The Cosmic Defect theory tested by observation

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Abstract: The Cosmic Defect (CD) theory describes the accelerated expansion of the universe in terms of a four-dimensional defect in the texture of space-time, similar to defects in crystals and material continua, and of its influence on the metric properties of the medium. What we, in our 3+1 view, interpret as a non uniform expansion rate, appears in this theory as a strained state of space-time. The presence of matter, as it is usual in General Relativity, adds its contribution to the intrinsic curvature induced by the presence of the defect, without modifying the general features of the model. Altogether CD accounts for an initial inflationary era, followed by a sequence of decelerated-accelerated-decelerated expansion; the expansion asymptotically ends at time infinity.
The theoretical predictions of CD are contrasted with observational data from type Ia Supernovae, and compared with the predictions of the Λ-cold-dark-matter theory (ΛCDM). The result is positive, showing a good correspondence with the data at the same level as for ΛCDM. Further implications and the correspondence with other pieces of evidence are discussed.

1. Introduction.

A relevant discovery of the last decade has been the accelerated expansion of the universe. The initial conclusion drawn from the observation of the luminosity versus redshift of the type Ia supernovae (SnIa) [1] has been corroborated by other independent pieces of evidence [2]. Today, the picture which seems to emerge from the data is that of an universe which has undergone a transition from a decelerated to an accelerated phase, with a relatively recent turning point located at $z \approx 0.46$ [3].This new feature was not part of the standard model of the evolution of the universe, and represented a challenge for theoretical cosmology, so many people have started to elaborate possible explanations for the newly discovered acceleration. The simplest remark is that a positive cosmological constant would indeed imply the acceleration. Things go as if the universe was filled up with a perfectly homogeneous and isotropic fluid which does not give origin to any aggregate and consequently does not manifest any direct gravitational effect, but only determines the acceleration. This peculiar "fluid", that does indeed never flow, is also called "dark energy". Its more sophisticated variants, often worked out well before this sudden revival, introduce various types of equations of state for the fluid [4][5][6][7][8]. However the theory which best fits the data is indeed the simplest one, now called the Λ-cold-dark-matter theory (ΛCDM), which makes use of a real mere cosmological constant Λ, furthermore surmising the presence in the universe of one more ingredient, actually able to produce gravitational effects: dark matter. The problem with ΛCDM is not in a poor capacity to reproduce observed facts, but rather in the difficulty to interpret the nature of the dark energy (and dark matter). "Dark" ingredients are always suspect; of course one may wait for some new fact or experiment shedding light on them, or look for alternative explanations or at least interpretation paradigms. Many attempts are under way, or have been tried in the past, to work out alternatives or extensions of the otherwise successful general relativity theory (GR). Many of these attempts are essentially heuristic and/or formal: people try and modify for instance the Einstein-Hilbert action for space time (as the so called f(R) theories do [9]) or introduce new appropriately chosen fields [10]. The weakness again, more than in the failure of any given such theory, is the, at least initial, lack of a physical motivation or interpretation of the introduced extensions or changes, other than the practical need to cope with the experimental facts.

Here we expose a theory of our own, trying to move from a physical start, inspired by known physics. The inspiration is from the properties of physical continua and the effects produced in them by the presence of texture defects. Actually this is an old theory for three-dimensional continua [11]; we extend it to the four-dimensional continuum represented by space-time; attempts in this direction had already been tried [12] without drawing cosmological consequences. In our view the initial singularity (the Big Bang) is interpreted as a defect in the space-time continuum, inducing everywhere a strained state which manifests itself as what we read as a non-uniform expansion rate of the universe; a complete exposition of the theory is available in [13]. As we shall see, this approach will lead, with a number of free parameters not bigger than for ΛCDM, to a comparatively good fit of the luminosity data from SnIa, without calling in dark actors, or, at least, providing a consistent interpretation of the observed phenomena and of the tools used to explain them.

