On a possible quantum contribution to the red shift

© K. Urbanowski [1,2]

[1] University of Zielona Gora, Institute of Physics, ul. Prof. Z. Szafrana 4a, 65-516 Zielona Gora, Poland
[2] Email: K.Urbanowski@proton.if.uz.zgora.pl

Abstract: We consider an effect generated by the nonexponential behavior of the survival amplitude of an unstable state in the long time region: In 1957 Khalfin proved that this amplitude tends to zero as t goes to the infinity more slowly than any exponential function of t. This effect can be described in terms of time-dependent decay rate $\gamma(t)$ and then the Khalfin result means that this $\gamma(t)$ is not a constant for long times but that it tends to zero as t goes to the infinity. It appears that a similar conclusion can be drawn for the energy of the unstable state for a large class of models of unstable particles: This energy should be much smaller for suitably long times t than the energy of this state for t of the order of the lifetime of the considered state. Within the given model we show that the energy corrections in the long ($t \to \infty$) and relatively short (lifetime of the state) time regions, are different. It is shown that these corrections decrease to $\varepsilon = \varepsilon_{min} < \varepsilon_u$ as $t \to \infty$, where ε_u is the energy of the system in the state $|u\rangle$ measured at times $t \sim \tau_u = \hbar/\gamma$. This is a purely quantum mechanical effect. It is hypothesized that there is a possibility to detect this effect by analyzing the spectra of distant astrophysical objects. The above property of unstable states may influence the measured values of astrophysical and cosmological parameters.

1. Introduction

Within the quantum theory the state vector at time t, $|\psi(t)\rangle$, for the physical system under consideration which initially (at $t = t_0 = 0$) was in the state $|\psi\rangle$ can be found by solving the Schödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H\,|\psi(t)\rangle, \quad |\psi(0)\rangle = |\psi\rangle, \tag{1}$$

where $|\psi(t)\rangle, |\psi\rangle \in \chi$, χ is the Hilbert space of states of the considered system, $\| \; |\psi(t)\rangle \| = \| \; |\psi\rangle \| = 1$ and H denotes the total selfadjoint Hamiltonian for the system. If one considers an unstable state $|\psi\rangle \equiv |u\rangle$ of the system then using the solution $|u(t)\rangle$ of Eq. (1) for the initial condition $|u(0)\rangle = |u\rangle$ one can determine the decay law, $P_u(t)$ of this state decaying in vacuum

$$P_u(t) = |a(t)|^2, \tag{2}$$

where $a(t)$ is the probability amplitude of finding the system at the time t in the initial state $|u\rangle$ prepared at time $t_0 = 0$,

$$a(t) = \langle u | u(t)\rangle. \tag{3}$$

We have

$$a(0) = 1. \tag{4}$$

From basic principles of quantum theory it is known that the amplitude $a(t)$, and thus the decay law $P_u(t)$ of the unstable state $|u\rangle$, are completely determined by the density of the energy distribution $\omega(\varepsilon)$ for the system in this state [1],

$$a(t) = \int_{Spec.(H)} \omega(\varepsilon)\, e^{-\frac{i}{\hbar}\varepsilon t}\, d\varepsilon. \tag{5}$$

where $\omega(\varepsilon) > 0$.

Note that (5) and (4) mean that there must be

$$a(0) = \int_{Spec.(H)} \omega(\varepsilon) d\varepsilon = 1. \tag{6}$$

From the last property and from the Riemann-Lebesgue Lemma it follows that the amplitude $a(t)$, being the Fourier transform of $\omega(\varepsilon)$ (see (5)), must tend to zero as $t \to \infty$ [1].

In [2] assuming that the spectrum of H must be bounded from below, $(Spec.(H) > -\infty)$, and using the Paley-Wiener Theorem [3] it was proved that in the case of unstable states there must be

