Universe driven by the vacuum of scalar field: VFD model

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Abstract: It is shown that in the Vacuum Fluctuations Dominated model (VFD), where the vacuum fluctuations of scalar fields dominate over matter and radiation during the all history of the Universe expansion (gr-qc/0604020, gr-qc/0610148), the acceleration parameter evolves monotonically from zero to the present day negative value. That is according to this model, the Universe has no decelerating past, and the conventional radiation domination and matter domination epochs are absent. Theoretical predictions for $z \sim 0-7$ are compared with those following from the SN type Ia and gamma ray bursts data.

Fast progress in accumulating and handling of the astrophysical data about the Universe expansion [1,2,3,4,5,6] clears the way to testing of different models of the Universe evolution. Although, the ΛCMD model is able to explain the observational data [7], it is necessary to provide a deeper insight into the cosmological constant problem [8,9,10,11,12,13,14,15]. Among numerous approaches to the cosmological constant problem, the quantum field theory (QFT) approach may suggest some solutions.
It is well known that the covariant removing of all divergent terms from the energy-momentum tensor by some regularization procedure leads to the vacuum energy density $\rho_{vac} \sim 1/L^4$, where L is the radius of the Universe curvature [16]. This quantity is too small[1] to explain the observed Universe acceleration if one may identify L with the size of a present day Universe.
On the other hand, the direct ultraviolet (UV) momentum cut-off for evaluation of the vacuum energy provides the enormous quantity $\rho_{vac} \sim M_p^4$, where M_p is the Planck mass.
In our previous works [18,19], the accelerated expansion of Universe was attributed to the back-reaction of the vacuum fluctuations of massless scalar fields. It was found, that the use of UV cut-off at the Planck level in the equation of motion for the Universe scale factor instead of that in the Friedman equation allows explaining the observable value of Universe acceleration. In our approach, the effective density of dark energy is proportional to the Hubble constant squared $\rho_{vac} \sim H^2\kappa_{max}^2 \sim H^2 M_p^2$, as it occurs in the holographic dark energy models [20,21,22,23,24,25] (here κ_{max} denotes the UV cut-off of the present day physical momentums).
Below our previous model is summarized and compared with the SN type Ia and gamma ray bursts data.

Let us write down the system of Friedman– Lemaotre equations for the Universe scale factor a, the density of matter ρ and the pressure p:

$$-\frac{1}{2}M_p^2\left(a'^2+Ka^2\right)+\rho a^4+\frac{1}{6}M_p^2\Lambda a^4=0,$$
$$M_p^2 a''=-M_p^2 Ka+(\rho-3p)a^3+\frac{2}{3}M_p^2 a^3\Lambda,$$
$$\rho'+3\frac{a'}{a}(\rho+p)=0, \tag{1}$$

where the conformal time η implying the metric $ds^2=a^2(\eta)(d\eta^2+d\sigma^2)$ is used (the reason will be explained below), Λ is the cosmological constant, K is the signature of space-time, and the Planck mass M_p should be read as $M_p=\sqrt{\frac{3}{4\pi G}}$.
The ΛCDM model can be obtained by setting $p=0$, $K=0$ and finally is reduced to the single equation

[1] For the flat expanding Universe and the self-interacting scalar field $V(\phi) \sim \lambda\phi^4$, it is $\rho_{vac} \sim \lambda H^4$ [17], where H is the Hubble constant. This quantity is too tiny even for $\lambda \sim 1$..

$$a'' = 2\frac{a'^2}{a} - \frac{3}{2}a_0 H^2 \Omega_m, \qquad (2)$$

where $a_0 = a(0)$ is the present day scale factor (this moment corresponds to $\eta = 0$ hereafter), $H = \frac{a'}{a}|_{\eta=0}$ is the conformal Hubble constant[2] and the constant Ω_m is connected with the matter density $\frac{1}{2}\Omega_m M_p^2 H^2 a_0 = \rho a^3 = \rho_0 a_0^3$.

