper motions, which is reflected in large rms errors and a low reliability of the coefficients. Obviously, this is not a consequence of the observational errors, because the random accuracies of both proper motions and radial velocities are very high, but reflects the stochastic nature of the motions of globular clusters. Formally, the table contains no coefficient greater than 3σ. The largest coefficient, t_1, exceeds the error approximately by a factor of 2.5. Comparing Tables 2 and 1, we understand that this coefficient is responsible for the rotation of the entire system of globular star clusters as a single whole around the Z axis. The angular velocity of this system will be

$$\omega_z = t_1/(2.89 \times 4.738) = -2.88 \pm 1.2 \text{ mas yr}^{-1}, \quad (6)$$

which leads to a rotation period of the entire system $\sim$450 $\pm$ 180 Myr. The direction of rotation coincides with the universally accepted direction of Galactic rotation. Other authors also arrived at such a result (Zinn 1985). However, this rotation can also be caused by the defects in constructing the Galactocentric coordinate system due to the errors in estimating the solar motion component V_Y in Eq. (4).

Lacking something better, we may consider the harmonics t_5, s_3, and s_8. However, they are already less than 2σ (but on the verge) and, therefore, their values are also unreliable. Only the harmonic s_3 responsible for the motion of the entire system along the X axis has a certain physical meaning. The remaining harmonics are unrelated to the linear kinematic model.

The picture for the radial velocities is similar. The rms. errors are very high and there are no significant coefficient even at the 2σ level.

Such low informativeness and physical considerations led us to the standard idea to separate the objects, in our case, by Galactocentric distance into "near" and "far" ones, despite their extremely small number. We divided the catalogue into two equal groups, each with 75 objects. The first group includes the clusters with Galactocentric distances less than 5.1 kpc (the accurate data from 480 to 5164 pc, the median of the distribution is 2603 pc, and the mean is 2800 pc) and the second group contains the clusters farther than 5.1 kpc (respectively, from 5164 to 144 770 pc, the median is 14 383 pc, and the mean is 21 703 pc). The distributions of both groups over the celestial sphere in the Galactocentric coordinate system are presented in Figs. 3 and 4.

