A Brief History of Large Scale Structures: from the 2D Sky to the 3D Maps

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Abstract: Key questions of the large scale structure studies are reviewed from a historical perspective. These include the first discoveries of very large structures of galaxies in the sky, the importance of distortion and projection effects, the slopes of complete and reduced correlation functions, and the spatial scale of homogeneity.

1. First discoveries of very large structures of galaxies in the sky

In his 1919 doctoral dissertation, Knut Lundmark, a pioneer in extragalactic studies in the era of The Great Debate, had expressed his conviction: "We can certainly expect to find a very complicated structure in the doubtlessly gigantic universal system, which is formed by the spiral nebulae." This prophecy was fulfilled: a major discovery of 20th century astronomy was the complex spatial distribution of galaxies, organized into superclusters, filaments, and voids. Here I review briefly a few central items in this development. Historical and technical details, and additional references, may be found e.g. in the books [1,2,3,4] and the review [5].

The telescope made us aware of nebulae. Evidently the nebulae were at different distances, but one had no means of measuring these. The ancients recognized various constellations, which are not real groupings of stars, but just nice projections on the sky. But there are also genuine stellar groups. Something similar took place with the nebulae. The comet hunter Charles Messier found a concentration of them in the constellation Virgo. William Herschel found a collection of many hundreds of nebulae in Coma Berenices and several other groupings. Then his son John obtained hints for the existence of what is now called the Local Supercluster, the Virgo concentration "being regarded as the main body of this system" and the Milky Way "placed somewhat beyond the borders of its densest portion". Photographic surveys of the sky from the early years of the 20th century revealed thousands of new nebulae, and later, in modern terminology, galaxies. It became clear that a band of nebulae goes through Virgo and is perpendicular to the nebula poor Milky Way.

In 1927 Lundmark plotted 55 clusters on the celestial sphere in a study in which he also recognized that galaxies are often found in pairs. The chart hinted that the clusters themselves may be clustered. Were there genuine clusters of clusters (superclusters)? It took decades of arguings before this issue was resolved. Already when galaxy catalogues gave just positions on the sky vault, and no distances, astronomers started to recognize superclusters of galaxies:

- Shapley's metagalactic clouds
- De Vaucouleurs: the Local Supercluster & the density-radius relation
- Abell's rich clusters and their superclusters
- Shane-Wirtanen clouds of galaxies from the Lick counts

Harlow Shapley initiated wide photographic surveys of galaxies [6]. Inspecting the distribution of galaxy clusters, he came to the conclusion that there are "metagalactic clouds", e.g. in Coma, Centaurus (Shapley's supercluster) and Hercules. Mainly through the efforts by de Vaucouleurs, the existence of the Local Supercluster became established in the 1950s. He presented evidence, from the Shapley-Ames catalog, for a flattened system centered at the Virgo cluster, and having an overall diameter of 30 Mpc. It causes the asymmetry in the numbers of bright galaxies in the two hemispheres.

An enormous increase in the number of known galaxy swarms came with George Abell's catalog of 2712 rich clusters of galaxies, published in 1958. It was one outcome of the important photographic survey of the northern sky, made by the 48 inch "Big Schmidt" at Palomar Observatory. Now the question was: Do Abell's rich clusters form superclusters? Abell concluded that his clusters were themselves clustered.

Not all astronomers were satisfied with the apparent clumping of clusters on the sky. Some warned about the dust (already in the 1940s Victor Ambartsumian made studies on how the cloudy interstellar dust would influence the apparent distribution of galaxies). When Neyman & Scott [7] introduced their statistical clustering model, in 1952, they pointed out two explanations of the observed clustering: 1) The actual spatial distribution of galaxies is clustered, or 2) the clustering is only apparent, caused by the extinction by dust clouds. Both factors were likely to play a role. But which factor dominates in superclustering?

Erik Holmberg, one of the pupils of Knut Lundmark, was worried about the role the dust may play. In 1974 he showed evidence that the regions of the sky containing many clusters are more transparent (less dusty) than the sky in general, so that "the random distribution of the galaxy clusters has thus been proved in an indirect way"'. Indeed, at the time the issue of superclusters could not yet be studied directly, by working in 3D space. Now that we know superclusters really exist, the worries about fluctuating extinction may sound historical curiosities. In the specific issue of superclusters this explanation, although reasonable and worth probing, was destined to fail. But there is no reason to ignore the tiny grains of dust altogether. Even though superclusters exist, Holmberg's result – more clusters in transparent regions – remains valid, and we should not forget that our lines of sight reach the galaxies and quasars through the patchy, dusty window.