Coming to the VFD model [18, 19] we set $\Lambda = 0$, $p = 0$, $K = 0$ and add a massless scalar field, which is characterized by the averaged pressure and the density:

$$\rho_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} + \frac{(\nabla\phi)^2}{2a^2} \right) d^3\mathbf{r},$$

$$p_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} - \frac{(\nabla\phi)^2}{6a^2} \right) d^3\mathbf{r}, \qquad (3)$$

where V is some volume, which will be set to unity hereafter. The second step is to turn to the quasiclassical picture, where the scalar field $\hat{\phi}(\eta,\mathbf{r})$ is quantum. The resulting master equations for the VFD model are

$$-M_p^2 \frac{a'^2}{2} + \rho a^4 + \int \left(\frac{a^2 <0|\hat{\phi}'^2|0>}{2} + \frac{a^2 <0|(\nabla\hat{\phi})^2|0>}{2} \right) d^3\mathbf{r} = const,$$

$$M_p^2 a'' = \rho a^3 - \int \left(a <0|\hat{\phi}'^2|0> - a <0|(\nabla\hat{\phi})^2|0> \right) d^3\mathbf{r}, \qquad (4)$$

$$\hat{\phi}'' + 2\frac{a'}{a}\hat{\phi}' - \Delta\hat{\phi} = 0,$$

where $<0|...|0>$ denotes a mean value over the vacuum state of scalar field. The first equation is the integral of motion for two last equations. However, it should be noted that it is not the Friedman equation because the constant on the right hand side is not zero. The point is that some renormalization is needed to avoid the cosmological constant problem, i.e. the huge QFT vacuum energy in the Friedman equation. Instead of determining the renormalization constant, one can consider two last equation and fix the constant assigning the initial condition for the equations. It is very important, that in conformal time a renormalization is not required for the second equation. The reason is that the equation contains exact difference of the kinetic and potential energies of the field oscillations. In the Minkowski space-time this difference is exactly zero by virtue of the virial theorem for an oscillator, which states that the kinetic energy is equal to the potential one in the virial equilibrium. In the expanding Universe this difference is proportional to the Hubble constant squared.

Scalar field can be decomposed in a complete set of the modes $\phi(\mathbf{r}) = \sum_{\mathbf{k}} \phi_{\mathbf{k}} e^{i\mathbf{kr}}$ and quantization of the modes consists in postulating [16]

$$\hat{\phi}_{\mathbf{k}} = \hat{a}^+_{-\mathbf{k}}\chi_k^*(\eta) + \hat{a}_{\mathbf{k}}\chi_k(\eta), \qquad (5)$$

where complex functions $\chi_k(\eta)$ satisfy the relations:

$$\chi_k'' + k^2\chi_k + 2\frac{a'}{a}\chi'_k = 0,$$

$$a^2(\eta)(\chi_k \chi_k'^* - \chi_k^* \chi_k') = i. \qquad (6)$$

The adiabatic approximation

$$\chi_k(\eta) = \frac{\exp\left(-i\int_0^\eta \sqrt{k^2 - \frac{a''(\tau)}{a(\tau)}}\, d\tau\right)}{\sqrt{2}a(\eta)\sqrt[4]{k^2 - \frac{a''(\eta)}{a(\eta)}}} \qquad (7)$$

[2] $H = H_0 a_0$, where H_0 is the present day Hubble constant.