$$|\, a(t)\,| \geq A e^{-bt^{q}}, \tag{7}$$

for $|t| \to \infty$. Here $A > 0,\; b > 0$ and $0 < q < 1$. This means that the decay law $P_u(t)$ of unstable states decaying in vacuum, (2), can not be described by an exponential function of time t if time t is suitably long, $t \to \infty$, and that for these lengths of time $P_u(t)$ tends to zero as $t \to \infty$ more slowly than any exponential function of t. The analysis of the models of the decay processes shows that $P_u(t) \approx \exp[-\gamma_u^0 t/\hbar]$, (where γ_u^0 is the decay rate of the state $|u\rangle$), to a very high accuracy for a wide time range t: From t suitably later than some $T_0 \approx t_0 = 0$ but $T_0 > t_0$ up to $t >> \tau_u = \gamma_u^0/\hbar$ and smaller than $t = t_{as}$, where t_{as} denotes the time t for which the nonexponential deviations of $a(t)$ begin to dominate (see eg., [2], [4-7]). From this analysis it follows that in the general case the decay law $P_u(t)$ takes the inverse power-like form $t^{-\lambda}$, (where $\lambda > 0$), for suitably large $t \geq t_{as} >> \tau_u$ [2], [4-6]. This effect is in agreement with the general result (7). Effects of this type are sometimes called the "Khalfin effect" (see eg. [8]).

The problem how to detect possible deviations from the exponential form of $P_u(t)$ in the long time region has been attracting the attention of physicists since the first theoretical predictions of such an effect [9, 10, 7]. Many tests of the decay law performed some time ago did not indicate any deviations from the exponential form of $P_u(t)$ at the long time region. Nevertheless, conditions leading to the nonexponential behavior of the amplitude $a(t)$ at long times were studied theoretically [11-13]. Conclusions following from these studies were applied successfully in the experiment described in [14], where the experimental evidence of the exponential decay law at long times was reported. This result gives rise to another problem which now becomes important: if (and how) deviations from the exponential decay law at long times affect the energy of the unstable state and its decay rate at this time region.

Note that in fact the amplitude $a(t)$ contains information about the decay law $P_u(t)$ of the state $|u\rangle$, that is about the decay rate γ_u^0 of this state, as well as the energy ε_u^0 of the system in this state. This information can be extracted from $a(t)$. Indeed, if $|u\rangle$ is an unstable (a quasi--stationary) state then

$$a(t) \cong e^{-\frac{i}{\hbar}(\varepsilon_u^0 - \frac{i}{2}\gamma_u^0)t}. \tag{8}$$

So, there is

$$\varepsilon_u^0 - \frac{i}{2}\gamma_u^0 \equiv i\hbar \frac{\partial a(t)}{\partial t}\,\frac{1}{a(t)}, \tag{9}$$

in the case of quasi-stationary states.

The standard interpretation and understanding of the quantum theory and the related construction of our measuring devices are such that detecting the energy ε_u^0 and decay rate γ_u^0 one is sure that the amplitude $a(t)$ has the form (8) and thus that the relation (9) occurs. Taking the above into account one can define the "effective Hamiltonian", h_u, for the one-dimensional subspace of states $\chi_{\|}$ spanned by the normalized vector $|u\rangle$ as follows (see, eg. [15])

$$h_u \stackrel{def}{=} i\hbar \frac{\partial a(t)}{\partial t}\,\frac{1}{a(t)}. \tag{10}$$

In general, h_u can depend on time t, $h_u \equiv h_u(t)$. One meets this effective Hamiltonian when one starts with the Schrödinger Equation (1) for the total state space χ and looks for the rigorous evolution equation for the distinguished subspace of states $\chi_{\|} \subset \chi$. In the case of one-dimensional $\chi_{\|}$ this rigorous Schrödinger--like evolution equation has the following form for the initial condition $a(0) = 1$ (see [15] and references one finds therein),

$$i\hbar \frac{\partial a(t)}{\partial t} = h_u(t)\, a(t). \tag{11}$$

Relations (10) and (11) establish a direct connection between the amplitude $a(t)$ for the state $|u\rangle$ and the exact effective Hamiltonian $h_u(t)$ governing the time evolution in the one--dimensional subspace $\chi_{\|} \ni |u\rangle$.

So let us assume that we know the amplitude $a(t)$. Then starting with this $a(t)$ and using the expression (10) one can calculate the effective Hamiltonian $h_u(t)$ in a general case for every t. Thus, one finds the following expressions for the energy and the decay rate of the system in the state $|u\rangle$ under considerations,

$$\varepsilon_u \equiv \varepsilon_u(t) = \Re(h_u(t), \qquad \gamma_u \equiv \gamma_u(t) = -2\Im(h_u(t), \tag{12}$$

where $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of z respectively.

As it was mentioned above the deviations of the decay law $P_u(t)$ from the exponential form can be described equivalently using time-dependent decay rate. In terms of such $\gamma_u(t)$ the Khalfin observation that $P_u(t)$ must tend to zero as $t \to \infty$ more slowly than any exponential function means that $\gamma_u(t) << \gamma_u^0$ for $t >> t_{as}$ and $\lim_{t\to\infty}\gamma_u(t) = 0$.