2. The cosmic defect theory

As declared in the introduction, we move from analogies with other branches of known physics. The first remark concerns the fact that the phase space of a spacely homogeneous and isotropic universe is bidimentional: the two parameters are the scale factor a and its expansion rate $\dot{a}$ and the evolution is represented by a simple line. Everything looks like the case of a point massive particle acted upon by an isotropic viscous force: there too the phase space is bidimensional with variables x and $\dot{x}$. Building on this simple correspondence and on the fact that the behaviour of a point particle moving across a fluid may be described in Lagrangian terms [13][14] we are able to conjecture an action integral for empty space-time in the form:

$$S = \frac{1}{\kappa}\int e^{-g_{\mu\nu}\gamma^{\mu}\gamma^{\nu}} R\sqrt{-g}\,d^4x, \tag{1}$$

where $\kappa = 16\pi G / c^4$ and γ^{μ}'s represent the components of a four-vector field. This vector field, as we shall see further on, is assumed to be related with the presence of a texture defect in space-time at some specific space-like three-dimensional hypersurface (or even single event), corresponding to what is usually called the "Big Bang". Introducing space isotropy and homogeneity we know that the line element must assume the Robertson Walker (RW) form

$$ds^2 = d\tau^2 - \frac{a(\tau)^2}{1-kr^2}dr^2 - a(\tau)^2 r^2\left(d\theta^2 + sin^2\theta d\varphi^2\right),$$

so that Eq.1 writes:

$$S = V\int_{\tau_1}^{\tau_2} e^{-\gamma^2}\left(a\ddot{a} + \dot{a}^2 + k\right)a\,d\tau\,. \tag{2}$$

Now V is a constant, k is the curvature parameter, and γ stays for the norm of the four-vector, which is in general a function of a; τ is the cosmic time (in units of length); the signature of the metric is $+---$.
An effective second order Lagrangian is immediately read out of S, $L = e^{-\gamma^2}\left(a\ddot{a} + \dot{a}^2 + k\right)a$; however it is convenient to integrate by parts the second derivative term in Eq.2 so that the action integral is transformed into:

$$S = S_0 + V\int_{\tau_1}^{\tau_2} e^{-\gamma^2}\left[\left(2\gamma\gamma' a - 1\right)\dot{a}^2 + k\right]a\,d\tau\,, \tag{3}$$

where S_0 is a "surface" term and primes denote derivatives with respect to a.
Let us introduce matter in the form of some homogeneous fluid, so that the total effective Lagrangian will be:

$$L = e^{-\gamma^2}\left[\left(2\gamma\gamma' a - 1\right)\dot{a}^2 + k\right]a + \kappa f a^3\,. \tag{4}$$

Now f is a function of a describing the mass-energy distribution of the fluid.
For the next steps it is convenient to introduce the Hamiltonian function of the system:

$$H = \dot{a}\frac{\partial L}{\partial \dot{a}} - L = e^{-\gamma^2}\left[\left(2\gamma\gamma' a - 1\right)\dot{a}^2 - k\right]a - \kappa f a^3\,. \tag{5}$$

This is a conserved quantity ($\partial H / \partial\tau = 0$, then $H = W$ = constant) so that from Eq.5 we write directly the evolution equation of the universe:

$$\dot{a} = \sqrt{\frac{\left(\kappa f a^3 + W\right)e^{\gamma^2} + ka}{a\left(2\gamma\gamma' a - 1\right)}}\,. \tag{6}$$

The + sign has directly been chosen since we know that we are dealing with an expansion rather than a contraction.
In the case of a space-time exempt of any defect we expect to recover the Friedmann-Robertson-Walker (FRW) solution, so letting $\gamma, W \to 0$ we should recover

$$\frac{\dot{a}^2}{a^2} = \frac{3}{2}\kappa\rho c^4 - \frac{k}{a^2},$$

being ρ the matter-energy density of the cosmic fluid. In practice we identify f with $-3\rho c^4/2$.
Let us then suppose that the cosmic fluid is made of a number of different non-interacting components, each with its own equation of state in the form $p_i = w_i\rho_i c^2$, where $w_i's$ are real positive numbers and p_i is the

partial pressure of the i-th component. The conservation laws for matter-energy and the principles of thermodynamics allow us to write the ρ_i's in terms of today's values (denoted with a 0 subscript), $\rho_i = \rho_{i0} a_0^{3(1+w_i)} / a^{3(1+w_i)}$, and Eq.6 is converted into:

$$\dot{a} = \sqrt{\frac{\left(W - \frac{3}{2}\kappa c^4 \sum_i \rho_{i0} \frac{a_0^{3(1+w_i)}}{a^{3w_i}}\right) e^{\gamma^2} + ka}{a(2\gamma\gamma' a - 1)}} . \tag{7}$$

