

# On the relativistic theory of gravitation and exact solutions for the interior of black holes

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## Abstract:

Arguments are given to abandon the general theory of relativity, e.g. the general need of a gravitational energy-momentum tensor and the infinite gravitational energy related with the horizon of the Schwarzschild metric. It is proposed to adopt the relativistic theory of gravitation with a positive cosmological constant. This theory brings new effects when the gravitational field is large, that is for black holes and for cosmology. The Schwarzschild metric gets deformed close to the would-be horizon. Modeling the interior by an equation of state, an exact solution for the interior and horizon metric is found. Starting from a scalar field theory for H-atoms in curved space, a Bose-Einstein condensation can take place; the metric can be solved self-consistently. For reasonable parameters this may describe supermassive black holes. The mass of the constituent H atoms does not uniquely fix the BH mass.

For flat space cosmology the  $\Lambda$ CDM model is covered, including a positive curvature-type term. There appears a form of inflation at early times. Both the total energy and the zero point energy vanish.

## 1. Introduction: the importance of a gravitational energy-momentum tensor

When Einstein started his search for the general theory of relativity (GTR) he wanted to improve the scalar field approach of Maxwell, since that led to the problem of negative masses, never observed. In his letters Einstein stressed the importance of having an energy-momentum tensor for the gravitational field itself (gEMT). He got stuck on this approach and, after interaction with Marcell Grossmann, continued with the Riemann approach. In doing so, he changed position: a gEMT was not demanded anymore and the Einstein equations were derived. Still he felt uncomfortable about the physics of his approach and later proposed a gEMT, the Einstein pseudotensor.

It was stressed by e.g. Guido Pizzella that the measurement of gravitational waves must rely on the exchange of energy between wave and detector. If one has to choose between either describing gravitation by a curved space or having an energy-momentum tensor (EMT) for gravitation itself, then this physical argument forces one to choose the second option: the gEMT is needed for consistency. The Noether theorem then demands that the gravitation is a field in Minkowski space – just like and next to Maxwell's electromagnetism. Anyhow, since the “linear” gEMT is well accepted for gravitational radiation by the Hulse-Taylor binary, a gEMT should exist also for strong gravitational fields, again for consistency. It should also be local, we come to this point in section 3.

The idea that the embedding space is just the flat Minkowski space is considered with skepticism by many. We consider this as a psychological issue. We have met senior researchers of undisputed reputation who admitted not to have understood the physics of GTR. The same people have considerably less trouble to accept electromagnetism or the standard model of elementary particles.

It is therefore proper to turn the question around: What are the hard arguments for taking Riemann space as fundamental space? Actually, we believe there are none. For stars and galaxies, only weak gravitation is needed, so Minkowski space is a good start. For light bending by them the role of the gravitational field is no different than the role of the material fields in glasses: a change in the dielectric constant and perhaps magnetic permeability – indicating another direct analogy between effects of matter and gravitation. During decades, and for good reasons, the belief has existed that cosmology should be described by a curved space. But modern experiments show that curvature is zero within the error bars.

We take these points as a good motivation to continue just with Maxwell's assumption: gravitation is a field in Minkowski space, next to electromagnetism and the weak and strong forces. This view, considering Minkowski space as primary space and Riemann space as an effective space, may not be so unnatural after all. An analogy exists in the problem of light transport through (cosmic) clouds, where the optical distance is a non-Minkowskian distance measure. It can be useful in certain calculations and arguments, with the

underlying distance between two given points still being the Minkowskian, that was there already before the cloud appeared. In this view the Riemann metric gives an effective distance between two given points in the presence of gravitation. The question: “In which space is has the Universe been inflating” is then no different from the question: “In which space has the Crab Nebula been inflating?”, with the same answer: a Minkowski space. The difficulty to determine distances in the Minkowskian embedding space, given the distance in Riemann space, is then in principle not different from the problem to determine the distance between two points that are separated by one optical unit: In the first case, the answer depends on the local gravitational field, in the second case on the humidity of the cloud. On the quantum level, a Minkowski space will induce no space-time foam, see the standard model of elementary particles.[3]

## **2. Derivation of the gEMT: the LLBG tensor**

The idea that gravitation is embedded in a Minkowski space goes back to “EPR” Nathan Rosen in 1940. [1] The formulation of gravitation as a field theory in Minkowski space-time has been worked by now: a field theoretic description for gravitation exists for which Minkowski space is the embedding space. This setup allows derive the proper gravitational energy-momentum tensor via the Noether theorem, a task was carried out by Babak and Grischchuk in 1999. [2] The result coincides in Cartesian coordinates with the Landau-Lifshitz pseudotensor, while it is also well behaved in arbitrary coordinates, hence we propose the name LLBG tensor. It is a true tensor in Minkowski space. The Einstein equations may now be written in the Minkowskian form: Acceleration tensor  $\propto$  Total EMT [3].

## **3. The LLBG tensor of the Schwarzschild black hole and its consequences**

We have applied the LLBG tensor to the Schwarzschild black hole. [4] The result has a quadratic singularity at the horizon, which cannot be integrated: The out-of-the-origin gravitational energy is minus infinity so there is a true, physical singularity, even though it is masked in the Riemann description. In our view, this means that one has to discard the Schwarzschild BH and even the theory that produces it, GTR. This avoids another uncomfortable problem we mentioned already: in GTR the (gravitational) energy is not localized. Instead of accepting this as a property of Nature, we see this as another reason to abandon GTR.