In the Santa Barbara 1961 meeting Neyman, Page & Scott stated in their conference summary that "Abell and de Vaucouleurs feel that superclustering is established beyond doubt, and that the dimensions of 2^{nd} –order clusters ... are 30 to 60 Mpc. This means that clusters cannot be treated as isolated systems embedded in an isotropic homogeneous medium of field galaxies". But the doubts continued about the reality of the very large structures during this "2D-era", when the use of redshift as a distance indicator for massive galaxy samples was not yet actual.

2. The shift from two to three dimensions

In his *The Inner Metagalaxy* from 1957 Shapley [6] summarized the work on the clustering of galaxies made at the Harvard observatory. Discussing deep surveys he writes: "The distribution of galaxies on the surface of the sky is easily examined on any uniform collection of long-exposure photographs." However, a study of the distribution in the line of sight "is complicated by the difficulties of nebular photometry as well as by uncertainties introduced through the considerable dispersion in the intrinsic luminosities of galaxies. Systems side by side in space can differ by five or more magnitudes in apparent brightness, as for example the Andromeda nebula and its companions; and a pair with equal apparent brightness may differ in distance by a factor of ten. ... In the study of the radial distribution of population it is necessary to use photometric methods for estimating distances, relative or absolute."

A modern reader in the middle of large redshift catalogs is stricken by the fact that nowhere is the redshift mentioned as a possible distance indicator. But at that time the local universe was not yet a subject of redshift surveys, the precious telescope time went to extending redshift measurements to deeper space. After the 1^{st} and 2^{nd} Reference Catalogues by de Vaucouleurs and collaborators were published in 1964 and 1976, with their compilations of hundreds and thousands of redshifts, many people started experimenting with the redshift in order to see the galaxy distribution in the radial direction.

For years, astronomers could make only indirect conclusions about the distribution of galaxies on the basis of their locations on the celestial sphere. The situation was completely changed when it became possible to measure the 3D distribution of galaxies using data from surveys of galaxy redshifts.In 1977, there was the landmark event, the Tallinn IAU symposium on Large Scale Structure of the Universe, which heralded the real shift from 2- to 3-dimensional investigations. In fact, the application of the Hubble law as a distance indicator for galaxies with redshifts from de Vaucouleur's 2^{nd} Reference Catalog led to Mikhel Joeveer's and Jaan Einasto's break-through work [8] and surprising results: galaxies are arranged in filaments surrounding empty voids. They interpreted the discovered pattern in terms of a cell structure, with the mean dameter of voids about 50 Mpc. In Tallinn large structures of various sizes were also demonstrated e.g. by de Vaucouleurs, Tully, Abell and Tifft.

It was necessary to produce well-planned redshift surveys, uniformly covering large regions of the sky, in order to check the Estonians' findings. The first such project was carried out at the Harvard-Smithsonian Center for Astrophysics (CfA), requiring redshifts for 1900 new galaxies. In 1986 V. Lapparant, M. Geller, and J. Huchra confirmed the existence of shell-like galaxy clustering and found still more variety in the realm of galaxies [9]. Their map "A Slice of the Universe" became a symbol of the complexity of the distribution of galaxies in space (Fig.1).

Inspired by these early results, several extensive redshift surveys have been performed for the new 3D cartography of the universe. The technique of multi-object spectrographs is used to measure redshifts of many galaxies simultaneously. During the last two decades many extensive redshift surveys have been completed, among them what are known by the abbreviations SSRS, LCRS, ESP and 2dF. The two-degree field survey 2dF has 220 000 galaxies measured, while the wide-angle (Sloan) SDSS survey is measuring one million redshifts over a quarter of the sky, with a median depth of the survey $300/h_{100}$ Mpc The special telescope at Apache Point Observatory in New Mexico is not a giant (its mirror is 2.5 m in size), but its state-of-the-art spectrograph can measure the redshifts of over 600 galaxies in a single observation.

