Inertial Frame Transformation Based on Lobachevsky Function and Some Optical Phenomena (Michelson-Morley experiment and Doppler Effect)

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Abstract: The inertial frame transformation based on Lobachevsky function and some of its properties are reminded. The results of introducing them into a physical consideration of the Michelson and standard Doppler effect in optics, are presented. It is that the new transformation also approves the negative result obtained in the Michelson experiment as the Lorentz one. But the crucial result is expected for the measurements performed with unequal lengths of the interferometer optical arms and it will be in favour of the new one. In contrast to Lorentz contractions the relativity requirement of the new transformation again leads to two different formulas for the Doppler effect in optics as in acoustics

Introduction: two possible ways to express the speed of light constancy principle

It is used to consider that the invariance of an interval serves as mathematical tool to express the principle of the speed of light constancy and this is the reason of great importance of the interval invariance in the special theory of relativity and generally in the theoretical physics. The world famous Lorentz transformation can be obtained by the requirement for the interval to have the same form in any two inertial frames. In its turn the Lorentz transformation leads to the relativistic velocity summation law and the relativistic mechanics (see Fig.1).

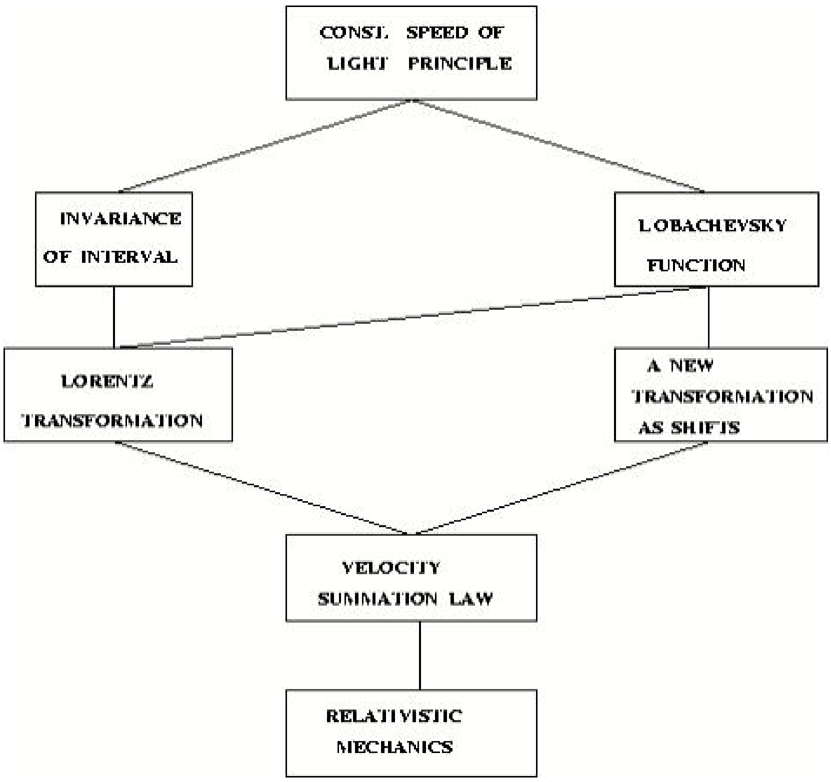

Fig.1 Two possible ways to express mathematically the principle of the speed of light constancy.

A close relation between the special theory of relativity and the theory of Lobachevsky parallel lines (LPL) was noticed soon after A.Einstein's publication [1]. Further development of high energy physics made this relation more evident because the full correspondence between relativistic kinematics and Lobacvhevsky geometry was established [2]. Due to that the so called Lobachevsky velocity space is widely used to present particle scattering processes in high energy physics. The two-photon pion decay $\pi^o \to \gamma\gamma$ was claimed as the physical equivalent for the main Lobachevsky geometrical axiom violating Euclidean V-th postulate[3].

This decay being represented in the velocity space due to its kinematics leads to the LPL in the plane of this space and visually demonstrates the principle of the speed of light constancy.

Further investigations in the physical foundation of LPL revealed full identity of the both theories. On the base of Lobachevsky function the synchronous process of the particle and light beams propagation has been found. It has turned out that one of the main features of this process is quite similar to the optical model used by I.M.Frank in 1942 for Doppler effect consideration in refracting medium [4]. This synchronous process can be considered, on the one hand, as a new physical equivalent for the LPL and, on the other hand, as a physical foundation for solving the main relativity problems including the time relativity in a new way [5]. In this investigation Lobachesky function expresses the principle of the speed of light constancy instead of the interval invariancy and a new inertial frame transformation in the form of shifts has been obtained. It leads to the same relativistic velocity summation law and to the same kinematics and mechanics(see Fig.1).

