Opik's method, Eddington's luminosity and Hubble's constant

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Abstract: Opik derived in 1918 the distance to the Andromeda nebula using its rotation velocity, apparent luminosity and angular size, together with a M/L ratio. We describe briefly this historic method, write down the relevant formulae in an instructive way, and use it as a starting point for a study of the cosmic distance scale via AGNs for which the mass has been measured. Considering the relation between central mass and quasar luminosity, $M_{BH} \sim L^{\text{б}}\ FHWM^2$, we point out that as it is calibrated at low z using distance independent reverberation mapping to get the BLR size, the derived M_{BH} does not depend on H_0, while L/L_{Edd} is sensitive to H_0. This means that high-z objects shining near the Eddington limit could be used to study the Hubble constant in a new way, bypassing the local distance ladder. The method could be practical if the factors (e.g. FHWM-to-velocity factor f, bolometric correction k) needed for deriving M_{BH} and L_{bol} were well known and objects with $L \approx L_{Edd}$ could be identified independently. To illustrate, we take a sample of tranquil luminous quasars at redshifts 0.5–1.6. Normalized to the usual values of k and f, h_{100} becomes $0.6\ e^{-1/2}\ (k / 9.5)^{1/2} / (f / 1)$, where $e = L_{bol} / L_{Edd}$. Formally, $h_{100} \approx 0.6 \pm 0.1$. Especially, the case $e \leq 1$ gives a tentative lower limit to $H_0 \approx 50\ kms^{-1}/Mpc$.

1. Introduction: Opik's method and relative and absolute distance indicators

In 1922 (and reported already in 1918 at a meeting in Moscow, that is 90 years ago) Ernst Opik in his paper *An estimate of the distance of the Andromeda nebula* [1] made a remarkable determination of the distance to the Andromeda nebula. He used the rotation velocity of the nebula to derive the mass within the radius where the rotation was measured. From the data available, Цpik first calculated the distance of the Andromeda nebula to be about 770 kpc, which value he dropped to 450 kpc in the 1922 ApJ paper. It was the era of the Great Debate, and he stated that "the coincidence of results obtained by several independent methods increases the probability that this nebula is a stellar universe, comparable with our Galaxy".

It is interesting and useful to write out in a transparent manner the principle of Цpik's dynamical method which uses the mass-to-luminosity ratio. Assuming that the object's mass is distributed in a spherically symmetric manner up to the point where we take the rotation velocity (the angle θ) we write for the mass, using the rotation of the Earth around the Sun as a meter stick (d is the unknown distance):

$$M/M_{sun} = (d/1AU)\ \theta\ (V_{rot} / 30\ kms^{-1})^2 . \qquad (1)$$

Similarly, in terms of the luminosity of the Sun:

$$L/L_{sun} = (d/1AU)^2\ (f / f_{sun}). \qquad (2)$$

Here f refers to the flux over all wavelengths. Denoting the mass-to-luminosity ratio of the considered object by γ (in the solar units), we can now write for the unknown distance d:

$$d = (1/\gamma)\ \theta\ (V_{rot} / 30\ km\ s^{-1})^2\ (f_{sun} / f)\ \ AU \qquad (3)$$

or in terms of more convenient units, and written with M31 in mind:

$$d_{M31} = 24.0\ (1/\gamma_{M31})\ (\theta_{deg}/2.5°)\ (V_{rot} / 225\ km\ s^{-1})^2\ (f_{sun} / f_{M31})\ 10^{-12}\ \ Mpc . \qquad (4)$$

It is instructive to put here first the mass-to-luminosity ratio equal to unity, corresponding to a system made wholly of Sun-like stars. A modern value for the flux ratio f_{sun} / f_{M31} would be about $10^{11.73}$ (from the galactic and internal extinction corrected V magnitude difference 2.53 – (–26.8) = 29.33) and the rotation velocity at the horizontal part of the rotation curve is about 225 km/s [2], starting at θ ≈ 2.5 deg. These values would give the distance $d_{M31} \approx 13$ Mpc for $\gamma_{M31} = 1$, $d_{M31} \approx 4.3$ Mpc for $\gamma_{M31} = 3$, while the "wanted" distance of 0.77 Mpc would require $\gamma_{M31} \approx 17$ within the radius considered. We see here directly the need for a lot of dark matter, as the needed mass-to-luminosity ratio is inversely proportional to the desired distance (as derived from other, accurate methods which currently give 770 kpc as a widely accepted value). Alternatively, for those of you who experiment with the modified Newtonian gravity and little dark matter, the derived too large distance would reflect the inadequate Newtonian expression (1).