The aim of this note is to examine the long time behaviour of $\varepsilon_u(t)$ using $a(t)$ calculated for the given density $\omega(\varepsilon)$. We show that $\varepsilon_u(t) \to 0$ as $t \to \infty$ for the model considered and that a wide class of models has similar long time properties: $\varepsilon_u(t)\,|_{\,t\to\infty} \neq \varepsilon_u^0$. It seems that in contrast to the standard Khalfin effect [2] in the case of the quasistationary states belonging to the same class as excited atomic levels, this long time properties of the energy $\varepsilon_u(t)$ have a chance to be detected by analyzing spectra of very distant stars.

2. The model

Let us assume that $Spec.(H)=[\varepsilon_{min},\infty)$, (where, $\varepsilon_{min}>-\infty$), and let us choose $\omega(\varepsilon)$ as follows

$$\omega(\varepsilon)=\frac{N}{2\pi}\Theta(\varepsilon-\varepsilon_{min})\frac{\gamma_u^0}{(\varepsilon-\varepsilon_u^0)^2+(\frac{\gamma_u^0}{2})^2}, \tag{13}$$

where N is a normalization constant and $\Theta(\varepsilon)=\{1$ for $\varepsilon\geq 0$, and, 0 for $\varepsilon<0\}$. For such an $\omega(\varepsilon)$ using (5) one has

$$a(t)=\frac{N}{2\pi}\int_{\varepsilon_{min}}^{\infty}\frac{\gamma_u^0}{(\varepsilon-\varepsilon_u^0)^2+(\frac{\gamma_u^0}{2})^2}e^{-\frac{i}{\hbar}\varepsilon t}\,d\varepsilon. \tag{14}$$

Formula (14) leads to the result

$$a(t)=Ne^{-\frac{i}{\hbar}(\varepsilon_u^0-i\frac{\gamma_u^0}{2})t}\{1-\frac{i}{2\pi}[e^{\frac{\gamma_u^0 t}{\hbar}}E_1(-\frac{i}{\hbar}(\varepsilon_u^0-\varepsilon_{min}+\frac{i}{2}\gamma_u^0)t)-E_1(-\frac{i}{\hbar}(\varepsilon_u^0-\varepsilon_{min}-\frac{i}{2}\gamma_u^0)t)]\}, \tag{15}$$

where $E_1(x)$ denotes the integral-exponential function [16, 17].

Using (14) or (15) one easily finds that

$$i\hbar\frac{\partial a(t)}{\partial t}=h_u^0\,a(t)\;+\;\Delta h_u(t)a(t), \tag{16}$$

where

$$h_u^0\equiv\varepsilon_u^0-\frac{i}{2}\gamma_u^0, \tag{17}$$

$$\Delta h_u(t)=+\frac{\Delta a(t)}{a(t)}, \tag{18}$$

and

$$\Delta a(t)=\frac{N}{\pi}\frac{\gamma_u^0}{2}e^{-\frac{i}{\hbar}(\varepsilon_u^0+\frac{i}{2}\gamma_u^0)t}\quad E_1(-\frac{i}{\hbar}(\varepsilon_u^0-\varepsilon_{min}+\frac{i}{2}\gamma_u^0)t), \tag{19}$$

Making use of the asymptotic expansion of $E_1(x)$ [17],

$$E_1(z)|_{|z|\to\infty}\sim\frac{e^{-z}}{z}(1-\frac{1}{z}+\frac{2}{z^2}-\ldots), \tag{20}$$

where $|\arg z|<\frac{3}{2}\pi$, one finds

$$h_u(t)|_{t\to\infty}\;\sim\;\varepsilon_{min}\;-\;i\frac{\hbar}{t}\;-\;2\frac{\varepsilon_u^0-\varepsilon_{min}}{|h_u^0-\varepsilon_{min}|^2}\;(\frac{\hbar}{t})^2+\ldots \tag{21}$$

for the considered case (13) of $\omega(\varepsilon)$.