In framework of the new approach the Lorentz formulas can be also derived, but here they loose previous meaning as coordinates of the given event in the moving frame. They look as coordinates relative to the point, which is called the center of projectivity in the projective geometry [6], that does not coincide with the origin of the moving frame (and due to that they could be hardly considered as coordinates of an event). But the both sets of values are related in a simple way through the γ factor.

Thus, there are two possible ways to express the principle of the speed of the light constancy and, respectively, there are two sets of values: the well known Lorentz transformation and the new one - shifts. Some physically interesting properties of the transformation based on Lobachevsky function published in [5] are reminded here and the results of introducing them into the physical consideration of some optical phenomena, namely, the Michelson-Morley experiment and standard Doppler effect in optics, are presented in this paper.

2. Some properties of transformation based on Lobachevsky function

Lorentz transformation [7]:

$$\begin{array}{lcl} x' = \gamma(x \mp ct\cos\theta_L) & | & x = \gamma(x' \pm ct'\cos\theta_L) \\ ct' = \gamma(ct \mp x\cos\theta_L) & | & ct = \gamma(ct' \pm x'\cos\theta_L) \\ y' = y & | & y = y' \\ z' = z & | & z = z' \end{array} \tag{1}$$

and the inertial frame transformation based on Lobachevsky function [5]:

$$\begin{array}{lcl} x^s = x \pm ct\cos\theta_L & | & x = \gamma^2\left(x^s \pm ct^s\cos\theta_L\right) \\ ct^s = ct \mp x\cos\theta_L & | & ct = \gamma^2\left(ct^s \pm x^s\cos\theta_L\right) \\ y^s = y/\gamma & | & y = \gamma\, y^s \\ z^s = z/\gamma & | & z = \gamma\, z^s \end{array} \tag{2}$$

can be written through parallel angle $\theta_L \equiv \Pi(\rho/k)$, defined by Lobachrvsky function:

$$\cos\Pi(\rho/k) = th(\rho/k) = V/c \equiv \beta, \quad (k=c), \quad \Pi(\rho/c) = 2arctg\left(e^{-\rho/c}\right) = 2arctg\sqrt{\frac{1-\beta}{1+\beta}}, \tag{3}$$

$$\cos\theta_L = \beta, \quad \sin\theta_L = \sqrt{1-\beta^2}, \quad \gamma = 1/\sin\theta_L, \tag{4}$$

here x, y, z, ct are coordinates of an event in a rest frame, x', y', z', ct' and x^s, y^s, z^s, ct^s are coordinates of the same event in a frame moving with velocity V along x-axes (the upper sign) or opposite x-axes (the lower sign) , β is the velocity V in units of the light velocity c, ρ / k is a value of rapidity in units of $k = c$ (k is the Lobachevsky constant):

$$\rho / c = \frac{1}{2} \ln \frac{1+\beta}{1-\beta} \tag{5}$$

It is seen from (3) that for any rapidity (and its velocity) there is a definite angle θ_L.. For the negative argument of the Lobachevsky function the parallel angle θ_L changes to $\pi - \theta_L$ [2], which corresponds to the same velocity but for the opposite direction.

Let us consider some obvious properties of a new inertial frame transformation.

First of all, one can see from (1) and (2) that:

$$\frac{dx^s}{cdt^s} = \frac{dx'}{cdt'} = \beta_x^{'}, \quad \frac{dy^s}{cdt^s} = \frac{dy'}{cdt'} = \beta_y^{'}, \quad \frac{dz^s}{cdt^s} = \frac{dz'}{cdt'} = \beta_z^{'}, \tag{6}$$

i.e., the new transformation leads to the same relativistic velocity summation law known from the Lorentz transformation.

II. This velocity summation law leads to the relativistic particle energy E and momentum P definition:

$$\beta = th(\rho / c) = \frac{m\, sh(\rho / c)}{m\, ch(\rho / c)}, \quad P / c = m\, sh(\rho / c) = \frac{m \beta}{\sqrt{1-\beta^2}}, \quad E / c^2 = m\, ch(\rho / c) = \frac{m}{\sqrt{1-\beta^2}}, \tag{7}$$

and to the Lorentz transformation for them (for simplicity it is better to use units $c = 1$).