Why then could Opik obtain such a good value for the distance? Partly it must have been due to good luck, but it should also be noted that his not-so-good data referred to the innermost parts of the nebula (within 2.5 arcminutes and not 2.5 degrees from the centre!). There the dark matter is not important.

This Цpik's method, if applied to nearby extragalactic objects, would bypass some lower rungs of the extragalactic distance ladder. However, as we saw it requires the knowledge of the M/L–ratio, for which an estimate must be obtained in some way. Thus though it is based on Newtonian mechanics, it is not purely "physical" or "absolute" [3,4], where one can get the distance in terms of physical units.

Note that in the usual "relative" methods, the distance is obtained in units of the calibrator distance. When Knut Lundmark in 1919 and Edwin Hubble in 1923 also derived large distances to M31, their "standard candles" (novae and Cepheids, respectively) were calibrated in the Milky Way. Similarly in a modern variant of Цpik's method, the Tully-Fisher relation for rotating galaxies, one bypasses the need for an explicit mass-to-luminosity ratio using calibrator galaxies. Opik's idea as a whole illustrates a definition once given: a distance indicator is a method where a galaxy is placed in 3D space so that its observed properties agree with what we know about galaxies, their constituents and the propagation of light [5].

If we can find objects radiating at a known mass-to-luminosity ratio and have a way to measure their mass, then we can use an analogous method to derive distances and study the distance scale. The Eddington luminosity could be such a quantity as it depends only on physical constants and on the mass of the radiating objects: L_{Edd} = 1.26 Ч 10^{38} (M_{BH}/M_{sun}) $ergs^{-1}$. A problem is how to find objects at the Eddington limit, i.e. having the Eddington ratio L/L_{Edd} equal to one. A good thing with such a limit is that one is concerned with the most luminous (isotropic) sources within a class of a fixed mass, hence visible from faw away.

2. Masses of compact AGN nuclei and the Eddington luminosity

Recent years have made it possible to infer masses of the compact nuclei in galaxies and quasars. The mass M_{BH} has been determined by primary methods for nearby objects (such as reverberation mapping giving a size of the broad line region R_{BLR}) and by secondary methods for more distant AGNs (such as the relation between R_{BLR} and optical luminosity; see [6]). The masses for quasars come mostly from the $L_{opt} - R_{BLR}$ relation, with the needed velocity parameter given by an emission line width FHWM:

$$M_{BH} = a\ [\text{л}\ L_{\text{л}}(5100E)/10^{44}\ \text{erg s}^{-1}]^{\text{б}}\ \text{FHWM}^2\ . \qquad (5)$$

Exponent б has got values from 0.5 to 0.7 [7]. Here we point out that the way the BLR size vs. luminosity relation is calibrated, has an interesting implication when deriving BH masses and Eddington ratios within a Friedmann model, and radiators near the Eddington limit might give information on the global distance scale.

The relation between the size R_{BLR} and luminosity L is calibrated at low redshifts (< 0.2) using an assumed value of H_0, [8]. *It is important to note that as the size is obtained from a light travel time argument ("reverberation mapping"), it does not depend on the distance scale*, and the R_{BLR} vs. L relation from the calibrator sample just shifts along the luminosity axis by a factor of $h^{-2} = (H/H_0)^{-2}$. As the inferred luminosity of a higher-z sample quasar is also changed by this same factor, a change of H_0 does not change BH masses at all (nor the constant L_{Edd}), but it does alter Eddington ratios L/L_{Edd} by a factor of h^{-2}:

$$M_{BH} \sim h^0, \quad L/L_{Edd} = 4p\ r_L^2\ f_{bol} / bM_{BH} \sim h^{-2}. \qquad (6)$$

It is this sensitivity to H_0 which makes Eddington ratios interesting for the distance scale. If the size in the calibration were calculated, as usually, from an angle, then L/L_{Edd} would change slower, as h^{-1}.