2. Meaning of the vector field

The vector field, as it appears in the action integral of Eq.1 or Eq.3, is not a dynamical quantity. It is there because space-time contains a defect and no defect is deduced from variational principles. Can we say more and give some better interpretation of γ? Actually the idea we have in mind is inferred from the theory of defects in material continua. Without expounding here too many technical details, the essential may be grasped looking at (Fig1), where for simplicity a three-dimensional point defect is considered.

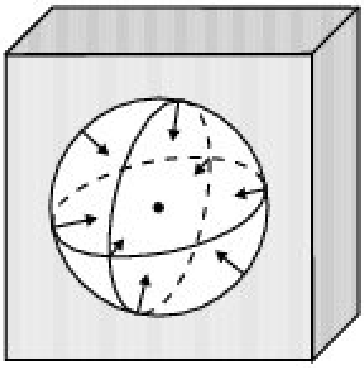

Fig. 1 A point defect in a three-dimensional continuum can be thought of as a spherical hole, being closed by pulling inwardly on the surface. This action induces a strain in the continuum, whose points are displaced radially towards the defect.

An intuitively simple vectorial quantity is represented by the displacement vector induced by the defect. Actually the situation is more complicated [13] and we need to consider the strain tensor. However we arrive again to a vector, by contracting at each point the strain tensor with the radial unit vector. Of course in the case of space-time we have to do with four rather than three dimensions and with a Lorentzian rather than Euclidean signature. In practice, for us "radial" means time-like and distances along the "radial" direction coincide with the cosmic time. Generally speaking, time-like geodesics in a defected space-time should be orthogonal to the geometrical locus of the defect (point, line, surface, hypersurface…). What matters here is that the situation can be described in terms of a vector field and the global symmetry tell us that the vector must be time-like and radial: in typical RW coordinates the only non-zero component of the vector is the time component and its value coincides with the norm of the vector (with a possible difference in sign): $\gamma^0 = \gamma$. Another important feature of the vector field is that it is divergence-free except in correspondence of the defect; the flow lines of γ cannot cross at any place, because this would imply the presence of one more defect at that place. This divergence condition is in fact a substitute for the dynamic equations for the vector field, and is formally $\gamma^\mu{}_{;\mu} = 0$ or $\left(\sqrt{-g}\gamma^\mu\right)_{,\mu} = 0$. The metric induced by the strained state in the given symmetry conditions is indeed the RW one, so that we immediately obtain:

$$\gamma = \gamma^0 = \frac{1}{a^3} . \tag{8}$$

The integration constant has been equaled to 1, properly scaling *a*.
We can now use the result in Eq.8 introducing it into Eq.7 and producing

$$\dot{a} = \sqrt{a^5 \frac{\left(\frac{3}{2} \kappa c^4 \sum_i \rho_{i0} \frac{a_0^{3(1+w_i)}}{a^{3w_i}} - W \right) e^{1/a^6} - ka}{6 + a^6}} . \tag{9}$$

We see that for $a \to \infty$ the expansion rate tends to zero: in all cases at the end the expansion asymptotically slows down to zero. On the contrary for $a \to 0$ the expansion rate diverges exponentially, which implies an initial inflationary behaviour.

It is easy and convenient to express the expansion rate as a function of the redshift parameter *z* rather than of *a*. It is known that the equation $a = a_0 / (1 + z)$ holds, so that

$$\dot{a} = \sqrt{a_0^5 (1+z) \frac{\left(\frac{3}{2} \kappa c^4 \sum_i \rho_{i0} (1+z)^{3w_i} a_0^3 - W \right) e^{(1+z)^6 / a_0^6} - k \frac{a_0}{(1+z)}}{6(1+z)^6 + a_0^6}} . \tag{10}$$

Eq.10 is what we need in order to start testing the theory.

