

Effects of cosmology on solar and stellar systems

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Abstract: The gravitational action of either dark energy or a smooth matter component filling in the universe can affect the orbits in a stellar system. Changes are related to the acceleration of the cosmological scale size. Actual bounds on the cosmological deceleration parameters from accurate astrometric data fall short of ten orders of magnitude with respect to estimates from observational cosmology but future radio-ranging measurements of outer planets in the Solar system could improve actual bounds by five orders of magnitude. Limits on either local dark matter or deviations from the gravitational Newton's law can be derived as well.

1. Introduction

The local effect on particle dynamics from the global expansion of the universe is a classical topic in general relativity. The picture is pretty clear for a test particle moving in the homogeneous and isotropic Robertson-Walker metric [1, and references therein]. A general result is that its peculiar velocity with respect to the Hubble flow declines in magnitude as the scale factor of the universe, R , expands, alike to any photon energy. However, this characteristic does not assure that all of the features of a generic trajectory in the expanding space will approach those of a trajectory in the Hubble flow.

Matter inhomogeneities complicate the picture. As far as gravitationally bound systems are concerned, we have to consider the gravitational effect of the (expanding) energy-matter fluid wherein the test particles move. It should be not the expansion of the universe that causes gravitational bound systems to separate or evolve. The effect, if any, should stem from the gravitational action of the homogeneous background in which the system is embedded.

The two main candidates to fill the universe are the cosmological constant and a dark matter background. In what follows we will discuss the effects of such components on a gravitationally bound system and how the expansion of the universe affects the dynamics of local systems. This is achieved by considering the two-body problem in a cosmological background.

2. The cosmological constant

The understanding of the cosmological constant Λ is one of the most outstanding topic in theoretical physics. On the observational side, the cosmological constant is motivated only by large scale structure observations as a possible choice for the dark energy. In fact, when fixed to the very small value of $\sim 10^{-46} \text{ km}^{-2}$, Λ , together with dark matter, can explain the whole bulk of evidence from cosmological investigations. In principle, the cosmological constant should take part in phenomena on every physical scale but due to its very small size, a local independent detection of its existence is still lacking. Measuring local effects of Λ would be a fundamental confirmation and would shed light on its still debated nature, so it is worthwhile to investigate Λ at any level.

The influence of the cosmological constant on the gravitational equations of motion of bodies with arbitrary masses can be discussed with a perturbation approach and eventually the two-body problem can be solved [2, and references therein]. Due to the anti-gravity effect of the cosmological constant, the barycenter of the system drifts away. The relative motion is like that of a test particle in a Schwarzschild-de Sitter space-time with a source mass equal to the reduced mass of the two-body system. Hence, we can use this last metric to consider local effects of Λ .

The main effect of the cosmological constant on a bound gravitational system is the precession of the pericenter on the orbital motion. We determined observational limits on the cosmological constant from measurements of the periastron advance in stellar systems, in particular binary pulsars and the solar system [2]. Based on accurate planetary ephemerides properly accounting for the quadrupole moment of the Sun and for major asteroids, the best constraint comes from Mars and Earth, $\Lambda \leq 1.2 \times 10^{-36} \text{ km}^{-2}$. Due to the experimental accuracy, observational limits on Λ from binary pulsars are still not competitive with results from interplanetary measurements in the solar system. Accurate pericenter advance measurements in wide systems

with orbital periods ≥ 600 days could give an upper bound of $\Lambda \leq 10^{-34}-10^{-33} \text{ km}^{-2}$, if determined with the accuracy performed for B1913+16. For some binary pulsars, observations with an accuracy comparable to that achieved in the solar system could allow to get an upper limit on Λ as precise as one obtains from Mars data.

The effect of Λ on the precession of a gyroscope, the change in the mean motion of a massive body and the gravitational redshift can be analyzed as well in the framework of the Schwarzschild-de Sitter metric [3]. As it could be expected from a dimensional argument, relative variations due to Λ always goes as $\sim \Lambda (a^3/r_g) (a/r_g)^i$, $i=\{0,1,\dots\}$, with a the typical physical length of the system and $r_g = GM/c^2$ the typical gravitational radius of the massive source. An analysis of anomalies in the mean motion provides limits at the same order of magnitude as those from perihelion precession. On the other hand, measurements of gyroscope precession of the Moon, via laser ranges, or of satellite, such as the Gravity Probe B mission, fall short in constraining Λ . Beyond the solar system, limits competitive with Earth precession data could come from gravitational redshift measurements in white dwarfs.