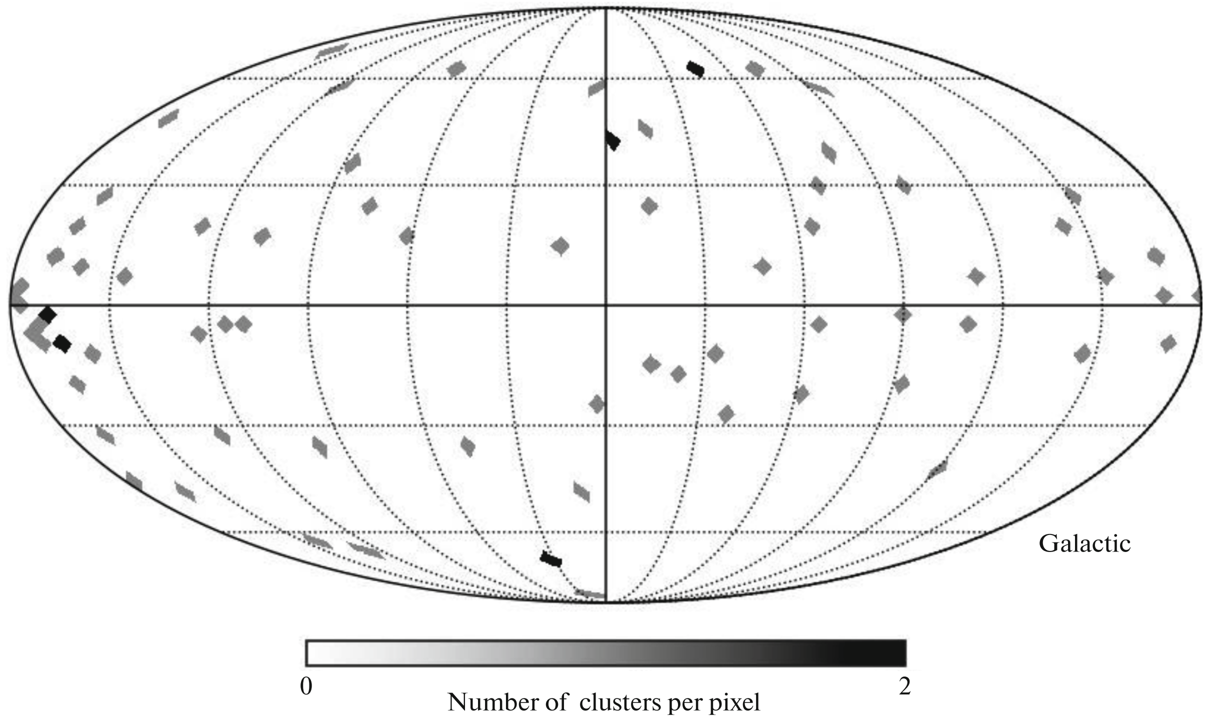

Fig. 3. Distribution of globular star clusters close to the Galactic center (0–5 kpc) on the celestial sphere in the Galactocentric coordinate system.

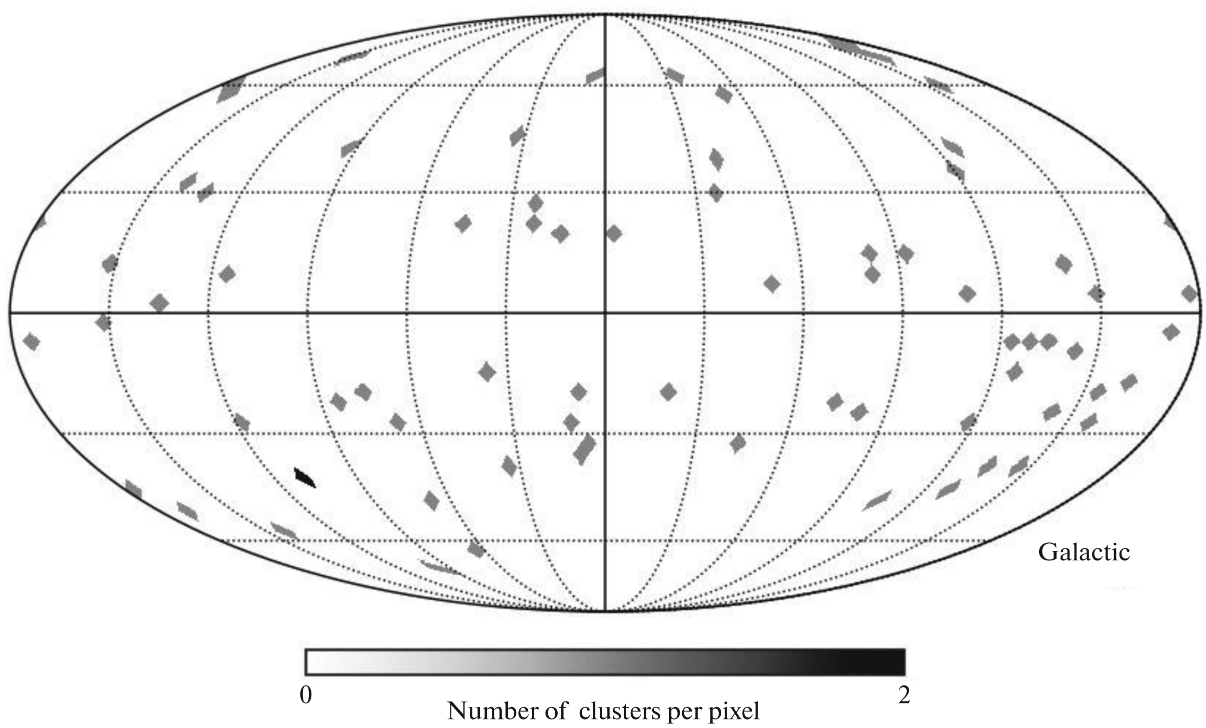

Fig. 4. Distribution of globular star clusters far from the Galactic center (5–124 kpc) on the celestial sphere in the Galactocentric coordinate system.