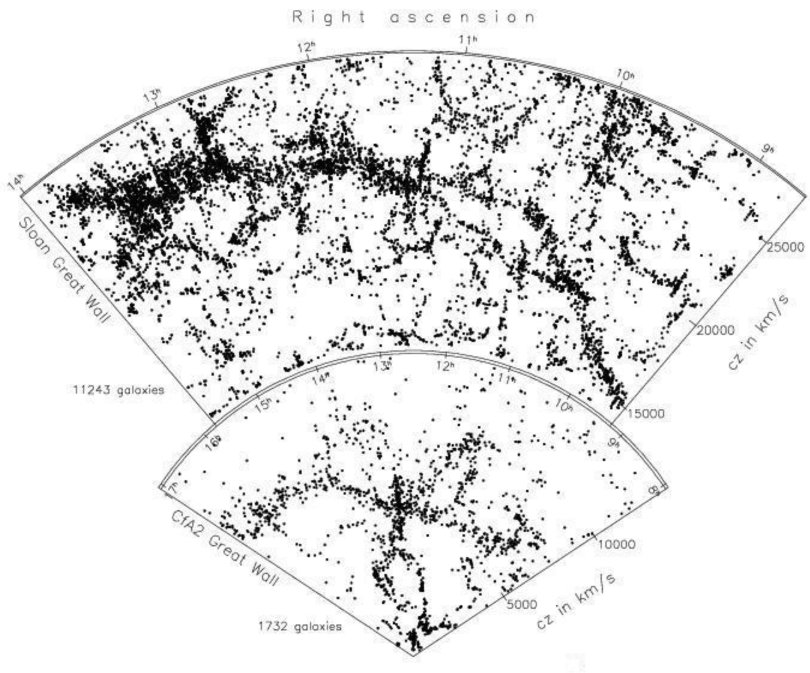

Fig.1 A comparison of the CfA map (below) and the slice from the larger SDSS survey (above). The Great Wall of CfA and the Sloan Great Wall extend across these maps. (Courtesy of J. Richard Gott III and Mario Juric).

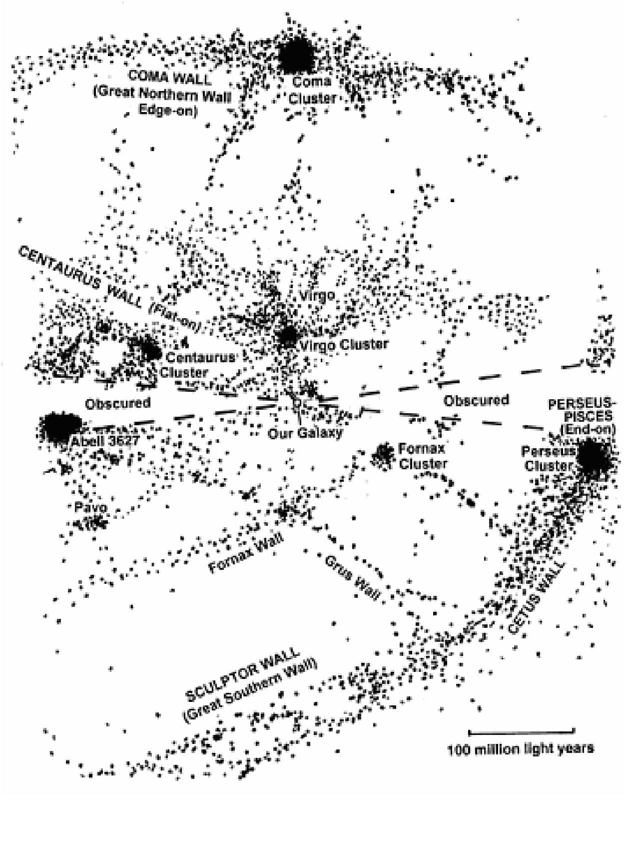

Fig.2 A sketch of the local galaxy universe within about 100 Mpc with some structures named (Courtesy of Anthony Fairall & Praxis Publishing Ltd, Chichester, UK.)

One may say that after Tallinn, and especially after the subsequent CfA redshift project, the large-scale galaxy distribution became also a visually fascinating topic, along with being a subject of important, but dry (though interesting!) statistical analysis. We became really aware of the strange new realm of galaxies, the beautiful structures of which superseded any previous expectations (Fig.2)