allows calculating the difference of the kinetic and potential energies of field oscillators up to second-order terms:

$$
\begin{aligned}
&\int \left(a < 0 | \hat{\phi}'^2 | 0 > - a < 0 | (\nabla \hat{\phi})^2 | 0 > \right) d^3\mathbf{r} = \\
&\sum_{\mathbf{k}} a < 0 | \hat{\phi}_k' \hat{\phi}_{-k}' | 0 > - k^2 a < 0 | \hat{\phi}_{\mathbf{k}} \hat{\phi}_{-\mathbf{k}} | 0 > = \sum_{\mathbf{k}} a(\chi_k'^* \chi_k' - k^2 \chi_k^* \chi_k) \approx \\
&\frac{1}{2}\left(-\frac{a''}{a^2} + \frac{a'^2}{a^3} \right) \sum_{\mathbf{k}} \frac{1}{k} + O(a'^3) + O(a'a'') + O(a'''),
\end{aligned}
\tag{8}
$$

where it was implied that a' is the first-order quantity, a'' is the second-order one, a''' is the third-order one and so on.
Using (8) in (4) leads to the master equation of VFD model in the form:

$$
M_p^2 a'' = \frac{1}{2} \Omega_m M_p^2 H^2 a_0 + \frac{1}{2}\left(\frac{a''}{a^2} - \frac{a'^2}{a^3} \right) \sum_{\mathbf{k}} \frac{1}{k}. \tag{9}
$$

Eq. (9) can be integrated up to the equation[3]

$$
a'^2 = a_0^2 H^2 \frac{S_0 - 1 - \Omega_m \left(a/a_0 - 1 \right)}{S_0 a_0^2/a^2 - 1}, \tag{10}
$$

where the parameter S_0, from the one hand, is determined by the UV cut-off κ_{max} of the physical momentums $\kappa = k/a_0$

$$
S_0 = \frac{1}{2M_p^2 a_0^2} \sum_k \frac{1}{k} = \frac{1}{M_p^2 a_0^2 (2\pi)^3} \int \frac{d^3\mathbf{k}}{2k} = \frac{\kappa_{max}^2}{8\pi^2 M_p^2}
$$

and, from the other hand, is connected with the present day deceleration parameter q_0 as $S_0 = \frac{2q_0 - 2 + \Omega_m}{2q_0}$. It was shown [18,19] that the UV cut-off of the present day physical momentums k/a_0 in the sum $\sum_k \frac{1}{k}$ at the Planck level $\kappa_{max} = k_{max}/a_0 \sim M_p$ can explain the observed value of Universe acceleration. In principle, the exact value of the UV cut-off has to result from the Planck scale physics.
Validity range of Eqs. (9), (10) is defined by the next terms in the expansion (8). According to Refs. [18,19], the next terms contain additional multiplier $a'/(ak_{max})$ as compared with the main term, where k_{max} is the UV cut-off $k_{max}/a_0 \sim M_p$ [18,19]. Thus Eqs. (9), (10) are valid if $\frac{a'}{a} \ll M_p a_0$, or $\dot{a} \ll M_p a_0$.
Eq. (10) can also be rewritten in the cosmic time $dt = a\, d\eta$

$$
H^2 = \left(\frac{\dot{a}}{a} \right)^2 = H_0^2 \frac{(S_0 + \Omega_m - 1) a_0^4 a^{-4} - \Omega_m a_0^3 a^{-3}}{S_0 a_0^2 a^{-2} - 1}, \tag{11}
$$

which gives $a(t) \approx a_0 H_0 \sqrt{\frac{S_0 + \Omega_m - 1}{S_0}}\, t$ in the vicinity of $t = 0$ (i.e. in the conformal time $a(\eta) = H \sqrt{\frac{S_0 + \Omega_m - 1}{S_0}} \exp\left(H \sqrt{\frac{S_0 + \Omega_m - 1}{S_0}}\, \eta \right)$). One can compare the theoretical dependence (11) with the observations of the apparent magnitude of the SN type Ia. If Universe is flat, the luminosity distance is given by $d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}$, where $z = a_0/a - 1$ is the red shift.. Measured value is the difference of the apparent and absolute magnitudes $\mu(z) = M - m = 5 \log \left(\frac{d_L(z)}{10\ \mathrm{pc}} \right)$. However, it is more convenient to con-

[3]This equation can be also deduced from the first of Eq. (4), when the corresponding normalization constant is chosen.

sider the quantity $\Delta\mu(z) = \mu(z) - \mu_{empty}(z)$, where $\mu_{empty}(z)$ corresponds to the empty closed Universe for which $d_L(z) = \frac{1}{H_0}(1+z)\sinh\left(\int_0^z \frac{dz'}{1+z'}\right) = \frac{z(1+z/2)}{H_0}$.