Table 3. Coefficients of the expansion of the Galactocentric proper motions $k\mu_l \cos b$ and $k\mu_b$ and radial velocities V_r/r for globular clusters in terms of vector and scalar spherical harmonics and their rms errors for objects close ($r < 5.1$ kpc) to the Galactic center (km s^{-1} kpc^{-1})

N	t_i	σ	s_i	σ	v_i	σ
0					15.0	34.9
1	−66.5	36.0	12.8	36.0	−49.5	35.0
2	7.2	36.2	34.8	36.2	59.5	36.1
3	20.7	37.3	24.3	37.3	−28.0	32.2
4	79.1	36.9	8.8	36.9	20.0	33.2
5	64.9	35.1	−22.3	35.1	12.4	38.4
6	60.2	36.2	21.1	36.2	32.5	29.9
7	−34.2	33.1	33.6	33.1	4.6	36.0
8	13.6	37.0	−51.3	37.0	8.1	28.7
9	−30.8	37.0	−25.0	37.0	15.5	31.4
10	−36.0	35.2	51.7	35.2	8.2	39.3
11	20.6	33.2	66.0	33.2	−42.9	31.0
12	−49.7	34.7	20.2	34.7	−24.2	38.8
13	−51.2	36.7	−10.1	36.7	9.7	29.3
14	−17.2	32.3	−20.4	32.3	−12.8	31.7
15	−31.9	34.2	−3.8	34.2	0.4	29.2

The results for the "near" group are presented in Table 3. We see that the clusters close to the Galactic center have significant stochastic motions; errors of the coefficients are very large (this is partly because of the smaller number of objects). Even the t_1 coefficient is less than 2σ.

A completely different picture is observed for the proper motions of far clusters (Table 4). The coefficient t_i stands out absolutely clearly already at a 3σ confidence level. The values of the remaining coefficients t_i and s_i point to the absence of other systematic motions. Note also the low errors of these coefficients, suggesting a decrease in the random components reflected in the proper motions. The rotation period for this group of clusters is 1170 ± 50 Myr.

Among the expansion coefficients of the radial velocities, the 2σ confidence level is reached by the coefficient v_7 responsible for the velocity field deformation in the XY plane.

CONCLUSIONS

To summarize, we can say that although the method of spherical harmonics did not find a large number of significant harmonics, as is usually the case when analyzing various groups of stars, nevertheless, it persistently showed the presence of rotation of the entire system of clusters coaxial with the Galactic one and the absence of rotations relative to the other axes. This is observed particularly clearly for far clusters. We can say that, to our surprise, virtually no other systematic motions were revealed, only the deformation in the XY plane remains in question. This may suggest a weak causal connectivity of the entire system of clusters. The rotation of the system as a whole can also be due to the Galactocentric coordinate system being noninertial as a result of the errors in taking into account the solar rotation around the Galactic center. This problem needs a further study.

Returning to the formulation of the problem, we want to emphasize that in this paper we described a

Table 4. Coefficients of the expansion of the Galactocentric proper motions $k\mu_l \cos b$ and $k\mu_b$ and radial velocities V_r/r for globular clusters in terms of vector and scalar spherical harmonics and their rms errors for objects far ($r > 5.1$ kpc) from the Galactic center (km s^{-1} kpc^{-1})

N	t_i	σ	s_i	σ	v_i	σ
0					1.8	5.8
1	−15.2	4.8	6.3	4.8	−6.3	5.9
2	−3.3	4.9	4.2	4.9	−0.2	5.7
3	0.4	5.4	3.7	5.4	8.2	4.8
4	−0.2	4.8	1.3	4.8	2.3	6.5
5	−1.4	5.1	−0.4	5.1	10.9	5.7
6	−1.2	5.4	−3.4	5.4	0.4	5.0
7	−5.0	4.8	−3.2	4.8	−11.1	5.1
8	4.3	4.7	1.6	4.7	3.3	5.5
9	2.4	5.3	1.1	5.3	5.5	6.1
10	3.4	5.2	−3.3	5.2	−5.2	6.2
11	−5.6	5.1	0.6	5.1	7.7	5.4
12	6.5	5.0	0.7	5.0	7.7	4.9
13	−4.3	4.9	−7.1	4.9	−3.6	5.3
14	−7.8	4.6	−1.3	4.6	−5.8	5.3
15	3.1	4.7	5.7	4.7	7.6	5.0

technique that would be used to analyze big data. The next paper will be devoted to this subject.

ACKNOWLEDGMENTS

We are grateful to the anonymous referees for their very useful critical remarks that allowed us to rewrite the original version of the paper.

We used the data from the Gaia mission of the European Space Agency (ESA) (Gaia Collaboration et al. 2016) processed by the Gaia Data Processing and Analysis Consortium (DPAC, Gaia Collaboration et al. 2018). The funding for the DPAC was provided by national institutions, in particular, the institutions participating in the Gaia Multilateral Agreement.

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Translated by N. Samus'