3. The slopes from reduced (ξ) and complete (Γ) correlation functions

Both the 2D angular maps and the 3D spatial (redshift) maps have been extensively analyzed during the last decades. The main tools have been the reduced correlation function $\xi(r)$, developed especially by Jim Peebles and his associates [10], and the complete correlation function $\Gamma(r)$, introduced to extragalactic astronomy by Luciano Pietronero and his team [11]. $\xi(r)$ is defined using the reduced two-point correlation function $C_2(r) = <(n(\mathbf{r}_1) - n_0)(n(\mathbf{r}_2) - n_0)> = R_{nn} - n_0$ normalized to the squared mean value $n_0 = <n(\mathbf{r})>$:

$$\xi(r) = C_2(r)/ n_0^2, \quad (1)$$

while the gamma function is defined using the complete correlation function $R_{nn}(r) = <n(\mathbf{r}_1)(n(\mathbf{r}_2)>$ as:

$$\Gamma(r) = R_{nn}(r)/ n_0 . \quad (2)$$

The two-point correlation function $\xi(r)$ has the usual interpretation that it gives the fractional excess of galaxies at the distance r from a galaxy, relative to the average value expected for a random uniform distribution. The "conditional density" $\Gamma(r)$ which has the dimension of number density describes the behaviour of the density of galaxies as a function of the distance r from a galaxy. Though the relation between these correlation functions is rather simple $\xi(r) = \Gamma(r) /n_0 - 1$ they are different in that $\Gamma(r)$ measures the behaviour of the density insides spheres within a sample, while $\xi(r)$ describes density fluctuations relative to the average density which is assumed to be valid for all space outside the finite sample. The relation between them also shows at once that if one of them is a power law, then the other cannot be. E.g. within a fractal distribution $\Gamma(r)$ is a power law, but $\xi(r)$ isn't, except within limited r-intervals. On the other hand, when estimating $\xi(r)$ (and earlier the corresponding 2D angular correlation function) it has been usually approximated as a power-law for some range of scales r:

$$\xi(r) = (r/r_0)^{-\gamma}. \quad (3)$$

Here r_0 is the unit scale (also called the correlation length) and γ is the correlation exponent. This form appeared already in the pioneering 1969 work by Totsuji & Kihara who from the 2D Lick galaxy counts derived the power law angular correlation function $w(\theta) = A\ \theta^{-0.8}$, which they transformed into the 3D power law form with $\gamma = 1.8$ and $r_0 = 4.7\ h_{100}^{-1}$ Mpc.[1] In 1973 Peebles started a program analysing all angular galaxy and cluster catalogs, and concluded that these all are characterized by almost the same power law with $\gamma = 1.77$ and $r_0 = 5\ h_{100}^{-1}$ Mpc in the scale range 0.1 to 10 Mpc [10].

During the "3D-era" many teams have applied both the ξ-function and the Γ-function in the analysis of large redshift catalogs. As the redshift is not a flawless distance indicator (peculiar velocities!), the analysis in the redshift space does not directly give the corresponding result in the real space. The classical study by Davis & Peebles of the CfA data in 1983 accounted for the peculiar velocities using a procedure [10] for the restoration of both the real space $\xi(r)$ and the relative peculiar velocity distribution $f(v)$ from the observed correlation function $\xi_z(r_\perp , r_{||obs})$ where $r_\perp$ and $r_{||obs}$ are the observed perpendicular and parallel to the line-of-sight components of the separation $s = (r_\perp^2 + r_{||obs}^2)^{1/2}$. By integrating $\xi_z(r_\perp , r_{||obs})$ along the line sight (a redshift range) one first derives a projected correlation function $w(r_\perp)$. The wanted inverse, the real space $\xi(r)$, is the Abel integral which in the case of a power law $w(r_\perp) = A\ r_\perp^{1-\gamma}$ obtains the power law form $\xi(r) = (r/r_0)^{-\gamma}$, where r_0 becomes expressed via A and γ.

The thus derived spatial correlation function from CfA turned out to be surprisingly close to earlier results from 2D angular catalogs, with $\gamma = 1.74$ and $r_0 = 5.4\ h_{100}^{-1}$ Mpc on the scales 0.01 to 10 Mpc. In recent years people have analysed the large 2dF and (partially completed) SDSS surveys with basically similar conclusions about the practically universal γ and r_0, in the real-space correlation function, e.g. $\gamma = 1.67$ and $r_0 = 5.1\ h_{100}^{-1}$ Mpc from 2dF [12] and $\gamma = 1.75$ and $r_0 = 6.1\ h_{100}^{-1}$ Mpc from SDSS [13]. Such results are inferred from the redshift space correlation, correcting for peculiar velocities, as described above.