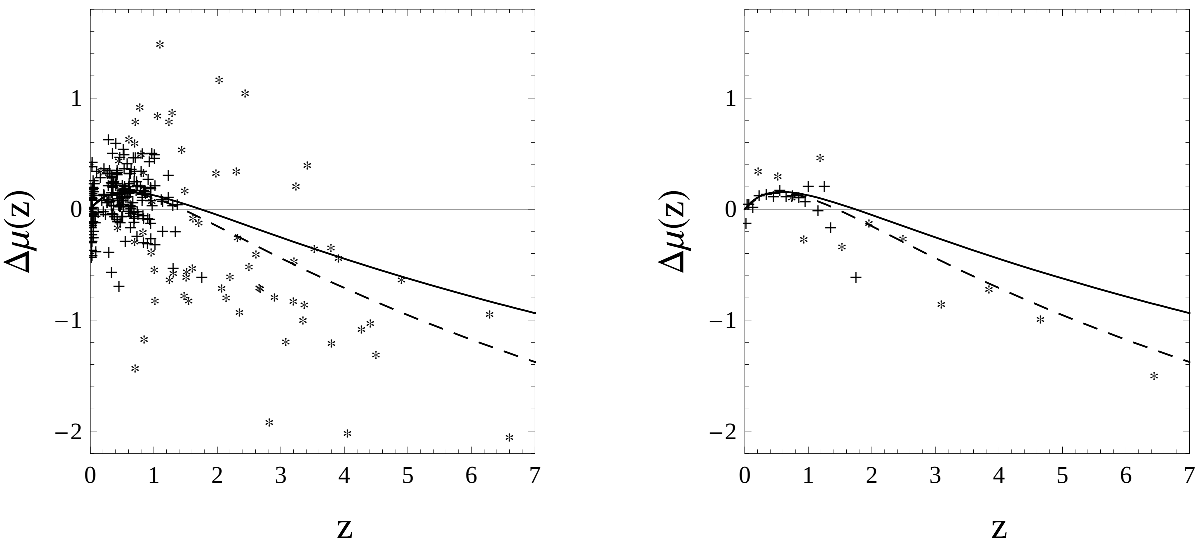

Fig. 1 The VFD curves (bold, $\Omega_m = 0.27$) of the $\Delta\mu(z)$ evolution and those of the ΛCDM (dashed, $\Omega_m = 0.27$). Asterisks and pluses denote the GRB and 115 SN Ia experimental data, respectively.. Original data and binned data correspond to the left panel and right panels, respectively.

Experimental data are given in Refs [26-28]. Figure 1 shows the original data (one can add some constant quantity to $\Delta\mu(z)$ because the value of the absolute magnitude M is not known exactly). The left panel shows the original data whereas the right panel shows the "binned" data, where averaging of $\Delta\mu$ over some interval of *z* is made and is attributed to the average *z* for this bin. To extend the range of *z*, one can try using the gamma ray bursts (GRB) as the standard candle. We combine the GRB data [29] with the SNIa ones. One can see that without using some statistical methods, it is difficult to prefer the standard Λ CDM model to the VFD model.
It should be noted that the deceleration parameter

$$q(z) = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1+z}{H}\frac{dH(z)}{dz} - 1 =$$

$$\frac{q_0\left(\Omega_m(2q_0+\Omega_m-2)z^2 + 2\left(\Omega_m^2 - 3\Omega_m + 2\right)z + (\Omega_m-2)^2\right)}{(\Omega_m + z(z+2)(2q_0+\Omega_m-2)-2)(\Omega_m + z(2q_0\Omega_m+\Omega_m-2)-2)}, \qquad (12)$$