Another notable point is that changing the other cosmological parameters (Щ_m, $\text{Щ}_{\text{Л}}$) has a smaller influence on L/L_{Edd}. Then comoving distances change, depending on z, and one must correct L individually ($L_c = (r_2/r_1)^2$ L), which affects the Eddington ratio only as $(r_2/r_1)^{2(1-\text{б})}$ (for б = 0.6 this is $(r_2/r_1)^{0.8}$). A similar tiny effect on the calibration at low redshifts may be generally ignored.

3. The Eddington luminosity and the distance scale

If one has reasons to think that some quasars radiate at L_{Edd}, one may infer which luminosity distance and, hence, which value of H_0 leads to this efficiency (for adopted Щ_m, $\text{Щ}_{\text{Л}}$). If the objects actually radiate below L_{Edd}, then the inferred H_0 is a lower limit, and even this usual case has bearing on the distance scale.

The method would have some positive sides: (1) independence of the local distance ladder, (2) probes the global distance scale, (3) not influenced by the usual Malmquist bias, (4) sensitive indicator of H_0, and

(5) rather robust to changes in $\text{Щ}_{\text{Л}}$ in the Л-dominated flat model. The last two items relate, respectively, to the low-z calibration of the BLR size vs. luminosity relation using the distance-scale independent reverberation mapping and to the good exponent $0.5 \leq \text{б} < 1$ in the relation.

Here the usual Malmquist bias (e.g. [5]) is absent: the calibration and derivation of the BLR size is made against luminosity, so a given log L predicts an unbiased size, hence unbiased M_{BH} and L_{Edd}.

At medium redshifts some quasars may radiate around L_{Edd}, while at $z < 0.5$ quasars work at lower accretion rate ([9,10]). However, there is now no sure way to tell, say from the spectrum, without a measured mass, if a quasar really shines near L_{Edd}. This forms an obstacle for practical use of the method, in addition to uncertainties in numerical factors.

Errors in the derived L/L_{Edd} may be due to the mass estimate M_{BH}, involving the exponent б, the factor f in f Ч FWHM, and the estimate of L_{bol}.

The factor f that transforms the Doppler-broadened line width FWHM into an orbital velocity affects the derived M_{BH} as f^2 ([7, 11,12]). For an isotropic distribution $f = \sqrt{3}/2 \approx 0.9$ ([13]). McLure & Dunlop [14] suggest a larger factor f =3/2, combining isotropic and disk components, and they also use f = 1 [10]. They [15] showed modelling the FWHM distribution for an AGN sample that f depends on the Hв line width (inclination effect). For widths over 2800 km/s (83% of the sample) f was rather close to the isotropic value $\sqrt{3}/2$. The model got some support from 10 Seyfert galaxies for which stellar velocity dispersion data exist.

If the mean value of f were much in error, say by a factor of 2, one would expect a shift in the log M_{BH} vs. M_R(bulge) relations for a sample with masses from Hв line widths and one with actual dynamical estimates. However, these agree in [15].

Going from M_{BH} to $\log L_{bol} / L_{Edd}$, one needs the luminosity L_{bol}, usually calculated from a constant correction log k to log л $L_{л}(5100)$, about log 9.5. There are large variations from-quasar-to-quasar in this bolometric correction ([16]). For a uniform AGN class the variations may be lower, and in any case, it is the average value of the correction k and its error that matter here. Judging from recent studies a systematic error, due to an incorrect average continuum, is narrowing to within ±0.2 ([8, 6, 17]).

Even if systematic errors were in control, a large enough sample of Eddington radiators is needed to give useful limits to H_0. If for a single object one may optimistically expect a future 1y accuracy of 0.3 in log L/L_{Edd}, this transforms into 0.15 in log H_0. E.g., 25 Eddington radiators would thus fix log H_0 within ±0.03 (1y), forgetting the small uncertainty due to the Щ parameters of the Friedmann model. With $h_{100} \approx 0.6$, this would imply 1y error bars of about ± 5 kms^{-1}/Mpc.