Now there are also for comparison studies which have used the complete Γ-correlation function in redshift space, providing a complementary view of the nature of the spatial distribution of galaxies. Here it is relevant to mention, along with the correlation exponent γ, the fractal dimension $D = 3 - \gamma$ (for a fractal-like

[1] For a power law spatial correlation function with the correlation exponent γ the angular correlation function $w(\theta)$ is also a power law and the correlation exponent is $\gamma - 1$, valid when $\gamma > 1$.

distribution the Γ function is a power law). Earlier studies of Pietronero's team [14] gave the value D = 2.0 ± 0.2 (γ = 1.0) on scales up to the radius of the largest sphere that can be contained in the sample. (In the Γ-function analysis one does not assume a constant average number density – in fact, one wants to know the behaviour of the density! – and the analysis is done within the sample of galaxies.) This result was in apparent conflict with those obtained from the ξ-function and gave rise to a "fractal debate" around the value of the slope and the homogeneity scale (and also the reality of the short correlation length $r_0 = 5\ h_{100}^{-1}$ Mpc).

Vasilyev et al. [15] analyzed galaxy correlations from samples taken from the 2dF final release, using the Γ-function method in redshift space. They found a power-law behaviour with the exponent γ = 0.8±0.2 (i.e. D = 2.2) in the interval 1 to 40 h_{100}^{-1} Mpc (see Fig. 3a). In a study on the 4th data release of the SDSS survey the same group [16] found γ = 1.0±0.1 (D = 2.0) in the interval 0.5 to 30 h_{100}^{-1} Mpc. These results were thus consistent with each other and with the previous studies using the Γ-function. On the other hand, it has been noted [6, 15] that when one inspects the ξ-function results in redshift space from these same surveys [12,13], one sees that on small scales from about 0.1–0.5 to 3–5 Mpc/h the ξ-function is described by γ ≈ 0.75–1, while on larger scales one needs γ ≈ 1.57–1.8. Thus on small scales the o-function and the Γ-function deliver about similar results (see Fig.3b). However, on larger scales, the o-function result diverges from the Γ-function power-law (and has essentially the "standard" γ there).

Next we list some possible factors whose influence should be further studied in order to understand the differences in the results from the different methods.

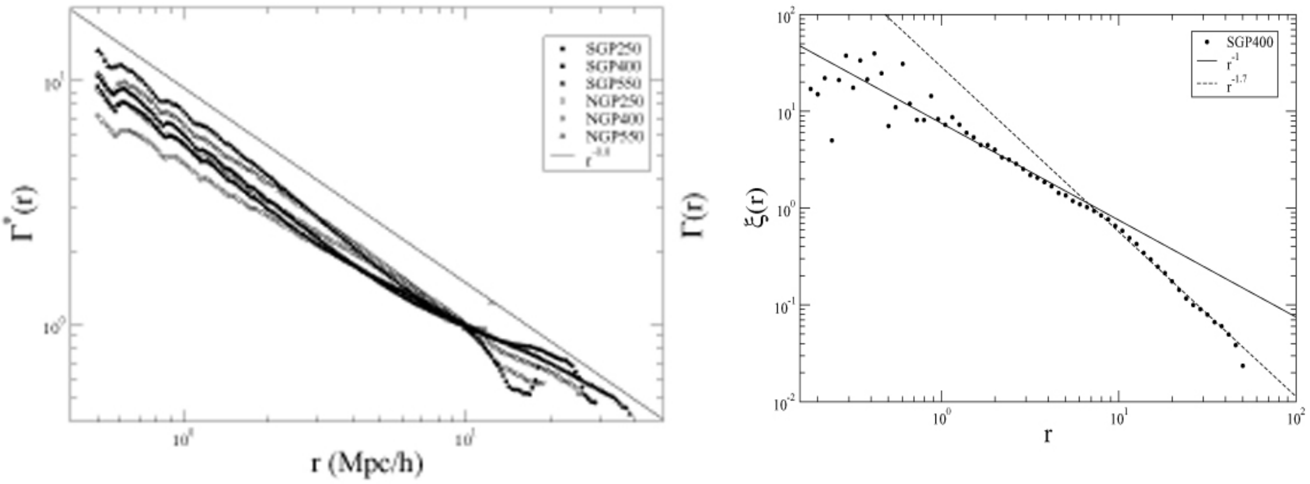

Fig.3 (left) Γ-function determinations for the 2dF survey for volume-limited samples of different depth. The inserted line has the slope γ = 0.8. (right) An example of estimated o-function (redshift space) for a VL sample (SGP400). The lines have the slopes of γ = 1.0 and 1.7. Taken from [15]. Γ-functions for the SDSS survey for volume-limited samples of different depth are discussed elsewhere in the conference by N. Vasilyev.