The bound on Λ from Earth or Mars perihelion shift is nearly $\sim 10^{10}$ times weaker than the determination from observational cosmology but it still gets some relevance. The cosmological constant might be the non perturbative trace of some quantum gravity aspect in the low energy limit. Λ is usually related to the vacuum energy density, whose properties depends on the scale at which it is probed. So that, in our opinion, it is still interesting to probe Λ on local scales. In fact, these tests can probe the universal origin of the cosmological constant on very different scales. Any detection of perturbations in the orbital motion in a bound gravitational system, either the solar system or a binary pulsar, probes Λ on a scale of the order of the astronomical unit. On the other hand, the relevant length scale in measurements of gravitational redshift is the distance to the source, which is of order of 10^2 pc for galactic white dwarfs. The experiments we have considered cover a range in distance of nearly seven orders of magnitude, which help in filling the gap between local systems and the cosmological distances.

3. Dark matter

Accurate planetary astrometric data can be also used to study either dark matter or modifications of the Newtonian inverse-square law in the solar system [4, and references therein]. The gravitational inverse-square law and its relativistic generalization have passed significant tests on very different length- and time-scales. Precision tests from laboratory and from measurements in the solar system and binary pulsars provide a quite impressive body of evidence, considering the extrapolation from the empirical basis. First incongruences seem to show up only on galactic scales with the observed discrepancy between the Newtonian dynamical mass and the directly observable luminous mass and they are still in order for even larger gravitational systems.

Two obvious explanations have been proposed: either large quantities of unseen dark matter dominate the dynamics of large systems or gravity is not described by Newtonian theory on every scale. In the dark matter scenario, the Milky Way is supposed to be embedded in a massive dark halo. Realistic models of the Milky Way based on adiabatic compression of cold DM haloes can be built in agreement with a full range of observational constraints. The local DM density at the solar circle is then expected to be $\rho_{\text{DM}} \sim 0.2 \times 10^{-21} \text{ kg/m}^3$, in excess of nearly five orders of magnitude with respect to the mean cosmological dark matter density. Best constraints on the local ρ_{DM} come from perihelion precession of Earth and Mars, with similar results from modifications of the third Kepler's law. The upper bound on the local dark matter density, $\rho_{\text{DM}} \leq 3 \times 10^{-16} \text{ kg/m}^3$ at the 2- σ confidence level, falls short to estimates from Galactic dynamics by six orders of magnitude.

High precision solar system tests can provide model independent constraints on possible modifications of Newtonian gravity. The solar system is the larger one with very well known mass distribution and can offer tight confirmations of Newtonian gravity and general relativity. Any deviation emerging from classical tests would give unique information either on dark matter and its supposed existence or on the nature of the deviation from the inverse-square law. Variations in the $1/r^2$ behavior can be considered in several forms. Up to scales of 10^{11} m , scale-dependent deviations in the gravitational acceleration are really small. We examined the MOND interpolating function μ in the regime of strong gravity. A large class of MOND interpolating function is excluded by data in the regime of strong gravity. The onset of the asymptotic $1/r$ acceleration should occur quite sharply at the edge of the solar system, excluding the more gradually varying μ suggested by fits of rotation curves. On the other hand, the standard MOND interpolating function $\mu(x) = x/(1+x^2)^{1/2}$ is still in place.

Studies on planetary orbits should be complemented with independent observations in the solar system. In combination with constraints from galactic rotation curves and theoretical considerations on the external field effect, the absence of any significant deviation from inverse square attraction in the solar system makes the range of acceptable interpolating functions significantly narrow. Future radio ranging observations of outer planets with an accuracy of few tenths of a meter could either give positive evidence of dark matter or disprove modifications of gravity.