## **4. Logunov's Relativistic Theory of Gravitation (RTG) and RTG+**

To proceed we have adopted Logunov's theory named Relativistic Theory of Gravitation (RTG).[5] It has the Hilbert-Einstein action, with additional bimetric term consisting of the contraction of the product of the Riemann and Minkowski metrics. Historically, this term has been chosen to impose the harmonic gauge. Seen as a field theory in Minkowski space, this implies the Lorentz gauge for the gravitational field. If in this theory the bimetric coupling equals the cosmological constant, then it allows Minkowski space as an empty space solution. For weak field gravitational effects, such as light bending or perihelion shifts, the new term leads to negligibly small effects. It does, however, force one to work within a specific gauge, the harmonic gauge. Actually, this approach makes the harmonic gauge well defined in arbitrary coordinates.

Whereas Logunov et al consider a negative cosmological constant, we propose to study the case with a positive one. The implied tachyonic nature of the graviton only shows up at the Hubble scale, where clearly single gravitons are irrelevant. Let us call this theory RTG+.

The bimetric term breaks the equivalence principle. With its strength being of Hubble size, its effects can only be relevant in cosmology or for very large gravitational fields. Indeed, though there appears a scalar component of the gravitational field, this does not radiate. [6] The argument is neat: in GTR a scalar mode is absent in the radiation, since is a gauge artifact. In RTG, being essentially the same in the harmonic gauge, it is then also absent. [4] However, role of the constraint is still strong in those cases, since prevents most of the gauge freedom of GTR. Within the class of spherically symmetric solutions, we have verified that the residual gauge group is actually empty: gauge transformations within this sector would lead to infinite energy, so we discard them. This makes the energy density a bona fide local property, at least within this spherically symmetric, static sector, and hopefully within the whole RTG+.

### 3. Black holes with asymptotic Schwarzschild metric in RTG+

In RTG+ one can reconsider the Schwarzschild BH. Far enough away from matter the Schwarzschild metric still holds. Near the horizon there appears a deformation, leading to a redshift of order  $10^{23} M_{\text{sun}}/M$ . So a true horizon is absent, and time keeps its role into the interior. If the interior also behaves properly, then, like for stars, Hawking radiation does not occur and Bekenstein-Hawking entropy has no bearing. [4]

#### *Equation of state approach*

To describe the interior, an explicit equation of state has to be considered. The idea to have all matter at or very near the center is inconsistent. Matter has just to be spread out within the horizon, as usual. For a combination of the vacuum and the stiff equation of state we have derived an explicit shape of the internal metric. [4] The infinite density of the Schwarzschild metric gets regularized, and the energy is  $Mc^2$ .

#### *Field theory in curved space approach*

It is easy to verify that a BH, in which matter consists of H atoms (mass:  $m$ ) smeared out homogeneously within the horizon, and in which the density takes the Bohr value (mutual distance of H atoms equal to the Bohr radius), is supermassive, with mass  $M_c$  of  $2 \times 10^8$  solar masses.

We have started from a quantum field theory for H atoms (scalar field, rotating wave approximation).[7,4] We have self-consistently solved this problem almost analytically by assuming that the H-atoms sit in a Bose-Einstein condensed ground state.

For such a field theory in curved space, it is well known that renormalization generates a coupling  $\xi R\phi^2$  between the Riemann curvature and the scalar field. In our approach we take  $\xi$  as a phenomenological parameter. Consistency requires the large value  $10^{58} M^2 / M_c^2$  implying that  $\xi R$  is of order  $m^2$ .

It was realized that the zero point energy of the constituents atoms,  $Nmc^2$ , can be much larger than the energy  $Mc^2$  seen from infinity, or, equivalently, that the binding energy can be any fraction, even near 100%, of  $Nmc^2$ . [3] Thus in RTG+ the Schwarzschild-type BH formed of Bose-Einstein condensed H atoms is described not only by its mass at infinity and but also by the total mass of its constituents.

Physically such might have been anticipated, since gravitational radiation due to external perturbations can increase the BH binding energy by changing the metric in which the condensate is embedded.

### 4. Cosmology in RTG+

For the flat-space Friedmann model in RTG the total energy density vanishes since the gravitational part cancels the material part. Technically speaking this occurs because this condition coincides with the Friedmann equation. [7] The zero point energy vanishes by Logunov's choice for the bimetric constant. [5] The bimetric term brings two extra terms in the Friedmann equation.[5,3] In RTG+ the first is a positive curvature-type term: If the empirical curvature is non-zero, then RTG+ predicts that its value is positive. Still, space is Minkowskian. The second term is relevant at very early times, and allows an arbitrarily small initial scale factor. The positive cosmological constant brings exponentially large values at late times.

Because in the LLBG tensor the material EMT gets multiplied by  $-g$ , the material energy in a coordinate element  $d^3x$  becomes  $\rho a^6 d^3x$ . The extra factor  $a^3$  poses another problem for GTR, at least in its field theoretic description. However, in RTG there is a difference between cosmic time and conformal (cosmic) time, and there exists [7] a natural interpretation for the cosmic material energy in a volume element  $d^3x$ .

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