4. Impact of distortion and projection effects

In astronomy we are constantly in a familiar, but inconvenient situation: stuck at practically one point in space and time from which to make observations of the surrounding deep world via light coming along lines of sight. This gives rise to all sorts of nasty selection effects which tend to distort our view of and conclusions about cosmic phenomena. For example, Malmguist-like effects usually creep into cosmological tests using standard candles. Similarly, the study of the large-scale structure of the galaxy universe has its own share of observational problems caused by the fact that we make observations from one point, select galaxies in the first place from the sky and not from space, and do not have absolutely accurate distance indicators. As these things may play a role in the mentioned different results from the ξ- and Γ-correlation functions, we make here a summary of some involved factors, generally requiring further study.[2]

Nature has given the astronomer, in the form of the linear redshift-distance law, a way to measure extragalactic distances, which is generally more precise than photometric methods. However, it is clear that, say, inside and near to clusters of galaxies redshifts may have large components from peculiar velocities,

[2]Other problems arise as we see distant things as they were at different cosmic times, via redshifted light. The galaxies and the structures have evolved during the look-back times. When we make volume-limited samples for a LSS analysis, we have to include the luminosity evolution of galaxies, our tracers of the structures. There is also the technical K(z)-effect in the measured magnitudes. Finally, as we have to calculate the locations of galaxies from their redshifts and angles between them (we occupy the common tip of those angles), we need a cosmological model.

decreasing the accuracy. In the o-function analysis this problem is usually approached using the method above (as a part of the method, high pair-wise peculiar velocity dispersions, around 600 km/s, appear on scales of several megaparsecs and less). In fact, the thus restored spatial o-function has been the classical power-law ($\gamma \approx 1.7$) from below 1 Mpc up to about 20 Mpc [12,13]. Ref. [17] discusses the corresponding problem in the context of the 3D power spectrum from the SDSS survey.

Interestingly, the Γ-function in a similar wide range of spatial scales is a power law with $\gamma \approx 1$, in redshift space. Preliminary tests using cosmological N-body simulations suggest that the Γ-function, when derived from the redshift-space, is not very sensitive to the distance errors caused by peculiar velocities [18]; however, this clearly requires further study (see also [19]). A related question is the true velocity dispersion in different environments. In our vicinity the dispersion is quite low (< 50 km/s); however, this refers to dark energy dominated regions, while the scales where the dispersion is ~ 500 km/s coincide with gravitation dominated regions [20].

As was mentioned above, the reduced o correlation function can't be a power law (excepting limited ranges of scale; especially, there is a distance where o(r) = 0), while the complete Γ-function can (if the distribution is fractal on the scales considered). In fact, one expects changes of slope in the o-function. E.g., for a true power-law with $\gamma = 1$, one would infer an apparent slope ≈ 2 for the o-function, if measured on scales close to the unit scale r_0. It has been noted that the behaviour of the (redshift-space) o-function tends to display this "doubling " of the correlation exponent [21, 5].

We select galaxies from the sky and not from space (a nearly similar phrase, applied to stars, was used by Arthur Eddington in his 1914 book *Stellar movements and the structure of the universe*). This fact, also related to our unique position, creates a lot of selection effects and biases in astronomy (e.g. the Malmquist and related effects). The inevitable magnitude limit is one factor determining how deep a volume of galaxy space is projected on the inspected part of the celestial sphere. The projection itself can cause unexpected problems for the analyzer of the structures, both in the angular 2D and spatial 3D cases. The origin of the problem is best seen in terms of fractal structure.

The properties of orthogonal projections and intersections of a fractal structure influence the analysis of galaxy samples with different volume geometries. Let an object (structure) with a fractal dimension D, embedded in an Euclidean space of dimension d = 3, be orthogonally projected onto an Euclidean plane (d' = 2). Then according to a general theorem of fractal projections [22,23], the projection is a fractal object and has the fractal dimension D_{pr} so that

$$D_{pr} = D, \quad \text{if } D < 2 \quad \text{and} \quad D_{pr} = 2, \ \text{if } D \geq 2 \ . \qquad (4)$$

So, in 3D space a cloud having the fractal dimension $D \approx 2.5$ gives rise to a homogeneous shadow ($D_{pr} = 2$) on the ground. Therefore the projection is expected to hide from view fractal structures with D > 2.