4. Cosmological expansion

Cosmological background can affect the orbit of a planetary system through the gravitational action of the smooth energy-matter components in which the system is embedded [1, and references therein]. Changes in the mean motion and anomalous periastron precession are sensitive to the acceleration of the scale factor, R''/R , whereas variations in the semi-major axis and eccentricity come from its time variation. In principle, a very detailed knowledge of the orbital parameters of a bound gravitational system would allow to put constraints on the cosmological parameters through two diagnostics, i.e. qH^2 and $d(qH^2)/dz$, that are independent of the main ways in which observational cosmology usually investigate the energy content of the universe, i.e. via either the distance-redshift relation or the growth of structure.

In a universe with significant dark matter, a gravitational system expands or contracts according to the amount and equation of state of the dark energy. At present time, the Solar system, according to the Λ CDM scenario emerging from observational cosmology, should be expanding if we consider only the effect of the cosmological background. Its fate is determined by the equation of state of the dark energy alone.

Actual bounds on the cosmological deceleration parameters q_0 from accurate astrometric data of perihelion precession and changes in the third Kepler's law in the Solar system fall short of ten orders of magnitude with respect to estimates from observational cosmology. Even if we consider other tests of gravity physics, such as geodetic precession and gravitational redshift, the situation does not improve substantially. Constraining the properties of the dark energy in local systems can be appealing on a theoretical side with its promise in breaking degeneracies which affect the other methods, but it seems an unlikely tool if we consider the observational prospects.

5. Conclusions

The paradigm of cold DM when complemented with a positive cosmological constant (the so called Λ CDM scenario) is successful in explaining the whole range of galactic and extra-galactic body of evidence, from flat rotation curves in spiral galaxies to large scale structure formation and evolution. Local dynamics can support the paradigm with independent and well-understood data.

Measurements of periastron shift should be much better in the next years. New data from space-missions should get a very high accuracy. Near-future technology should allow to improve bounds on cosmological parameters by nearly five orders of magnitude, the crucial step being radio ranging observations of solar system outer planets. Beyond the solar system, together with future measurements of periastron advance in wide binary pulsars, gravitational redshift of white dwarfs could provide bounds competitive with Earth and Mars data. A proper consideration of the gravito-magnetic effect in these analyses could also play a central role to improve the limit on Λ by several orders of magnitude. Such improvements could be significant but still not competitive with observational cosmology.

Apart from the technological challenge required to reach the experimental accuracy needed to study cosmology in the Solar system, two other issues should be considered. As a matter of fact, planetary systems are embedded in galactic dark matter haloes whose density is in excess of several orders of magnitude with respect to the mean cosmological dark matter. Furthermore, we have considered an homogeneous background but tidal forces from the environment could easily play a role. These two effects on the galactic scale should easily overcome the effects of the acceleration of the cosmological scale factor. Any future observed deviation in the measured values of perihelion precession or in the third Kepler's law should then be seen as an indication for Galactic dark matter rather than an effect of an accelerated expanding universe.

Debate between dark matter and departures from inverse-square law is still open. Considering both theoretical and observational aspects, dark matter seems to be slightly preferred. If on a galactic scale the two hypotheses match, on the cosmological side only DM can give a consistent framework. This might shortly change with the steady improvements in relativistic generalization of the MONDian paradigm. So, in our

opinion, it is of interest to examine results on a very different scale, that of the solar system. Solar system data have been confirming predictions from the general theory of relativity without any need for dark matter and it is usually assumed that deviations can show up only on a larger scale. We have explored what we can learn from orbital motion of major planets in the solar system. Results are still non-conclusive but nevertheless interesting. As a matter of fact, fits to galactic rotation curves, theoretical considerations on the external field effects and solar system data could determine the shape of the interpolating function with a good accuracy on a pretty large intermediate range between the deep Newtonian and MONDian asymptotic behaviors.

Future experiments performing radio ranging observations of outer planets could greatly improve our knowledge about gravity in the regime of large accelerations. The presence of dark matter could be detected with a viable accuracy of few tenths of a meter on the measurements of the orbits of Neptune or Pluto, whereas an uncertainty as large as hundreds of meters would be enough to disprove some pretty popular MOND interpolating functions.

Acknowledgments

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