From (21) it follows that

$$\Re(h_u(t)|_{t\to\infty}) \stackrel{def}{=} \varepsilon_u^\infty \approx \varepsilon_{min} - 2\frac{\varepsilon_u^0 - \varepsilon_{min}}{|h_u^0 - \varepsilon_{min}|^2}\,(\frac{\hbar}{t})^2 \underset{t\to\infty}{\to} \varepsilon_{min}, \tag{22}$$

where $\varepsilon_u^\infty = \varepsilon_u(t)|_{t\to\infty}$, and

$$\Im(h_u(t)|_{t\to\infty}) \approx -\frac{\hbar}{t} \underset{t\to\infty}{\to} 0. \tag{23}$$

The property (22) means that

$$\Re(h_u(t)|_{t\to\infty}) \equiv \varepsilon_u^\infty < \varepsilon_u^0. \tag{24}$$

For different states $|u\rangle = |j\rangle$, ($j = 1,2,3,\ldots$) one has

$$\varepsilon_1^\infty - \varepsilon_2^\infty = -2[\frac{\varepsilon_1^0 - \varepsilon_{min}}{|h_1^0 - \varepsilon_{min}|^2} - \frac{\varepsilon_2^0 - \varepsilon_{min}}{|h_2^0 - \varepsilon_{min}|^2}](\frac{\hbar}{t})^2 \neq \varepsilon_1^0 - \varepsilon_2^0 \neq 0. \tag{25}$$

Note that

$$\Im(h_1(t)_{t\to\infty}) = \Im(h_2(t)_{t\to\infty}), \tag{26}$$

whereas in general $\gamma_1^0 \neq \gamma_1^0$.

The most interesting relation seems to be the following one

$$\frac{\varepsilon_1^\infty - \varepsilon_2^\infty}{\varepsilon_3^\infty - \varepsilon_{u_4}^\infty} = \frac{\dfrac{\varepsilon_1^0 - \varepsilon_{min}}{|h_1^0 - \varepsilon_{min}|^2} - \dfrac{\varepsilon_2^0 - \varepsilon_{min}}{|h_2^0 - \varepsilon_{min}|^2}}{\dfrac{\varepsilon_3^0 - \varepsilon_{min}}{|h_3^0 - \varepsilon_{min}|^2} - \dfrac{\varepsilon_4^0 - \varepsilon_{min}}{|h_4^0 - \varepsilon_{min}|^2}} \neq \frac{\varepsilon_1^0 - \varepsilon_2^0}{\varepsilon_3^0 - \varepsilon_4^0}. \tag{27}$$

The relation (27) is valid also when one takes $|1\rangle$ instead of $|3\rangle$ or $|2\rangle$ instead of $|4\rangle$. It seems to be interesting that the relation (27) does not depend on time t.

Note that the following conclusion can be drawn from (25): For suitably long times $t > t_{as}$ there must be

$$|\varepsilon_1^\infty - \varepsilon_2^\infty| < |\varepsilon_1^0 - \varepsilon_2^0|. \tag{28}$$

3. Some generalizations

To complete the analysis performed in the previous Section let us consider a more general case of $a(t)$.

Namely, let the asymptotic approximation to $a(t)$ have the form

$$a(t) \underset{t \to \infty}{\sim} e^{-i\frac{t}{\hbar}\varepsilon_{min}} \sum_{k=0}^{N} \frac{c_k}{t^{\lambda+k}}, \tag{29}$$

where $\lambda > 0$ and c_k are complex numbers. The simplest case occurs for $\varepsilon_{min} = 0$. Note that the asymptotic expansion for $a(t)$ of this or a similar form is obtained for a wide class of densities of energy distribution $\omega(\varepsilon)$ [2, 4, 5, 6, 8], [11-13].

From relation (29) one concludes that

$$\frac{\partial a(t)}{\partial t} \underset{t \to \infty}{\sim} e^{-i\frac{t}{\hbar}\varepsilon_{min}} \{-\frac{i}{\hbar}\varepsilon_{min} - \sum_{k=0}^{N}(\lambda + k)\frac{c_k}{t^{\lambda+k+1}}\}. \tag{30}$$

Now let us take into account the relation (11). From this relation and relations (29), (30) it follows that

$$h_u(t) \underset{t \to \infty}{\sim} \varepsilon_{min} + \frac{d_1}{t} + \frac{d_2}{t^2} + \frac{d_3}{t^3} + \ldots, \tag{31}$$

where $d_1, d_2, d_3, \ldots$ are complex numbers with negative or positive real and imaginary parts. This means that in the case of the asymptotic approximation to $a(t)$ of the form (29) the following property holds,

$$\lim_{t\to\infty} h_u(t) = \varepsilon_{min} < \varepsilon_u^0. \tag{32}$$

It seems to be important that results (31) and (32) coincide with the results (21) - (25) obtained for the density $\omega(\varepsilon)$ given by the formula (13). This means that the general conclusion obtained for the other $\omega(\varepsilon)$ defining unstable states should be similar to those following from (21) - (25).