3. The luminosity in an expanding universe

In order to verify the correspondence between our theory and the experiment we make reference to the observed luminosities of type Ia supernovae. These supernovae are usually thought of as being good standard candles, i.e. there are good reasons to think that their luminosity curve stays the same at all times. The total energy emitted during the collapse is indeed proportional to the quantity of “burnt” nickel, which is in turn proportional to the Chandrasekhar mass [15]. Chandrasekhar’s limit could vary at different cosmic times if the gravitational interaction (in practice Newton’s *G*) had to be replaced by an effective time depending value, as it is the case for a number of alternative theories. In standard theories, as well as for ΛCDM and CD, the gravitational coupling constant stays the same at all times and so does Chandrasekhar’s mass. In fact *G* is a measure of the strength of the coupling between geometry and matter and also for the CD theory this coupling per se is not affected locally by the presence of a defect; for small enough space volumes (as for instance the volume of a star) and short enough time spans (as the duration of a supernova explosion) the gravitational interaction happens in the same way at all times [16].

The luminosity of a celestial body from the viewpoint of a given observer, L_{obs}, is usually expressed in terms of luminosity distance d_l, whose implicit definition is in the formula $L_{obs} = 4\pi d_l^2 \Phi$, being Φ the energy flux density measured by the observer. At the cosmic scale one has to take into account both the redshift of the radiated energy and the time delay induced by the expansion, so that between the observed and the absolute luminosity of the source there is the relation $L_S = (1+z)^2 L_{obs}$. Furthermore in a RW universe the luminosity distance and the coordinated distance, r_S, between the source and the observer are related by $d_l = a_0 r_S (1+z)$; then the coordinated distance, expressed as the travel length of a light ray, is $r_S = \int_{\tau_S}^{\tau_o} \frac{d\tau}{a}$. Casting everything in terms of the redshift *z* one has [17]

$$d_l = (1+z) \int_0^z (1+z') \frac{da(z')}{\dot{a}(z')} = (1+z) \int_0^z \frac{dz'}{H(z')} . \tag{11}$$

The Hubble parameter $H = \dot{a}/a$ has explicitly been introduced.

The luminosity data are generally expressed in terms of magnitude *m* or, more conveniently, of distance modulus, i.e. as

$m - M_S = 25 + 5 \log d_l$,

which holds good if M_S is the absolute magnitude of the source and the distance is measured in Mpc. The luminosity distance of course depends on the theory one uses to describe the behaviour of the universe. We shall limit our analysis to an universe containing only dust ($w = 0$) and radiation ($w = 1/3$) and to the ΛCDM and CD theories. In the case of ΛCDM everything is expressed in terms of matter-energy density plus a cosmological constant, i.e. a homogeneous dark energy, in a spacely flat universe. In practice the expected distance modulus as a function of the *z* of the source is:

$$m - M_S = \mu + 5\,log(1+z) + 5\,log\left(\int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + 1 - \Omega_m}}\right) \qquad (12)$$

Both Ω_m (ratio between the matter-energy density and the critical density) and μ are parameters that can be used to optimize the fit of the experimental data.
Coming to the CD theory and introducing the flat space condition ($k = 0$), the distance modulus is [17]:

$$m - M_S = \mu + 5\,log(1+z) + 5\,log\left(\int_0^z \sqrt{\frac{e^{-(1+z')^6/a_0^6}\left[1 + 6(1+z')^6/a_0^{\,6}\right]}{(1+z')^3\left[1 + \varepsilon_0 (1+z') - b\right]}}\,dz'\right) \qquad (13)$$

Now ε_0 is the present ratio between the radiation energy density and the matter energy density, and in principle we are left with three adjustable parameters: μ, a_0 and b. The b parameter is related with the constant W and the present matter density ρ_{m0}, being

$$b = \frac{W}{\rho_{m0} c^4 a_0^{\,3}}.$$

In practice, considering that $\varepsilon_0 \approx 10^{-4}$ and limiting the analysis to a few units of z (as it is the case for SnIa's) we see that the term containing the redshift in the denominator of the integrand can be overlooked, so that the b parameter is incorporated into μ and the number of free parameters turns again to two as for ΛCDM.