III. By direct substitution of (2) into the square of interval s^2 definition one can find the following:

$$s^2 = (ct)^2 - x^2 - y^2 - z^2 = \gamma^2 \left((ct^s)^2 - x^{s2} - y^{s2} - z^{s2} \right), \tag{8}$$

i.e., s^2 is not invariant.

IV. The same way one can obtain noninvariance of the scalar product (xp):

$$(xp) = ct\,E - x\,P_x - y\,P_y - z\,P_z = \gamma \left(x^s\, P^{'} \right) = \gamma \left(ct^s E' - x^s P_x^{'} - y^s P_y^{'} - z^s P_z^{'} \right), \tag{9}$$

where P_x, P_y, P_z are x, y, z - components of a particle momentum in the rest frame.

V. The light ray ct from the origin of the rest frame under the parallel angle to the x-axes allows one to introduce the conception of the projectivity (or the projective correspondence between the 1-, 2- and 3-dimensions of multitudes) and its main invariant :

$$(x_1 x_2 x_3 x_4) \equiv \frac{x_3 - x_1}{x_2 - x_3} : \frac{x_4 - x_1}{x_2 - x_4} = \frac{ct_3 - ct_1}{ct_2 - ct_3} : \frac{ct_4 - ct_1}{ct_2 - ct_4} \equiv (t_1 t_2 t_3 t_4),$$

(10)

the complex fraction of any four corresponding points [6].

VI. Let us also remind the length Δx^s (at $\Delta t^s = 0$) and the time duration Δt^s (at $\Delta x^s = 0$) contractions due to the new transformation and the Lorentz one:

$$\Delta x = \gamma^2 \Delta x^s = \gamma \, \Delta x' , \quad \Delta t = \gamma^2 \, \Delta t^s = \gamma \, \Delta t' . \qquad (11)$$

It is important to note that coordinates marked by s ('shifted' coordinates) and the 'primed' values are related by the γ-factor: $x^s = x'/\gamma$, $ct^s = ct'/\gamma$, $y^s = y'/\gamma$, $z^s = z'/\gamma$ (it is seen from here and also from comparison of (1) and (2)). Since the γ is a constant for the both sets of values (1)-(2), then this relation means that Maxwell's equations are invariant with respect to the new coordinate transformation in the same way as with respect to the Lorentz one.

This contractions and transverse coordinates transformation will be used in the sections below.

3. Michelson-Morley optical experiment

It is well known that the negative result of Michelson-Morley experiment was understood on the base of the length contraction followed from the Lorentz transformation. The introduction of the relativity theory into the consideration of this experiment takes place at the last step [8]:

$$T_1 + T_2 = \frac{2l_1}{c} \frac{1}{1-\beta^2} , \quad 2T = \frac{2l_2}{c} \frac{1}{\sqrt{1-\beta^2}} , \qquad (12)$$

where $T_1 + T_2$ is the total time for light beam propagation (there and back) in the first interferometer's arm l_1 oriented parallel to the Earth's velocity V, $2T$ is the total time for light beam propagation (there and back) in the second arm l_2 oriented perpendicular to the first one, l_1 and l_2 are lengths of the corresponding optical arms and β is V/c.

For equal lengths of arms $l_1 = l_2 = l$ the value for time difference ΔT was expected to be as follows:

$$\Delta T = T_1 + T_2 - 2T = \frac{2l}{c} \left(\frac{1}{1-\beta^2} - \frac{1}{\sqrt{1-\beta^2}} \right) \approx \frac{l}{c} \beta^2 , \qquad (13)$$

but the experiment has shown the so called 'negative' result corresponding to $\Delta T = 0$.

To obtain agreement with the experimental result, one should correct the length l_1 of the arm (oriented according to the Eath's velocity) due to the relativity requirement. In the case of Lorentz contraction instead of l_1 one should use l_1/γ value, i.e.:

$$T_1 + T_2 = \frac{2l_1 \sqrt{1-\beta^2}}{c\left(1-\beta^2\right)} = \frac{2l_1}{c} \frac{1}{\sqrt{1-\beta^2}} , \qquad (14)$$

In this case for ΔT_{Lor} one has:

$$\Delta T_{Lor} = T_1 + T_2 - 2T = \frac{2\Delta l}{c}\frac{1}{\sqrt{1-\beta^2}}, \quad \Delta l = l_1 - l_2, \tag{15}$$

and for $\Delta l = 0$ (as it was in the experiment) the $\Delta T_{Lor} = 0$, as it was observed.