4. A simple heuristic illustration

In a sample of the first data release SDSS quasars in the z interval 0.5 – 2 “the Eddington luminosity is still a relevant physical limit to the accretion rate of luminous quasars” [10]. If so and in order to illustrate we consider a class “AI” of luminous radio quasars, initially proposed by us to exist without any consideration of L_{Edd}, and radiating around $M_{min} \approx -26.0 + 5 \log h_{100}$, a minimum brightness V magnitude. About 30 potential AI objects in the z range 0.5 – 1.7 are found in our list of radio quasars with UBV photometry [18], when limited to $M_{min} < -25.6$ (fainter quasars are more violent optically as measured with variability and polarization). These quasars are optically very luminous and at the same time rather inactive. It is likely that they do not contain a strong beamed optical component and thus may be suitable for the present experiment. Some of their properties are discussed in [18,19].

Of these objects 11 have an entry in the compilations of M_{BH} ([9, 20]) where the exponents 0.7 and 0.5, respectively, were used in Eq.(5). Here we have only made the adjustment to the adopted cosmology. Two quasars in Table 1 would lie well below the others in Fig.1: 0414-0601 and 1954-3853. The latter one has optical polarization of 11% and variability ≥ 0.8 mag (Table 1 in [18]), hence it is optically active. For these objects the Eddington ratio is much less than for the others. In the L_{bol} vs. M_{BH} diagram (Fig.1) for the remaining 9 objects we show the effect of H_0, for the standard model (Щ_m, $\text{Щ}_{\text{Л}}$) = (0.3,0.7). With $h_{100} = 0.45$ and 0.80, these AIs radiate above and below the Eddington value, respectively. There is just a vertical shift by the factor 2 log 80/45 and the masses M_{BH} remain the same (sect.2).

We note that such limits on H_0 would be excluded if the AIs were *known* to radiate at L_{Edd} and if there were no systematic errors (sect.3). The plausible assumption $L_{bol} \leq L_{Edd}$ makes $h_{100} = 0.45$ a strict lower limit. Normalized to the numerical factors used for calculating M_{BH} and L_{bol} (FHWM-to-velocity factor f, bolometric correction k) the Hubble constant, as tied to this small sample of quasars, may be written as

$$h_{100} = 0.6\, e^{-1/2}\, (k/9.5)^{1/2} / (f/1). \qquad (7)$$

Table 1. Data ($M_{min} < −25.5$ mag, $0.5 < z < 1.6$)

RA	д	z	M_{min}	log L_{bol}	log M_{BH}	ref.
0024	+2225	1.118	−26.0	47.40	9.45	2
0405	−1219	0.574	−26.0	47.69	9.58	1
0414	−0601	0.781	−25.7	46.97	9.52	2
0454	+0356	1.345	−26.7	47.60	9.39	2
0637	−7513	0.651	−26.3	47.46	9.54	1
0957	+1757	1.472	−25.9	47.41	9.23	2
1458	+7152	0.905	−25.7	47.28	9.14	1
1954	−38.53	0.626	−25.8	46.61	8.75	1
2216	−0350	0.901	−26.0	47.52	9.40	1
2255	−2814	0.926	−26.1	47.32	9.32	1
2344	+0914	0.677	−25.8	47.38	9.44	1

References: 1. Woo & Urry [9] 2. Shields et al. [20]

The value $h_{100} = 0.6 \pm 0.1$ minimizes the dispersion around the Eddington limit line in Fig.1 (in fact, $h_{100} = 0.62$, but there is no need for such accuracy). The ~ 2y error bars ± 0.1 take into account the scatter around the L_{Edd} line. For example, if the Eddington ratio e ≤ 1, then $h_{100} \geq 0.5$ at about 2y confidence level, for the used values of k and f.

If we change $Щ_{Л}$ by ±0.15 (keeping $Щ_{tot}$ =1), taking the ~ 2y error bars from SNIa's ([21, 22], one sees just a small dependence of the Eddington ratio on Д$Щ_{Л}$ in Fig.1. For two quasars we show the position shifts for Д$Щ_{Л}$ = ± 0.15. The steeper slope corresponds to б = 0.5, the shallower one to б = 0.7.