4. Fitting the data

In order to compare the above mentioned theories with the observed luminosities, we make use of the same set of data used recently by Davis et al. [18] and incorporating supernovae analyzed in four different groups: 60 objects from the ESSENCE (Equation of State: SupErNovae trace Cosmic Expansion) project [19], 57 from SNLS (SuperNova Legacy Survey) [20], 45 nearby supernovae, 30 Sn's detected by the Hubble Space Telescope and qualified as "golden" supernovae by Riess et al. [21]. Altogether we use the luminosity data from 192 SnIa [22]; the highest redshift is z = 1.78. The fit is performed optimizing the free parameters in Eq.12 and Eq.13 and using a multidimensional nonlinear minimization by means of the MINUIT engine [23]; the open source routine we used, due to G. Allodi of the university of Parma, is named fminuit, is called from within MATLAB, and may be retrieved from ftp://ftp.fis.unipr.it/pub/matlab/fminuit.mex . The optimization is obtained minimizing the reduced χ^2 of the fit.

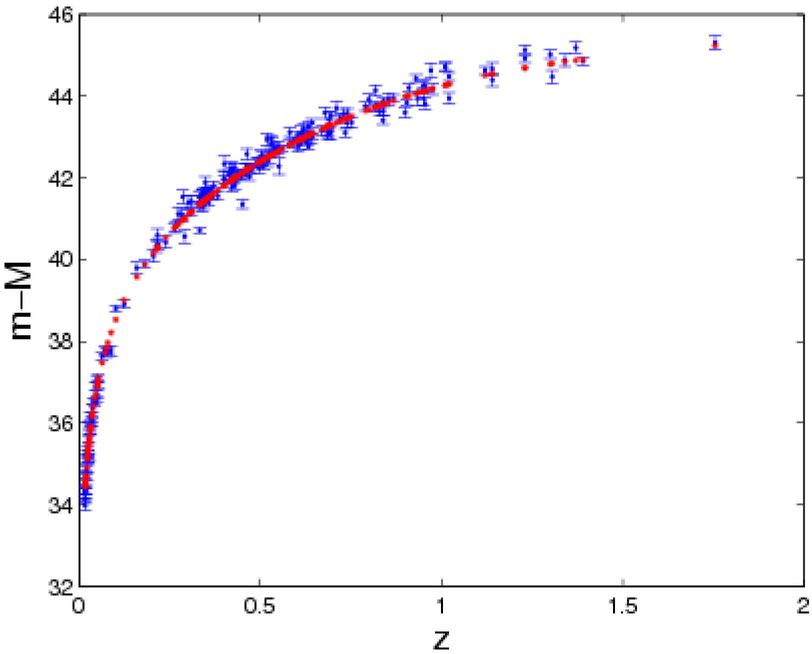

Fig. 2 Fit of the SnIa distance modulus data obtained using the CD theory. Theoretical predictions (dots) come from Eq.13. The experimental data are shown with their vertical error bars; the errors on the redshift z are unperceivable.

In the case of ΛCDM a known result is recovered. The best fit is obtained with μ = 43.30 ± 0.03, which corresponds to H_0 = (65.6 ± 0.9) km/s/Mpc, and Ω_m = 0.27 ± 0.03. As people know, in this theory 27% of the universe is matter (ordinary and dark), the rest is dark energy. The reduced chi square of the fit is χ^2 = 1.029. The fit obtained by means of the CD theory is shown in (Fig 2). The best fitting values of the parameters are μ = 46.26 ± 0.03 and a_0 = 1.79 ± 0.04; the reduced chi square is χ^2 = 1.092. The result is a bit worse than for ΛCDM but not dramatically so.

From the values obtained for the parameters it is possible to evaluate the Hubble constant, which is H_0 = (62.8 ± 1.7) km/s/Mpc, with a corresponding Hubble time of 15.6 Gyr. Also $\rho_{m0}(1-b) = (8.5 \pm 0.2) \times 10^{-27}$ kg/m^3 is obtained, which implies $b \sim 10$. An inconveniency is that computing the age of the universe directly from Eq.10 gives a poor (9.0 ± 0.2) Gyr. The latter unsatisfactory result is probably a signal of inadequacy of the theory at high z values.