The intersection of a fractal is another interesting case. If an object with fractal dimension D, embedded in a d =3 Euclidean space, intersects an object with the dimension D', then according to the law of codimension additivity [22,23] the dimension of the intersection D_{int} becomes

$$D_{int} = D + D' - d \ .$$

So, if a fractal structure with D = 2 in 3D space is intersected by a plane (D' = 2), then the dimension of the thin intersection is $D_{int} = 2 + 2 - 3 = 1$. This property can make a fractal structure with $D \approx 2$ look as a fractal with $D \approx 1$ when inspected on large scales from a sample coming from a slice-like galaxy survey.

Finally, let us mention an interesting theorem on the dimension of a subset *visible to the observer* [24]. Let F be a fractal set in R^3 with dimension D > 2. The visible part of the set F from a point P is the subset F_{VIS} of those points lit by a spotlight at P. Then the visible part F_{VIS} can in general not have a dimension more than 2. Here one does not speak about the projection of F_{VIS}, but about its spatial distribution. A related problem is due to the finite size of galaxies [25]: in deep space, part of galaxies will remain "behind" more nearby galaxies and will drop away from the sample. The remaining sample can have D at most 2 even if as a whole the galaxy distribution has $2 \leq D \leq 3$. It is good keep this in mind, even though the current galaxy samples are not yet so deep that the "shadowing" effect could influence essentially their constitution.

The critical fractal dimension 2 (or $\gamma = 3-D = 1$) appears in some steps of the LSS analysis. In study of angular catalogs, one goes from the directly measured 2D angular correlation function to the 3D spatial correlation function, which are linked by Limber's equation. In the case of a power law (and for small angles θ), the solution is $w(\theta) = A\ \theta^{-(\gamma-1)} \rightarrow \xi(r) = (r/r_0)^{-\gamma}$. However, this solution exists only if $\gamma > 1$ or $D < 2$ which means that angular catalogs may miss information about a $D \geq 2$ structure (e.g. [26, 5]), whose projection on the sky approaches homogeneous (D = 3) for deep samples (i.e. faint magnitude limits). For example, if the real galaxy distribution is characterized by a value of D close to the critical value 2, one should bear in mind the possibility that the 2D structure analysis is influenced by other effects than the real 3D distribution. The work by Montuori & Sylos Labini [27] is a very illuminating practical study in this respect.

The critical dimension D = 2 (γ =1) appears also in the method, mentioned above, for the restoration of the real space $\xi(r)$ distribution from the observed correlation function $\xi_z(r_\perp , r_{||obs})$ which is integrated along the line sight in order to derive the projected correlation function $w(r_\perp)$. The wanted inverse, the real space $\xi(r)$ obtains, in the case of a power law $w(r_\perp) = A\, r_\perp^{\,1-\gamma}$, the form $\xi(r) = (r/r_0)^{-\gamma}$. Also here the question on the influence of the real spatial γ (or D) on the analysis arises and should be clarified by computer simulations.

As a summary, the analysis of the true spatial correlation and the understanding of the differences between the ξ- and Γ-function results requires that attention be paid to several factors, such as the problem of peculiar velocities, the non-power law nature of the ξ-function (and the resulting distortion – changes of the slope, artificial homogeneity scales – if the spatial distribution is fractal-like), and the problem of projections of structures with power-law density distributions close to or below the critical exponent $\gamma = 1$ ($D \geq 2$). A relevant question is also the luminosity dependence of the spatial distribution (do brighter galaxies present longer correlation lengths r_0?) which could be a real effect as usually thought, or, entirely or partially, an artifact of the statistical analysis (e.g. [28]).

To repeat, an especially interesting question hidden behind all these complications concerns the main result of the Γ-function analysis, i.e. no slope of $\gamma = 1.77$, but a slope of $\gamma = 1 \pm 0.2$ within a wide range of scales. Is this valid in real space and why the difference with respect to the classical ξ-function analysis?

5. Scale of homogeneity and observed super-large structures

Together with the boundary to homogeneity, the centre of the universe has moved from Earth to Sun, and to the Milky Way. The Great Debate about the scale of the universe heralded the change of the Milky Way into an ordinary galaxy, whereby the cosmic centre finally disappeared into the realm of galaxies. Meanwhile, a new debate around the nature of galaxy clustering and on the outer limit of the large-scale inhomogeneity emerged. As always in astronomy, the scarcity of available observations at any moment tends to lead to uncertain interpretations of the data.The modern phase of the LSS debate concerns the spatial scale where the galaxy universe becomes homogeneous. This item has always been at the centre of cosmological thinking, because of its link to cosmological principles and to the problem of structure formation mechanisms on different scales in the universe.