comes from zero to the present day negative value as it is shown in Fig. 2. The parameter Ω_m amounts 0.27 for both models and $q_0 = -0.8$ for the VFD model. These values are chosen to fit the curves within a thin waist of the experimental data channel near z=0.2.
It is interesting that the VFD model is highly insensitive to the dark matter content. We see that two curves in Fig.2 corresponding to $\Omega_m = 0.27$ and $\Omega_m = 0.04$ (pure baryonic matter) almost coincide.

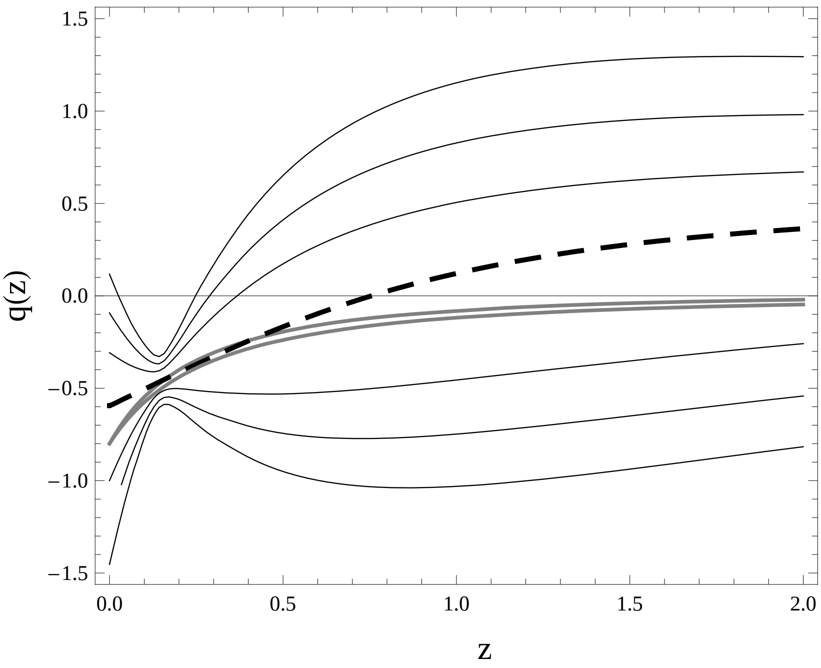

Fig. 2. The VFD curves (bold grey, $\Omega_m = 0.27$ and Ω_m =0.04) of the acceleration parameter evolution and those of the ΛCDM (dashed, $\Omega_m = 0.27$) put on the 1σ, 2σ, 3σ error channels (thin lines) of the deceleration parameter reconstructed from the 115 SN Ia data [30].

It should be noted that in the case of $S_0 = 0$ our model turns formally into the conventional model of flat Universe filled with dust and relativistic matter. However, the "matter domination epoch" and the "radiation domination epoch" are absent, because they lie in the non physical region after Big Rip, where the Hubble constant becomes infinite at some finite a and t, when denominator in Eq. (11) tends to zero.

To summarize, we have considered the VFD model offered in our previous works [18,19]. In this model, the Universe acceleration results from the vacuum fluctuations of fundamental scalar fields[4].

Main feature of the VFD model is that it does not predict the change from a deceleration to an acceleration in the past. If the father observations will insist on such a change, some modification of VFD should be required, because it has no tuning parameters. Some possibility of such a modification is a theory based on the truncation of physical momentums $k/a(\eta) \sim M_p$ rather than that of static (comoving) momentums $k \sim a_0 M_p$.. This would require a consideration in a system of reference, in which Universe looks like the Hoyle-Narlikar one [31,32]. Another feature of the VFD model is that in principle the dark matter is not needed.

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