In the case of using Lobachevsky transformation instead of l_1 one should use the l_1/γ^2 value and instead of the transverse length (coordinate) l_2 should use l_2/γ, i.e., for ΔT_{Lob} one has:

$$\Delta T_{Lob} = \frac{2l_1}{c}\frac{1-\beta^2}{1-\beta^2} - \frac{2l_2}{c}\frac{\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}} = \frac{2\Delta l}{c}, \tag{16}$$

and for $\Delta l = 0$ has again $\Delta T_{Lob} = 0$.

It is important to note that the result (15) is still a function on β instead of the new result (16). The both transformations are adequate to the observed value $\Delta T = 0$ due to the equal lengths of the interferometer arms $\Delta l = 0$.

It is interesting to compare the result of this experiment obtained for $\Delta l \neq 0$ for the both predictions (15) and (16).

3. Doppler effect in optics

As in the previous section the introduction of the relativity theory to consider the standard Doppler effect in optics, can be also done at the last step of deriving formulas for the frequency shift in acoustics [8]:

a) for the case when the source is moving and the receiver/observer is still:

$$N = \nu_o \tau = \nu \vartheta, \quad \vartheta = \tau(1 \pm \upsilon/u), \quad \nu = \frac{\nu_o}{1 \pm \beta_u}, \tag{17}$$

b) for the case when the observer is moving and the source of waves is still:

$$N = \nu_o \tau = \nu \vartheta, \quad \vartheta = \frac{\tau}{1 \mp \upsilon/u}, \quad \nu = \nu_o(1 \mp \beta_u), \tag{18}$$

where N is the number of waves emitted by the source during the time τ, ν_o and ν are emitted and registered frequencies (of light here), respectively, $\beta_u = \upsilon/u$, υ is the relative velocity between the source and the device (observer), u is the speed of the signal, ϑ is the time of waves registering, the upper sign refers to the object moving away, the lower sign refers to the object moving in the opposite direction.

Now one should take into account the time contraction in the moving frame and $u = c$, $\beta = \upsilon/c$. Let us make it firstly for Lorentz transformation ($\Delta t' = \Delta t\sqrt{1-\beta^2}$) and use ϑ from (17) and (18), respectively:

a) $\nu_o \tau \sqrt{1-\beta^2} = \nu \vartheta \quad \Rightarrow \quad \nu = \frac{\nu_o \sqrt{1-\beta^2}}{1 \mp \beta_u} = (u=c) = \nu_o \sqrt{\frac{1 \mp \beta}{1 \pm \beta}}$, (19)

b) $\nu_o \tau = \nu \vartheta \sqrt{1-\beta^2} \quad \Rightarrow \quad \nu = \frac{\nu_o (1 \pm \beta_u)}{\sqrt{1-\beta^2}} = (u=c) = \nu_o \sqrt{\frac{1 \mp \beta}{1 \pm \beta}}$. (20)

The same can be done for the new transformation (one should use $\Delta t^s = \Delta t(1-\beta^2)$):

a) $\nu_o \tau (1-\beta^2) = \nu \vartheta \quad \Rightarrow \quad \nu = \nu_o (1 \mp \beta)$, (21)

b) $\nu_o \tau = \nu \vartheta (1-\beta^2) \quad \Rightarrow \quad \nu = \frac{\nu_o}{1 \pm \beta}$. (22)

It is seen from (19) and (20) that Lorentz contraction leads to the same result for the registered frequency in a) and b) cases in optics (at $u=c$). But the relativity requirements from the new transformation leads again to two different formulas as in acoustics, but with a) $\Leftrightarrow$ b).

Thus, for the same relative velocity there are three different formulas for the registered frequency (19) - (22).

4. Conclusions

- A new inertial frame transformation has been obtained by using Lobachevsky function as a mathematical tool to express the principle of the speed of light constancy.

A pair of light beams chosen according to the Lobachevsky relation between the given velocity and parallel angle, makes the concept of the time relativity to be natural and transparent.

A new transformation leads to the same relativistic kinematics as the Lorentz one, but it violates the invariance of both - the interval and the scalar product (xp).

A new transformation leads to the length and time contractions by γ times more than that for the Lorentz one. It also leads to transverse coordinate shifts.

- New length contractions and transverse coordinate shifts in a moving frame completely explain the negative result of the Michelson-Morley experiment.

Moreover, if this experiment is performed with nonequal lengths of the interferometer optical arms, then the results will be crucial for one of the two transformations. The predictions based on the both transformations differ in principle and the results are in favour of the new one.

- Michelson-Morley experiment with $\Delta l \neq 0$ should be performed.
- A new time contraction leads to two formulas for the standard Doppler frequency shift in optics (as in acoustics).
- The experiment for accurate measurements Doppler effect should be repeated.

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