5. Conclusions

We have fitted the apparent luminosity data from SnIa's with the values predicted by the ΛCDM and the CD theories. The result is of course not new in the case of ΛCDM, but we see now that also CD gives a fit comparable with the one of ΛCDM. Using the same data and the same number of parameters we obtained similar values of the reduced χ^2's suggesting the idea that CD also is a viable theory. It is however true that the apparently small difference between the reduced χ^2's of the fits corresponds to a rather big difference in the full χ^2 that, when analyzed on the light of statistical information criteria, such as the Akaike Information Criterion (AIC) [24] and the Bayesian Information Criterion (BIC) [25], enhances the distance between the two theories in favor of ΛCDM. It is however the case to remark that both reduced values of χ^2 are bigger than 1; furthermore the H_0 values obtained from observation using different methods are systematically higher than the ones of the 2-parameters best fits above. The most recent data from WMAP [26] yield $H_0 = \left(73.2^{+3.1}_{-3.2}\right)$km/s/Mpc which is consistent with a number of other results produced by different methods and indicators (like SnI, SnII, Cepheids in nearby galaxies, Sunyaev-Zeldovitch effect, X rays from clusters, gravitationally lensed systems) all quoted in [26]. The central values from these different observations range from 72 to 76 km/s/Mpc and in general the historical evolution of the estimated values of the Hubble constant seems to progressively converge towards something around 75 km/s/Mpc [27], which is ~ 15% more than the results got by means of the fits in this paper. If the "experimental" value of H_0 were used in the fits (so reduced to 1-parameter ones) the agreement with the data would consistently worsen both for ΛCDM and for CD. In practice there is something missing beyond the details of the theories and their interpretation, which deserves investigation and insight.

The ΛCDM is indeed different from the CD theory: the former assumes in the universe the presence of a cosmological constant corresponding to a sort of uniformly and homogeneously distributed dark energy; the latter interprets space-time as a continuum with a cosmic defect inducing a strained state containing both the symmetry and the non-uniform expansion rate. Besides this, we know that ΛCDM requires also that the matter content in the universe be one order of magnitude bigger than what expected from baryonic particles only. In the case of the CD theory, instead, we saw that the ordinary matter density is combined with the effect induced by the defect via the b parameter, so that, in a sense, it gives rise to an effective matter/energy density one order of magnitude bigger than the actual one. Adding the fact that one can interpret the strained state induced by the cosmic defect as being the equivalent of a non-uniform (in time) dark energy, we see that in fact the principle difference between ΛCDM and CD could not be that deep. However the CD theory produces, somehow unexpectedly, one additional result, which is an inflationary phase in the initial life of the universe, with no need for an ad hoc field [13]. This is not the case of the ΛCDM theory. The latter is of course mathematically simple and practically working, but it is not that simple on the side of the interpretation of what Λ actually is; furthermore it apparently implies a never-ending acceleration of the expansion. Our theory instead leads back to a final decelerated phase, which we think is a good feature.

On the formal side we may also remark that CD already proved to correspond to vector theories developed with different motivations and within a different scenario [28]. Of course there are many observational facts against which to test the theory. We have started with the most known and considered one, SnIa luminosity, with no pretence that this is the end of the story. In this test the range of z values is limited, and the poor result obtained for the age of the universe seems to indicate an inadequacy of the theory at high redshift values, where probably a better treatment of the matter content is in order. The exponential factor appearing in the Lagrangian for space-time in Eq.4, together with the inverse cube dependence of the norm of the four-vector consequent to the null divergence condition (Eq.8) produce an inflationary expansion, which however extends too much in cosmic time. On the other side the same picture (defect in a four-dimensional continuum) can be preserved with other choices of the action integral; for instance directly combining the Ricci tensor with an appropriate strain tensor, induced by the defect, without resorting to any exponentiation. The work is in progress; the result with the type Ia supernovae, summed with the other features of the theory, is anyway encouraging.

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