4. Final remarks.

Let us consider a class of unstable states formed by excited atomic energy levels and let these excited atoms emit the electromagnetic waves of the energies $\varepsilon_u^0 = \varepsilon_{n_{jk}}^0 \equiv h\nu_{n_{jk}}^0$ (where $\nu_{n_{jk}}^0$ denotes the frequency of the emitted wave, and $\varepsilon_{n_{jk}}^0$ is energy emitted by an electron jumping from the energy level n_j to the energy level n_k). Then for times $t > t_{as}$, according to the results of the previous sections, there should be

$$\varepsilon_{n_{jk}}^\infty < \varepsilon_{n_{jk}}^0. \tag{33}$$

So in the case of electromagnetic radiation in the optical range registered by a suitably this effect should manifest itself as a red shift. In a general case this effect should cause a loss of energy in the emitted electromagnetic radiation if the distance between an emitter and receiver is suitably long, that is if the emitted radiation reaches such a distance from the emitter that the time necessary for photons to reach this distance is longer than the maximal range of time of the validity of the exponential decay law for the excited atomic level emitting this radiation.

It can be easily verified that relation (27) does not depend on the redshift connected with the Doppler effect [18]. Therefore it seems that there is a chance to detect the possible effect described in this paper using relation (27) and analyzing spectra of distant astrophysical objects. It can be done using this relation if one is able to register and analyze at least three different emission lines from the same distant source. Another possibility to observe this effect is to modify the experiment described in [14] in such a way that the emitted energy (frequency) of the luminescence decays could be measured which could make possible to test relations (33) or (28).

The last conclusion. Cosmic distances and other parameters computed from the observed redshift of very distant objects emitting electromagnetic radiation [19] are calculated without taking into account the possible quantum long time energy redshift described in Sec. 2 and Sec. 3, so these distances as well as the values of these parameters need not reflect correctly the real picture.

References

1. S. Krylov, V. A. Fock, Zh. Eksp. Teor. Fiz. **17**, 93, (1947).
2. L. A. Khalfin, Zh. Eksp. Teor. Fiz. **33**, 1371, (1957); [Sov. Phys. - JETP **6**, 1053, (1958)].
3. R. E. A. C. Paley, N. Wiener, *Fourier transforms in the comlex domain*, American Mathematical Society, New York, 1934.
4. M. L. Goldberger, K. M. Watson, *Collision Theory,* Willey, New York 1964.
5. L. Fonda, G. C. Ghirardii and A. Rimini, Rep. on Prog. in Phys. **41**, 587, (1978).
6. A. Peres, Ann. Phys. **129**, 33, (1980).
7. P. T. Greenland, Nature **335**, 298, (1988).
8. D. G. Arbo, M. A. Castagnino, F. H. Gaioli and S. Iguri, Physica **A 227**, 469, (2000).
9. J . M. Wessner, D. K. Andreson and R. T. Robiscoe, Phys. Rev. Lett. **29**, 1126, (1972).
10. E. B. Norman, S. B. Gazes, S. C. Crane and D. A. Bennet, Phys. Rev. Lett. bf 60, 2246, (1988). E. B. Norman, B. Sur, K. T. 11. Lesko, R.-M. Larimer, Phys. Lett. **B 357**, 521, (1995).
12. J. Seke, W. N. Herfort, Phys. Rev. **A 38**, 833, (1988).
13. R. E. Parrot, J. Lawrence, Europhys. Lett. **57**, 632, (2002).
14. T. Jiitoh, S. Matsumoto, J. Sato, Y. Sato, K. Takeda, Phys Rev. **A 71**, 012109, (2005).
15. C. Rothe, S. I. Hintschich and A. P. Monkman, Phys. Rev. Lett. **96**, 163601, (2006).
16. K. Urbanowski, Phys. Rev. **A 50**, 2847, (1994).
17. K. M. Sluis, E. A. Gislason, Phys. Rev. **A 43**, 4581, (1991).
18. *Handbook of Mathematical Functions*, Natl. Bur. Stand. Appl. Math. Ser. No 55, eds. M. Abramowitz nad I. A. Stegun (U.S. GPO, Washington, D.C., 1964).
19. W. Heitler, *The Quantum Theory of Radiation,* Oxford University Press, London 1954 and Dover Publications, New York 1984.
20. E. W. Kolb, M. S. Turner, *The Early Universe*, Addison--Wesley Publ. Co., 1993.