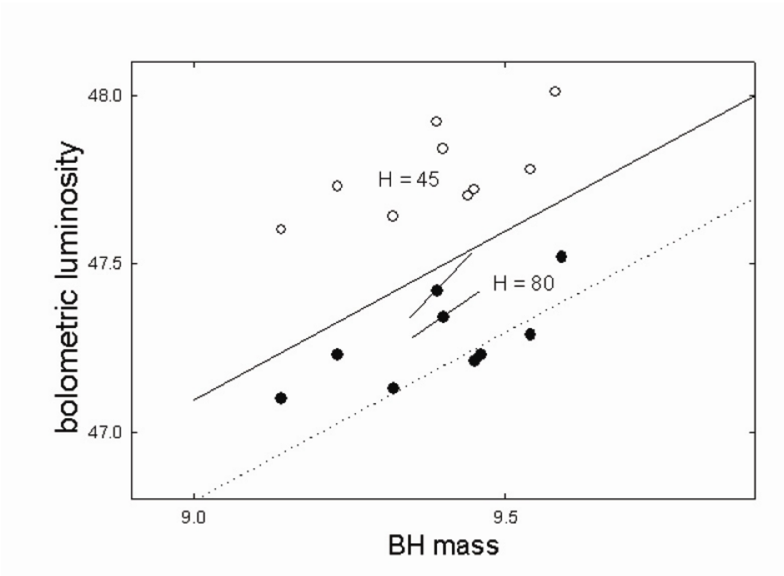

Fig. 1 Luminous AI quasars in the z interval 0.6–1.5 in the log L_{bol} vs. log M_{BH} diagram for two values of H_0 in the world model ($Щ_m$, $Щ_Л$) = (0.3,0.7). The upper line indicates the Eddington luminosity, the lower line is $0.5L_{Edd}$. A change of H_0 causes a vertical shift. For two quasars we show how a shift Д$Щ_{Л}$ = ± 0.15 in the flat model affects their positions. If these quasars are radiating at L_{Edd} or less, then for the numerical factors used for calculating L_{bol} and M_{BH}, 45 kms^{-1}/Mpc is a strict lower limit for the Hubble constant.

5. Concluding remarks

As we all know the Hubble constant has had a colourful history when its measured value has dropped tenfold from 625 kms^{-1}/Mpc, as first estimated by Georges Lemaotre in 1927 (before the Hubble law was discovered!), down to about 62.5, as derived by Sandage and his team in 2006 [3], based on the calibration of the Ia supernovae from Cepheid HST distances to the local host galaxies. Our work together with Georges Paturel (see his contribution in this conference and [23]) on the bias problems in the local Cepheid-based distance scale points at about such a value, too. In fact, the local distance scale has in recent years also become important in another way: the closeby Hubble flow carries valuable information on the enigmatic dark energy, making even the local Hubble parameter cosmologically relevant (e.g. [24]).

Although the “50 versus 100 duel” of the 1970s – 80s is over, there is still some uncertainty about the correct value of the global Hubble constant, as the HST Key Project [25] obtained in 2001 the value 72 ± 8. This agrees with a similar value extracted from the fluctuations of the cosmic background radiation [26], though it should be noted that the latter one still depends on assumed cosmological physics and dark-substance components, and it would be very important to have independent more direct measurements on very large scales. “Large scales” means that usual relative distance indicators based on standard candles are easily affected by Malmquist-like biases (for example, at large distances the samples contain over-luminous objects). Therefore there is still ample room for various physical high-z methods to study H_0, which are less sensitive to such selection biases (and which bypass the local distance ladder, another source of possible systematic errors). The method based on the Sunyaev-Zeldovich effect [27] and Refsdal’s method of the time delay in gravitational lens images [28] do have their own problems, causing rather wide error margins.

Although the approach via the Eddington luminosity sketched here is still rudimentary and does not yet add much truly independent information about the distance scale, it is noteworthy that the assumption $L = L_{Edd}$ for the considered powerful quasars leads to $h_{100} \approx 0.6$ when the usual values of the parameters are used in the calculation of the BH mass and bolometric luminosity. We hope that this instructive example has shown that with a large sample of well-studied quasars and more accurate numerical factors in the calculation of L_{bol}/L_{Edd}, the study of the Eddington efficiency is interestingly connected with the problem of the global distance scale.

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