Of course, our neighbourhood is very inhomogeneous and the border to the assumed uniformity must lie at a larger distance. Early counts of bright galaxies were close to the $10^{0.6m}$ law, pointing to smoothness at a quite short distance. As one might now guess from large-scale 3D maps, those results were spurious and modern LEDA galaxy counts in the B magnitude range 10 to 16 give significantly more shallow slopes, from 0.44 to 0.5, depending on the magnitude range [29, 30]. In the 1930s Hubble argued that galaxy clusters were the largest units in the distribution of matter, whereas Shapley presented evidence on still larger structures and turned out to be right, as narrated above.

The border to uniformity has gradually shifted outwards. In our times the observed superclusters of galaxies have put this border to a scale of at least 100 Mpc. On the other hand, the isotropy of the cosmic background radiation and other arguments make us expect that on sufficiently large scales the universe is homogeneous, so there should be a crossover scale to homogeneity. There are claims for finding this scale already in the available data, like the "luminous red galaxy" sample of the SDSS [31], where a simple power law with $\gamma = 1$ (D = 2) gives a good fit to the data up to at least 20 h_{100}^{-1} Mpc after which the decrease of the density gets slower, and there is flattening around 70 Mpc up to 100 Mpc, the largest scale probed by the sample (however, see [28] for a discussion of difficulties in showing whether the transition to a well-defined mean density has indeed been found, and [19] for another recent study).

Though the available data may not yet be sufficient to define the crossover scale to homogeneity, one can find evidence for occasional very large structures which may suggest some further surprises in this topic. In the local universe, there is Paturel's "hypergalactic plane" [32, 30], whose influence may have been present in the number density gradient found from the KLUN galaxy sample up to about 100 Mpc [33]. In a statistical study of the 2dF survey superclusters, their size spectrum extends from below 10 Mpc up to 100 h_{100}^{-1} Mpc (maximal diameters up to almost 200 Mpc) [34].

The Great Wall, discovered in the CfA survey, a 200 h_{100}^{-1} Mpc long filament of galaxy groups and clusters, found its winner in the 500 Mpc long Sloan Great Wall (from the SDSS survey [35]). As to the other side of the large-scale structure, the voids free of galaxies, there is also a wide spectrum of sizes, from small "pores" and "bubbles" (on Mpc-scale; [36]) through minivoids to big voids with sizes of tens of Mpc. Even the nearby Tully's Local Void has a diameter approaching 50 Mpc [37]. The recently suggested giant void at the location of the WMAP cold spot has a record diameter of about 300 h_{75}^{-1} Mpc [38].

Galaxies are the visible tracers of the large-scale structure, but in modern cosmology dark matter and dark energy are the greatly dominating substances. There are various indications of dark matter within galaxy systems of different scales. However, in order to determine the nature of the overall distribution of gravitating dark matter, the gravitational lensing method is the most promising one being based on the analysis of rays of light influenced by dark matter along trajectories passing through voids as well as superclusters. Recently first important steps towards revealing the 3D distribution of dark matter were made via an analysis of the weak gravitational lensing in the HST survey COSMOS [39]. The shapes, orientations, and photometric redshifts of half a million distant galaxies in a 1.3° Ч 1.3° field were measured. The main result was that the spatial distribution of the dark matter generally repeats that of the luminous matter both on small and large scales. If confirmed in future more extensive studies this will underline the relevance of the large-scale structure as measured using the luminous tracers.

The antigravitating dark energy is mysterious in many ways. Its substance and spatial and temporary distributions are open questions for the future. However, there are already ways to probe dark energy densities on different epochs and spatial scales. The first studies in the local galaxy universe (e.g. [40, 37]) seem to support the view that it fills the voids as well as the interiors of galaxy systems, consistent with the popular conception of uniformly distributed and constant vacuum density (Einstein's cosmological constant). It may represent, together with other relativistic components of the universe, the naturally homogeneous substance on which the Friedmann model is based.

In a review in *Nature*, Wu, Lahav & Rees summarized the situation with respect to the distribution of galaxies: "The Universe is inhomogeneous – and essentially fractal – on the scale of galaxies and clusters of galaxies, but most cosmologists believe that on larger scales it becomes isotropic and homogeneous." [41]. Indeed so, and it is also true that now almost a decade after that apt summary we are still not quite certain about the large scale beyond which the very lumpy ordinary and dark matter looks really smooth.

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